Previous IPE

SOLVED PAPERS

MARCH -2026(AP)

PREVIOUS PAPERS

IPE: MARCH-2026 (AP)

Time : 3 Hours **JR. PHYSICS** Max.Marks : 85

SECTION-A

I. Answer ALL the following questions. $9 \times 1 = 9$

1. Which of the following physical quantity has no dimensions?

   1) Angular Velocity 2) Angular Acceleration

   3) Angular Displacement 4) Stress

2. Why gases cannot support the propagation of transverse wave?

   1) gases are incompressibles 2) gases have no shearing stress

   3) density of gas is less 4) gases can flow

3. Among the following, the material with highest elasticity is

   1) Copper 2) Aluminium 3) Brass 4) Steel

4. The value of an acceleration of a freely falling body is __________

5. Hydraulic lift and hydraulic brakes are based on __________law.

6. A monoatomic gas molecule has ________degrees of freedom.

7. When are the two vectors said to be equal vectors?

8. State Kepler's law of areas.

9. Write the mathematical equation of Newton's Second Law.

SECTION-B

II. Answer ALL the following questions. $14 \times 2 = 28$

10. Write the physical quantities that are having Jm^{-1} and Jm^{-2} as their units.

11. The states of rest and motion are relative. Explain.

12. If $\vec{P} = 2\vec{i} + 4\vec{j} + 14\vec{k}$ and $\vec{Q} = 4\vec{i} + 4\vec{j} + 10\vec{k}$ find the magnitude of $\vec{P} + \vec{Q}$.

13. What is inertia ? What gives the measure of inertia ?

14. State the relation between the kinetic energy and momentum of a body.

15. We cannot open or close the door by applying force at hinges. Why?

16. What are the factors that make 'g' the least at the equator and maximum at the poles?

17. State Hooke's law of elasticity.

18. Give the expression for the excess pressure in the soap bubble in air.

19. The roofs of buildings are often painted white during summer. Why?

20. What thermodynamic variables can be defined by (a) Zeroth law (b) First law?

21. Define mean free path.

22. What happens to the energy of a simple harmonic oscillator if its amplitude is doubled?

23. What are 'beats' ?

SECTION-C

III. Answer ANY EIGHT of the following questions. $8 \times 4 = 32$

24. Explain the terms the average velocity and instantaneous velocity. When are they equal?

25. Show that the maximum height reached by a projectile launched at an angle 45° is one quarter of its range.

26. Mention the methods used to decrease friction.

27. Define vector product. Explain the properties of a vector product with two examples.

28. When 100J of work is done on a fly wheel, its angular velocity is increased from 60rpm to 180rpm. What is the moment of inertia of the wheel?

29. What is escape velocity? Obtain an expression for it.

30. Define Young's modulus, Bulk modulus and Shear Modulus.

31. What is atmospheric pressure and how is it determined using Barometer?

32. In what way is the anomalous behaviour of water advantageous to aquatic animals?

33. If the maximum intensity of radiation for a black body is found at 2.65 μm. Find the temperature of the radiating body. (Wien's constant = 2.9×10^{-3} mK)

34. What is the ratio of r.m.s speed of Oxygen & Hydrogen molecules at the same temperature?

35. Explain the modes of vibration of an air column in an closed pipe.

SECTION-D

IV. Answer ANY TWO of the following questions. $2 \times 8 = 16$

36. Define elastic and inelastic collisions? Derive expressions for the final velocities of bodies in one dimensional elastic collision, when one body collides with other body which is initially at rest.

37. Define two specific heat capacities of a gas and derive the relation between them on the basis of first law of thermodynamics. .

38. a) Define simple harmonic motion. Show that the motion of (point) projection of a particle performing uniform circular motion. On any diameter, is simple harmonic?

    b) A particle executes SHM such that, the maximum velocity during the oscillation is numerically equal to half the maximum acceleration. What is the time period?

IPE AP MARCH - 2026

SOLUTIONS

SECTION-A

1. Which of the following physical quantity has no dimensions?

1) Angular Velocity 2) Angular Acceleration

✓ **3) Angular Displacement** 4) Stress

2. Why gases cannot support the propagation of transverse wave?

1) gases are incompressibles ✓ **2) gases have no shearing stress**

3) density of gas is less 4) gases can flow

3. Among the following, the material with highest elasticity is

1) Copper 2) Aluminium 3) Brass ✓ **4) Steel**

4. The value of an acceleration of a freely falling body is **g= 9.8 m/s^2**

5. Hydraulic lift and hydraulic brakes are based on **Pascal's** law.

6. A monoatomic gas molecule has **three** degrees of freedom

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7. When are the two vectors said to be equal vectors?

A: Two vectors are said to be equal if they have the **same magnitude & direction.**

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8. State Kepler's law of areas.

A: **Law of areas:** The line that joins any planet to the sun sweeps **equal areas in equal intervals of time.**

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9. Write the mathematical equation of Newton's Second Law.

A: F=ma is the mathematical equation of Newton's Second Law.

SECTION-B

10. Write the physical quantities that are having Jm^{-1} and Jm^{-2} as their units.

A: **Force** has unit Jm^{-1} **Surface tension** has unit Jm^{-2}

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11. The states of rest and motion are relative. Explain.

A: **1) For a person travelling in a train**, the co-passengers appear to be at rest, but objects outside the train like electric poles, trees etc, appear to be moving backwards.

2) For a person on the platform the electric poles, trees etc. appear to be at rest but the train appears to be moving. So the states of rest and motion are relative.

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12. If $\vec{P} = 2\vec{i} + 4\vec{j} + 14\vec{k}$ and $\vec{Q} = 4\vec{i} + 4\vec{j} + 10\vec{k}$ find the magnitude of $\vec{P} + \vec{Q}$.

A: 1) Given that $\vec{P} + \vec{Q} = (2\vec{i} + 4\vec{j} + 14\vec{k}) + (4\vec{i} + 4\vec{j} + 10\vec{k}) = 6\vec{i} + 8\vec{j} + 24\vec{k}$

2) $\therefore \left|\vec{P} + \vec{Q}\right| = \sqrt{6^2 + 8^2 + 24^2} = \sqrt{676} = 26$ units.

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13. What is <u>inertia</u> ? What gives the measure of inertia ?

A: **1) Inertia :** Inertia is an <u>inherent property</u> of a body, by which it resists any change in its state.

2) **Mass** is the measure of inertia.

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14. State the relation between the kinetic energy and momentum of a body.

A: $K.E = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{1}{2}\frac{m^2v^2}{m} = \frac{P^2}{2m}$, where p = mv is momentum of the body.

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15. We cannot <u>open or close the door</u> by applying <u>force at hinges</u>. Why?

A: 1) Torque = force × lever arm of the force

2) When the force is applied at hinges, lever arm of the force will be zero. So there will be no torque and hence we cannot open or close the door.

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16. What are the factors that make 'g' the least at the equator and maximum at the poles?

A: The factors that make 'g', the least at the equator and maximum at the poles are

1) Non sphericity (shape of the earth) 2) Rotation of the earth.

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17. State Hooke's law of elasticity.

A: **Hooke's Law :** Within the elastic limit, the stress is directly proportional to strain.

Thus, stress $\propto$ strain $\Rightarrow$ stress = E (strain). Here E is called as modulus of elasticity.

18. Give the expression for the excess pressure in the soap bubble in air.

A: 1) Soap bubble in air contains two interfaces.

2) Hence, excess pressure in soap bubble $= P_{inside} - P_{outside} \Rightarrow P_{excess} = \frac{4T}{r}$

Where, r = radius of bubble; T = surface tension of liquid-air interface.

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19. The roof of buildings are often painted white during summer. Why?

A: **1) White paint** is a **good reflector of heat**. Good reflector is a bad absorber of heat.

2) So roof of buildings that are painted white to **'keep cool' during summer.**

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20. What thermodynamic variables can be defined by (a) Zeroth law (b) First law?

A: a) Thermodynamic variable defined by Zeroth law of thermodynamics is **temperature**.

b) Thermodynamic variable defined by First law of thermodynamics is **internal energy**.

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21. Define mean free path.

A: **Mean free path:** It is the **'average distance'** covered by a molecule between **'two successive collisions'.**

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22. What happens to the energy of a simple harmonic oscillator if its amplitude is doubled?

A: Total energy of a simple harmonic oscillator is $T.E = \frac{1}{2} m\omega^2 A^2$ **[MAR-26]**

where m = mass of the particle; ω = angular velocity; A = amplitude

When amplitude is doubled, the new T.E will be

$$T.E = \frac{1}{2} m\omega^2 (2A)^2 \Rightarrow 4 \times \frac{1}{2} m\omega^2 A^2 = 4 \times \text{initial T.E}$$

So total energy will be increased to 4 times.

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23. What are 'beats' ?

A: **1) Beats:** When two sounds of slightly different frequencies are travelling in same direction along the same line of action, the intensity of their resultant sound 'waxes and wanes' periodically. This phenomenon is called beats.

2) Beats are formed due to interference of sound waves in time.

SECTION-C

24. Explain the terms the average velocity and instantaneous velocity. When are they equal?

A: 1) **Average velocity:** Suppose a body is at a point x_1 at time t_1 and at some other point x_2 at time t_2, then the displacement of the body is $x_2 - x_1$. Hence its average velocity is given by

$$V_{avg} = \frac{\text{displacement}}{\text{time interval}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

2) **Instantaneous velocity:** The velocity of an object at a particular point of its path or at a particular instant of time is called as instantaneous velocity.

$$\text{Instantaneous velocity } v = \underset{\Delta t \to 0}{Lt} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

3) During **uniform motion**, the average and instantaneous velocity are always same because the velocity during uniform motion is same at each point of its path or at each instant.

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25. Show that the maximum height reached by a projectile launched at an angle 45° is one quarter of its range.

A: The horizontal distance travelled by a projectile during its time of flight is called Range.

1) We know that Range $R = \frac{u^2 \sin 2\theta}{g}$ and maximum height $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

2) It is clear that R is maximum, when $\theta = 45^\circ$ [we have $2\theta = 90^\circ$ and $\sin 90^\circ = 1$(max)]

3) Maximum height $H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2}{2g}(\sin^2 45^\circ)^2 = \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g}$ ....(1)

4) Maximum Range $R_{max} = \frac{u^2}{g}\sin 2\theta = \frac{u^2}{g}\sin 90^o = \frac{u^2}{g}(1) = \frac{u^2}{g}$ ....(2)

5) $\therefore$ From (1) & (2), $H_{max} = \frac{R_{max}}{4}$.

Thus, maximum Height is one quarter of the Range.

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26. Mention the <u>methods</u> used to <u>decrease friction</u>.

A: Methods used to decrease Friction:

1) <u>P</u>olishing: It reduces the frictional force of the polished surfaces.

2) <u>L</u>ubricants: They reduce the friction by forming thin layers between the surfaces in contact.

3) <u>B</u>all bearings: They reduce friction in the wheels of motor vehicles while revolving.

4) <u>S</u>tream lining: It reduces friction due to air when Aeroplanes and Cars are streamlined in their front surfaces.

27. Define vector product. Explain the properties of vector product with two examples.

A: **1) Vector Product:** The vector product of two vectors $\bar{a}, \bar{b}$ with angle θ between them, is $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}| \sin\theta \, \hat{n}$

Here, $\hat{n}$ is the unit vector normal to the plane of $\bar{a}, \bar{b}$.

2) Properties of Vector Product :

i) Commutative law is not satisfied: $\bar{a} \times \bar{b} \neq \bar{b} \times \bar{a}$

ii) Distributive law is satisifed : $\bar{a} \times (\bar{b} + \bar{c}) = (\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c})$

iii) Vector product of two parallel vectors is null vector.

Ex: $\bar{i} \times \bar{i} = \bar{j} \times \bar{j} = \bar{k} \times \bar{k} = \bar{o}$

iv) Vector product of two perpendicular unit vectors is unit normal vector.

Ex: $\bar{i} \times \bar{j} = \bar{k}$, $\bar{j} \times \bar{k} = \bar{i}$, $\bar{k} \times \bar{i} = \bar{j}$

3) Examples: Torque $\bar{\tau} = \bar{r} \times \bar{F}$; Velocity $\bar{v} = \bar{\omega} \times \bar{r}$

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28. When 100J of work is done on a fly wheel, its angular velocity is increased from 60rpm to 180rpm. What is the moment of inertia of the wheel? [MAR-26]

Sol: Given W=100

Initial angular velocity $\omega_1 = \dfrac{2\pi n_1}{t_1} = \dfrac{2\pi(60)}{60} = 2\pi \text{ rad/sec}$

Final angular velocity $\omega_2 = \dfrac{2\pi n_2}{t_2} = \dfrac{2\pi(180)}{60} = 6\pi \text{ rad/sec}$

We know, work done $W = \dfrac{1}{2} I(\omega_2^2 - \omega_1^2) \Rightarrow I = \dfrac{2W}{\omega_2^2 - \omega_1^2}$

$\therefore I = \dfrac{2 \times 100}{(6\pi)^2 - (2\pi)^2} = \dfrac{200}{36\pi^2 - 4\pi^2} = \dfrac{200}{32\pi^2} = 0.63$ kg-m^2.

29. What is escape velocity? Obtain an expression for it.

A: 1) **Escape Velocity(V_e) :** The **minimum velocity** required for an body to escape from the gravitational influence of a planet is known as "escape velocity".

2) **Derivation:** Consider an body of mass 'm' at rest on the surface of a planet of mass M and radius R.

3) The gravitational potential on the surface of a planet = $\dfrac{-GM}{R}$

The gravitational P.E of the system = Gravitational potential×mass of the body = $\dfrac{-GMm}{R}$ ....(i)

4) When a body of mass m is projected with a velocity V_e then its K.E = $\dfrac{1}{2}mV_e^2$ ....(ii)

After crossing the Gravitational limits, the total energy becomes zero.

5) Applying the Law of conservation of energy, from (i) & (ii) we have

$$\frac{1}{2}mV_e^2 = -\left(\frac{-GMm}{R}\right) \Rightarrow \frac{1}{2}mV_e^2 = \frac{GMm}{R} \Rightarrow V_e^2 = \frac{2GM}{R}$$

6) $$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2g\cancel{R^2}}{\cancel{R}}} \quad [\because GM = gR^2]$$

$\therefore V_e = \sqrt{2gR}$. (Its value for the earth is V_e= 11.2 km/s)

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30. Define Young's modulus, Bulk modulus and Shear Modulus.

A: 1) **Young's modulus (Y):** With in the elastic limit, Young's modulus is the ratio of longitudinal stress to the longitudinal strain.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{\mathbf{F/A}}{\mathbf{e}/l} = \frac{\mathbf{F}l}{\mathbf{Ae}}$$

2) **Bulk modulus (K)**: With in the elastic limit, Bulk modulus is the ratio of volume stress to volume strain

$$K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{\mathbf{F/A}}{\mathbf{-\Delta V/V}}$$

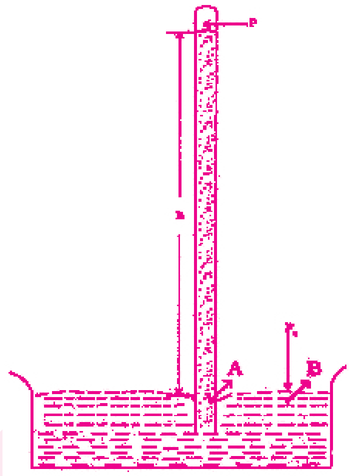
3) **Shear Modulus (Rigidity modulus (η))**: With in the elastic limit, **Rigidity modulus** is the ratio of Shear stress to the Shear strain

$$\eta = \frac{\text{Shear stress}}{\text{Shear strain}}$$

31. What is atmospheric pressure and how is it determined using Barometer?

A: The pressure of atmosphere at any point is equal to the weight of air column of unit cross-sectional area.

It extends from that point to the top of the atmosphere.



Barometer: Mercury barometer consists of a long glass tube closed at one end and filled with mercury. It is inverted into a trough of mercury as shown in figure. The space above the mercury column in the tube contains only mercury vapour whose pressure(P) may be neglected.

Points A and B are at same level.

Hence pressure at A = pressure at B

$P_a = h\rho g$

Where ρ = density of mercury

h = height of the mercury column in the tube

p_a = atmospheric pressure

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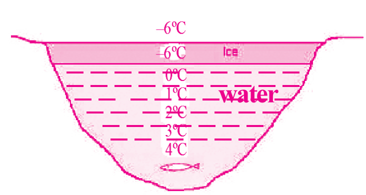
32. In what way is the anomalous behaviour of water advantageous to aquatic animals?

A: 1) **Anomalous behaviour:** Water attains **maximum density** at 4ºC.

2) So, when temperature increases from 0ºC to 4ºC water contracts instead of expansion. This is anomalous behaviour of water.



3) During winter, when the temperature of the atmosphere falls below 0ºC , the surface of lakes gradually freezes to ice.

4) But ice is bad conductor of heat. Under the frozen upper layers, the water remains in its liquid form and does not freeze. Thus aquatic animals are survived in this water.

33. If the maximum intensity of radiation for a black body is found at 2.65 μm. Find the temperature of the radiating body. (Wien's constant = 2.9 × 10⁻³ mK) [MAR-26]

Sol: Given $\lambda_m = 2.65\ \mu m = 2.65 \times 10^{-6}$ m

From Wein's displacement law $\lambda_m T = b$

Where b is Wein's constant , $b = 2.9 \times 10^{-3}$ m.K,

$$\therefore T = \frac{b}{\lambda_m} = \frac{2.9\times10^{-3}}{2.65\times10^{-6}} = 1.094 \times 10^3 \text{ K} = 1094 \text{ K}$$

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34. What is the ratio of r.m.s speed of Oxygen & Hydrogen molecules at the same temperature?

Sol: 1) Given $M_{H_2} = 2,\ M_{O_2} = 32$

2) We know r.m.s speed $C = \sqrt{\frac{3RT}{M}} \Rightarrow C \propto \frac{1}{\sqrt{M}}$ [when T is constant].

3) $\therefore \frac{C_{O_2}}{C_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$

4) ∴ The ratio of r.m.s speeds is 1 : 4

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35. Explain the modes of vibration of an air column in an closed pipe.

A: **Closed pipe:** A pipe closed at one end is called closed pipe.

1) First Harmonic: Here one node and one antinode is formed.

Thus, the length of the air column $l = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4l$

$\therefore$ Frequency of Vibration $\nu_1 = \frac{V}{\lambda_1} = \frac{V}{4l}$.

2) Third Harmonic: Here two nodes and two antinodes are formed.

Thus, the length of air column $l = \frac{\lambda_3}{4} + \frac{\lambda_3}{4} + \frac{\lambda_3}{4} = \frac{3\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4l}{3}$

$\therefore$ Frequency of Vibration $\nu_3 = \frac{V}{\lambda_3} = \frac{3V}{4l} = 3\left(\frac{V}{4l}\right) = 3\nu_1$

3) Fifth Harmonic:

Here three nodes and three antinodes are formed.

Thus, the length of air column $l = \frac{5\lambda_5}{4} \Rightarrow \lambda_5 = \frac{4l}{5}$

$\therefore$ Frequency of Vibration $\nu_5 = \frac{V}{\lambda_5} = \frac{5V}{4l} = 5\left(\frac{V}{4l}\right) = 5\nu_1$

$\therefore \nu_1 : \nu_2 : \nu_3 : ... = \nu_1 : 3\nu_1 : 5\nu_1 .... = 1:3:5:..$

4) Conclusion: The harmonics in a closed pipe are in the ratio of **odd numbers.**

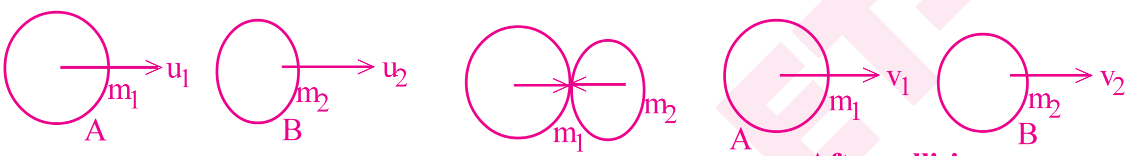
SECTION-D

36. Define elastic and inelastic collisions? Derive expressions for the final velocities of bodies in one dimensional elastic collision, when one body collides with other body which is initially at rest.

A: 1) **Elastic collision:** It is the collision in which both Momentum and Kinetic energy are conserved.
Ex: Collision between gas molecules

2) **Inelastic collision:** It is the collision in which only Momentum is conserved but not K.E.
Ex: Collision between a bullet and its target

3) **One dimensional elastic collision:** If the velocities of colliding bodies before and after collision are confined to the same straight line then such collisions are called one dimensional collisions.Consider two spheres A and B of masses m_1 and m_2 moving with initial velocities u_1 and u_2 undergo an elastic collision . Let v_1 ,v_2 be the velocities after collision .



Before collision **During collision** **After collision**

4) From the law of **conservation of momentum,** we have

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

But, m_2 is initially at rest. So $u_2 = 0$

$\therefore\ m_1u_1 = m_1v_1 + m_2v_2 \Rightarrow m_1(u_1 - v_1) = m_2v_2$ ....(1)

5) From the law of **conservation of K.E,** we have

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow m_1u_1^2 = m_1v_1^2 + m_2v_2^2 \Rightarrow\ m_1(u_1^2 - v_1^2) = m_2v_2^2 \quad ....(2)$$

$$\frac{eq.(2)}{eq.(1)} \Rightarrow \frac{\cancel{m_1}(u_1^2 - v_1^2)}{\cancel{m_1}(u_1 - v_1)} = \frac{\cancel{m_2v_2^2}}{\cancel{m_2v_2}} \Rightarrow \frac{\cancel{(u_1 - v_1)}(u_1 + v_1)}{\cancel{(u_1 - v_1)}} = v_2 \Rightarrow (u_1 + v_1) = v_2 \quad ....(3)$$

Now, substituting this value of v_2 in eq(1), we get $m_1(u_1 - v_1) = m_2\,(u_1 + v_1)$

$\Rightarrow m_1u_1 - m_1v_1 = \ m_2u_1 + m_2v_1\ \Rightarrow u_1(m_1 - m_2) = v_1(m_1 + m_2)$

$$\therefore\ v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]u_1 \quad ....(4)$$

Now from eq(3) we have $v_1 = v_2 - u_1$

substituting this value of v_1 in eq(1), we get $m_1\,(u_1 - v_2 + u_1) = m_2\,v_2$

$\Rightarrow m_1u_1 - m_1v_2 + m_1u_1 = m_2\,v_2 \Rightarrow m_1u_1 + m_1u_1 = m_2\,v_2 + m_1v_2$

$\Rightarrow 2m_1u_1 = v_2(m_1 + m_2)$

$$v_2 = \left[\frac{2m_1}{m_1 + m_2}\right]u_1 \quad ....(5)$$

Eq(4), (5) are the expressions for the final velocities.

37. Define two specific heat capacities of a gas and derive the relation between them on the basis of first law of thermodynamics.

A: i) **Specific heat at constant pressure (C_p):** At constant pressure, the quantity of heat necessary to increase the temperature of unit mass of a gas through one degree, is called specific heat of the gas at constant pressure.

ii) **Specific heat at constant volume (C_v):** At constant volume, the quantity of heat necessary to increase the temperature of unit mass of a gas through one degree, is called specific heat of gas at constant volume.

Relation between C_p & C_v:

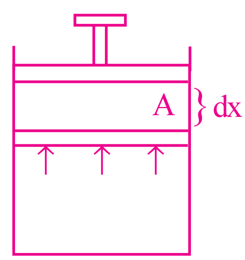
Consider one mole of Ideal Gas contained in a cylinder provided with a frictionless Piston. Let A be the area of cross section of the Piston and P, V, T be the pressure, volume and temperature of the gas.

When Gas is heated, it expands and Piston moves up a distance dx.

Case- I : Let the Gas be heated at constant volume, then Heat supplied is $dQ = C_v dT$

From first law of thermodynamics, $dQ = dU + dW$

But, in this case $dW = 0$ $\therefore$ $dQ = dU = C_v dT$ ....(1)



Case- II : Let the Gas be heated at constant pressure.

then Heat supplied is $dQ = C_p dT$ ....(2)

But, workdone by the gas is $dW = F.S = (PA)dx = PdV$, $[\because Adx = dV]$

$\therefore dW = PdV$ ....(3)

From first law of thermo dynamics, $dQ = dU + dW$..........(4)

Substitute equations (1), (2) , (3) in equation (4) we have $C_p dT = C_v dT + PdV$ .... (5)

From Ideal gas equation $PV = RT$

At constant pressure, $PdV = RdT$

Hence, from (5), we get $C_p dT = C_v dT + RdT$ $\Rightarrow C_p = C_v + R$

$\therefore$ $C_p - C_v = R$

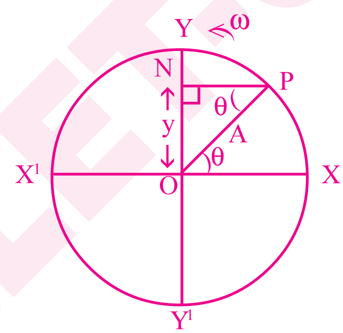
38. a) Define simple harmonic motion. Show that the motion of (point) projection of a particle performing uniform circular motion, on any diameter, is simple harmonic.

A: **1)** **Simple Harmonic Motion (SHM):** The 'to and fro motion' of a particle along a straight line, about a fixed point is said to be **Simple Harmonic motion**, when the acceleration is always proportional to its displacement, but in opposite direction.

2) **Proof:** Suppose a particle P is moving along the circumference of a circle of radius A.

Let N be the projection of P on the diameter Y-axis.

If P completes one revolution then its projection point N makes one oscillation on the diameter.



3) If θ is the angular displacement of P at time t and ω is uniform angular velocity then $\theta = \omega t$

4) From ΔOPN, $\sin\theta = \frac{ON}{OP} = \frac{y}{A} \Rightarrow y = A\sin\theta$

$\therefore$ Displacement $y = A\sin(\omega t)$........... (i)

5) Velocity is the 'rate of change of displacement'.

$$\therefore \text{Velocity } v = \frac{d}{dt}(y) = \frac{d}{dt}A\sin(\omega t) = A\frac{d}{dt}\sin(\omega t)$$

$$= A\omega\cos(\omega t) \quad \left[\because \frac{d}{dx}\sin(kx) = k\cos(kx)\right]$$

6) Acceleration is the 'rate of change of velocity'.

$$\therefore \text{Acceleration } a = \frac{d}{dt}(v) = \frac{d}{dt}A\omega\cos(\omega t) = A\omega\frac{d}{dt}[\cos(\omega t)]$$

$$= -A\omega(\omega)[\sin(\omega t)] = -\omega^2[A\sin(\omega t)] = -\omega^2 y, \text{ [from (i)] } \left[\because \frac{d}{dx}\cos(kx) = -k\sin(kx)\right]$$

7) $\boxed{\therefore a \propto -y}$ $(\because \omega \text{ is a constant})$

8) Hence, the motion of projection N on any diameter is S.H.M.

b) A particle executes SHM such that, the maximum velocity during the oscillation is numerically equal to half the maximum acceleration. What is the time period?

Sol: If A is the amplitude and ω is angular velocity of the particle in S.H.M. **[MAR-26]**

then maximum velocity, $v_{max} = A\omega$
and maximum acceleration, $a_{max} = A\omega^2$

But given $v_{max} = \frac{1}{2} a_{max}$ (numerically)

$$\Rightarrow A\omega = \frac{1}{2} A\omega^2 \quad \Rightarrow 1 = \frac{1}{2}\omega \quad \Rightarrow \omega = 2\,\text{rad/sec}$$

$$\text{But } \omega = \frac{2\pi}{T} \quad \Rightarrow \frac{2\pi}{T} = 2 \quad \therefore T = \pi \text{ sec}$$