Previous IPE

SOLVED PAPERS

MAY -2026(AP)

PREVIOUS PAPERS

IPE: MAY-2026 (AP)

Time : 3 Hours **JR. MATHS** Max.Marks : 100

SECTION - A

I. Answer ALL the questions. Each question carries ONE mark. 12 × 1 = 12

1. $A = \{1, 2, 3\}, B = \{a, b\}$ then $A \times B =$

   1) $\{(1, a), (2, b), (3, a)\}$ 2) $\{(a, 1), (b, 2), (a, 3), (b, 3)\}$
   3) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$ 4) $\{(1, a), (1, b), (2, b), (3, b)\}$

2. Let $x, y \in R$, then $x + iy$ is a non real complex number if

   1) $x = 0$ 2) $y = 0$ 3) $x \neq 0$ 4) $y \neq 0$

3. The number of arrangements that can be made out of the letters in expansion $a^4 b^3 c^5$ when written in full length is

   1) $\frac{9!}{3!2!4!}$ 2) $\frac{7!}{2!3!2!}$ 3) $\frac{5!}{4!3!2!}$ 4) $\frac{(12)!}{4!3!5!}$

4. The expansion of $(x + y)^{1000}$ is

   1) $\sum_{r=0}^{100} {}^{1000}C_r\, x^{r-1000} y^r$ 2) $\sum_{r=0}^{1000} {}^{1000}C_r\, x^{1000-r} y^r$ 3) $\sum_{r=0}^{999} {}^{1000}C_r\, x^{r-1000} y^r$ 4) $\sum_{r=0}^{999} {}^{1000}C_r\, x^{1000-r} y^r$

5. If the distance between the points P(0, a, 3) and Q(3, 0, 7) is $\sqrt{41}$ then a =

   1) –4 2) 4 3) ± 4 4) None of these

6. Let A and B be two events such that P(A) = 0.8, P(B) = 0.7, then $P(A \cap B)$

   1) ≤ 0.6 2) ≥ 0.5 3) ≥ 0.7 4) ≤ 0.4

7. $\{x : x \in R, -4 < x \leq 6\}$ write as interval.

8. Find the value of the $\sin\left(\frac{31\pi}{3}\right)$

9. Find the term a_9 in the following sequence whose n^{th} term is $a_n = (-1)^{n-1} n^3$

10. Find the equation of the line which is passing through the points (–1, 1) and (2,–4)

11. Find the eccentricity of hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

12. Find the derivative of the function $(ax+b)^n$

SECTION -B

II. Answer ALL the questions. Each question carries TWO marks. 10 × 2 = 20

13. If X= {a,b,c,d} and Y= { f, b, d, g), find (ii) X–Y (ii) Y–X (iii) $X \cap Y$

14. If $A = \{-1, 1\}$, find $A \times A \times A$.

15. Prove that sin (n+1) x sin (n+2) x +cos (n+1) x cos (n+2) x

16. Find the multiplicative inverse of $2-3i$.

17. If ${}^{18}P_{(r-1)} : {}^{17}P_{(r-1)} = 9:7$, find r

18. Write down and simplify 6^{th} term in $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$

19. Find the equation of the parabola with vertex (0, 0) passing through (5, 2) and symmetric with respect ot y-axis.

20. Three vertices of a parallelogram ABCD are A(3, – 1, 2), B (1, 2, – 4) and C (– 1, 1, 2). Find the coordinates of the fourth vertex.

21. Compute $\underset{x\to 0}{Lt} \frac{a^x - 1}{b^x - 1}$

22. Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12

SECTION-C

III. Answer ANY SEVEN the questions. Each question carries FOUR marks.7 × 4 = 28

23. If A = { 3, 5, 7, 9, 11 }, B = {7, 9, 11, 13}, C = {11, 13, 15}and D = {15, 17}; find

(i) $A \cap B$ (ii) $A \cap (B \cup D)$

(iii) $(A \cap B) \cap (B \cup C)$ (iv) $(A \cup D) \cap (B \cup C)$

24. A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x + 1}

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, codomain and range of R.

25. Show that the points in the Argand plane represented by the complex numbers

$-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.

26. Solve the inequalities and represent the solution graphically on number line:

$3x - 7 > 2(x-6),\ 6 - x > 11 - 2x$

27. Simplify ${}^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$

28. If n is a positive integer, then prove that $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots\ldots\ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

29. A(–1,1), B(5,3) are opposite vertices of a square in the XY-plane. Find the equation of the other diagonal

30. Find the equation of the ellipse, with major axis along the x-axis and passing through the points (4, 3) and (– 1,4).

31. Find thd derivative of the function (x+1)(x–2), from first principle.

32. A die is thrown, find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

SECTION-D

IV. Answer ANY FIVE the questions. Each question carries EIGHT marks. $5 \times 8 = 40$

33. If f= {(4, 5),(5,6),(6,–4)};g={(4,–4), (6,5), (8,5)}

    find (i) f+g (ii) f–g (iii) 2f+4g (iv) f+4 (v) fg (vi) $\frac{f}{g}$

34. Prove that $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$

35. Find the sum of the series .6 +. 66 +. 666+… up to n terms:

36. Show that the straight linex x+y=0, 3x+y–4=0 and x+3y–4=0 from an isosceles triangle.

37. Find the equation of the circle passing through the points (0,0), (2,0), (0,2).

38. (a) If $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$ For what values of a does $\underset{x\to a}{Lt} f(x)$ exists?

    (b) Find the derivative of $5\sin x+e^{x}\log x$.

39. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

40. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

IPE AP MAY - 2026

SOLUTIONS

SECTION-A

1. If A = {1, 2, 3}, B = {a, b} then A × B =

1) {(1, a), (2, b), (3, a)}
2) {(a, 1), (b, 2), (a, 3), (b, 3)}
✓ **3) {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)}**
4) {(1, a), (1, b), (2, b), (3, b)}

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2. Let x, y ∈ R, then x + iy is a non real complex number if

1) $x = 0$
2) $y = 0$
3) $x \neq 0$
✓ **4) $y \neq 0$**

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3. The number of arrangements that can be made out of the letters in expansion $a^4 b^3 c^5$ when written in full length is

1) $\frac{9!}{3!2!4!}$
2) $\frac{7!}{2!3!2!}$
3) $\frac{5!}{4!3!2!}$
✓ **4) $\frac{(12)!}{4!3!5!}$**

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4. The expansion of $(x + y)^{1000}$ is

1) $\sum_{r=0}^{100} {}^{1000}C_r\, x^{r-1000} y^r$
✓ **2) $\sum_{r=0}^{1000} {}^{1000}C_r\, x^{1000-r} y^r$**
3) $\sum_{r=0}^{999} {}^{1000}C_r\, x^{r-1000} y^r$
4) $\sum_{r=0}^{999} {}^{1000}C_r\, x^{1000-r} y^r$

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5. If the distance between the points P(0, a, 3) and Q(3, 0, 7) is $\sqrt{41}$ then a =

1) –4
2) 4
✓ **3) ± 4**
4) None of these

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6. Let A and B be two events such that P(A) = 0.8, P(B) = 0.7, then P(A ∩ B)

1) ≤ 0.6
✓ **2) ≥ 0.5**
3) ≥ 0.7
4) ≤ 0.4

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7. {x : x ∈ R, – 4 < x ≤ 6} Write as intervals :

Sol: i) $-4 < x \leq 6 = (-4, 6]$. [Here, – 4 is not included and 6 is included]

8: Find the value of $\sin\frac{31\pi}{3}$.

Sol: $\sin\frac{31\pi}{3} = \sin\left(\frac{30\pi+\pi}{3}\right) = \sin\left(10\pi+\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

9. Find the a_9 term in the sequence whose n^{th} term is $a_n = (-1)^{n-1}n^3$

Sol: Given that $a_n = (-1)^{n-1}n^3$ Put n = 9, then $a_9 = (-1)^{9-1}(9)^3 = 729$

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10. Find the equation of the line which passes through the points (–1, 1) and (2, –4).

Sol: Equation of the line through (–1, 1) and (2, –4) is given by $(y - y_1) = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$\Rightarrow (y-1) = \frac{-4-1}{2+1}(x+1) \Rightarrow (y-1) = \frac{-5}{3}(x+1) \Rightarrow 3(y-3) = -5(x+1)$$

$$\Rightarrow 3y - 3 = -5x - 5 \Rightarrow 5x + 3y + 2 = 0$$

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11. Find the eccentricity of hyperbola $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$

Sol: Instead of the standard equation $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$ here we have given $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$

$\therefore$ Eccentricity of the hyperbola $e = \dfrac{c}{b} = \dfrac{\sqrt{a^2 + b^2}}{b}$ [$\because$ Roles of a,b are interchanged]

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12. Find the derivative of $f(x)=(ax+b)^n$

Sol: $f(x) = (ax + b)^n$

$\Rightarrow f'(x) = n(ax + b)^{n-1} \dfrac{d}{dx}(ax + b) = n(ax + b)^{n-1}a$ $\left[\because \dfrac{d}{dx}f(x)^n = nf(x)^{n-1}\dfrac{d}{dx}f(x)\right]$

SECTION-B

13. If X = {a, b, c, d} and Y = {f, b, d, g}, find (i) X – Y (ii) Y – X (iii) X ∩ Y.

Sol: i) $X - Y = \{a, \cancel{b}, c, \cancel{d}\} - \{f, \mathbf{b}, \mathbf{d}, g\} = \{a, c\}$

ii) $Y - X = \{f, \cancel{b}, \cancel{d}, g\} - \{a, \mathbf{b}, c, \mathbf{d}\} = \{f, g\}$

iii) $X \cap Y = \{a, b, c, d\} \cap \{f, b, d, g\} = \{b, d\}$.

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14. If A = {– 1, 1}, find A × A × A.

Sol: $A \times A = \{-1, 1\} \times \{-1, 1\} = \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

$\therefore A \times A \times A = \mathbf{\{-1, 1\}} \times \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

$= \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$.

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15. Prove that sin[(n+1)x]sin[(n+2)x] + cos[(n+1)x]cos[(n+2)x] = cosx

Sol: Let $(n + 1)x = A$ and $(n + 2)x = B$

$L.H.S = \sin[(n+1)x].\sin[(n+2)x] + \cos[(n+1)x].\cos[(n+2)x]$

$= \sin A \sin B + \cos A \cos B = \cos A \cos B + \sin A \sin B = \cos(A-B)$

$= \cos[(n+1)x-(n+2)x] = \cos(nx+x-nx-2x) = \cos(-x) = \cos x = R.H.S \; [\because \cos(-x) = \cos x]$

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16. Find the multiplicative inverse of 2 – 3i.

Sol: Let $z = 2 - 3i$, then $z^{-1} = \dfrac{1}{2-3i} = \dfrac{2+3i}{(2-3i)(2+3i)} = \dfrac{2+3i}{2^2+3^2} = \dfrac{2+3i}{4+9} = \dfrac{2}{13} + \dfrac{3}{13}i$

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17. If $^{18}P_{(r-1)} : {}^{17}P_{(r-1)} = 9 : 7$, find r

Sol: Given that $^{18}P_{(r-1)} : {}^{17}P_{(r-1)} = 9:7 \Rightarrow \dfrac{^{18}P_{r-1}}{^{17}P_{r-1}} = \dfrac{9}{7}$

$$\Rightarrow \frac{(18)!}{(18-(r-1))!} \times \frac{(17-(r-1))!}{(17)!} = \frac{9}{7} \Rightarrow \frac{18!}{(19-r)!}\,\frac{(18-r)!}{17!} = \frac{9}{7}$$

$$\Rightarrow \frac{18 \times \cancel{17!}\,\cancel{(18-r)!}}{(19-r)\,\cancel{(18-r)!}\,\cancel{17!}} = \frac{9}{7} \Rightarrow 18 \times 7 = 9(19-r) \Rightarrow 14 = 19 - r \Rightarrow r = 19-14 = 5 \Rightarrow r = 5$$

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18. Find the 6th term in $\left(\dfrac{2x}{3} + \dfrac{3y}{2}\right)^9$

Sol: General term in $(x+y)^n$ is $T_{r+1} = {}^nC_r\, x^{n-r} y^r$.

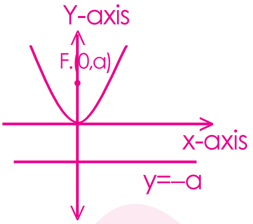
$$\therefore T_6 = T_{5+1} = {}^9C_5 \left(\frac{2x}{3}\right)^4 \left(\frac{3y}{2}\right)^5 = {}^9C_4 \left(\frac{2}{3}\right)^4 \left(\frac{3}{2}\right)^5 x^4 y^5 = \left(\frac{9\times8\times7\times6}{1\times2\times3\times4}\right)\left(\frac{2^4}{3^4}\right)\left(\frac{3^5}{2^5}\right) x^4 y^5 = 189\, x^4 y^5$$

19. Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis

Sol: The vertex is (0, 0) and the parabola is symmetric about the y-axis.

The parabola passes through point (5, 2), which lies in the first quadrant.

Hence the parabola is upward vertical.



$\therefore$ the equation of the parabola is of the form $x^2 = 4ay$.

The point (5, 2) should satisfy the equation $x^2 = 4ay$.

$\therefore 5^2 = 4 \times a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$

$\therefore$ the equation of the parabola is $x^2 = \cancel{4}\left(\frac{25}{\cancel{8}}\right)y \Rightarrow 2x^2 = 25y$

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20. Three vertices of a parallelogram ABCD are A(3, – 1, 2), B (1, 2, – 4) and C (– 1, 1, 2). Find the coordinates of the fourth vertex.

Sol: Given vertices are A=(3, – 1, 2), B =(1, 2, – 4), C =(– 1, 1, 2) and D = (x, y, z).

In the Parallelogram ABCD, we have Mid-point of AC = Mid-point of BD

$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$

$\Rightarrow (1, 0, 2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right) \Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \frac{z-4}{2} = 2$

$\Rightarrow x + 1 = 2 \Rightarrow x = 2–1 =1; \Rightarrow y + 2 = 0 \Rightarrow y = –2; \Rightarrow z – 4 = 4 \Rightarrow z = 4 + 4 =8$

$\Rightarrow x = 1, y = -2, z = 8$ $\therefore$ Fourth vertex D = (1, –2, 8).

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21. Compute $\lim_{x \to 0} \frac{a^x - 1}{b^x - 1}$ (a > 0, b > 0, b ≠ 1)

Sol: $\lim_{x \to 0} \frac{a^x - 1}{b^x - 1} = \lim_{x \to 0} \frac{\left(\frac{a^x - 1}{x}\right)}{\left(\frac{b^x - 1}{x}\right)} = \frac{\lim_{x \to 0}\left(\frac{a^x - 1}{x}\right)}{\lim_{x \to 0}\left(\frac{b^x - 1}{x}\right)} = \frac{\log_e a}{\log_e b} = \log_b a$ $\left(\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a\right)$

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22. Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12

Sol:
- Mean of the given data $\bar{x} = \frac{\sum x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$
- The deviations $(x_i - \bar{x})$ of the observations from the mean $(\bar{x})$ are
  $6 – 9 = –3; 7 – 9 = –2; 10 – 9 = 1; 12 – 9 = 3; 13 – 9 = 4; 4 – 9 = –5; 8–9 = –1; 12–9 = 3$
- The absolute values of the deviations are 3, 2, 1, 3, 4, 5, 1, 3
- Variance $\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{3^2 + 2^2 + 1^2 + 3^2 + 4^2 + 5^2 + 1^2 + 3^2}{8}$

  $= \frac{9+4+1+9+16+25+1+9}{8} = \frac{74}{8} = 9.25$

SECTION-C

23. If A = { 3, 5, 7, 9, 11 }, B = {7, 9, 11, 13}, C = {11, 13, 15}and D = {15, 17}; find

(i) $A \cap (B \cup C)$ (ii) $A \cap C \cap D$ (iii) $A \cap (B \cup D)$

(iv) $(A \cap B) \cap (B \cup C)$ (v) $(A \cup D) \cap (B \cup C)$

Sol: i) $A \cap (B \cup C) = \{3, 5, 7, 9, 11\} \cap [\{7, 9, 11, 13\} \cup \{11, 13, 15\}]$

$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$

ii) $A \cap C \cap D = (A \cap C) \cap D = [\{3, 5, 7, 9, 11\} \cap \{11, 13, 15\}] \cap \{15, 17\}$

$= \{11\} \cap \{15, 17\} = \varnothing$

iii) $A \cap (B \cup D) = \{3, 5, 7, 9, 11\} \cap [\{7, 9, 11, 13\} \cup \{15, 17\}]$

$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15, 17\} = \{7, 9, 11\}$

iv) $(A \cap B) \cap (B \cup C) = [\{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}] \cap [\{7, 9, 11, 13\} \cup \{11, 13, 15\}]$

$= \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$

v) $(A \cup D) \cap (B \cup C) = [\{3, 5, 7, 9, 11\} \cup \{15, 17\}] \cap [\{7, 9, 11, 13\} \cup \{11, 13, 15\}]$

$= \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11, 15\}$.

24: Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x + 1}

i) Depict this relation using an arrow diagram.

ii) Write down the domain, codomain and range of R.

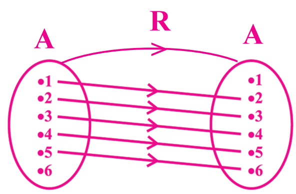
Sol: Given $R : A \to A$

i) $A = \{1, 2, 3, 4, 5, 6\}$ and elements in the range follow $y = x + 1$

$\therefore$ Range = {1+1, 2+1, 3+1, 4+1, 5+1, 6+1(×)} = {2,3,4,5,6}

$\therefore R = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$

The arrow diagram of R is shown here.



ii) We have R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

The domain = { Set of first elements} = {1, 2, 3, 4, 5,}

The range = { Set of second elements}= {2, 3, 4, 5, 6}

The codomain = {1, 2, 3, 4, 5, 6}. [$\because R : A \to A$]

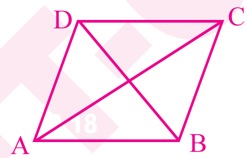
25. Show that the points in the Argand plane represented by the complex numbers $-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.

Sol: Let the given points are taken as $A(-2,7)$, $B\left(-\frac{3}{2},\frac{1}{2}\right)$, $C(4,-3)$; $D\left(\frac{7}{2},\frac{7}{2}\right)$

$$AB=\sqrt{\left(-2+\frac{3}{2}\right)^2+\left(7-\frac{1}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\frac{\sqrt{170}}{2};\quad BC=\sqrt{\left(4+\frac{3}{2}\right)^2+\left(-3-\frac{1}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\frac{\sqrt{170}}{2}$$

$$CD=\sqrt{\left(4-\frac{7}{2}\right)^2+\left(-3-\frac{7}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\frac{\sqrt{170}}{2};\ DA=\sqrt{\left(\frac{7}{2}+2\right)^2+\left(\frac{7}{2}-7\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\frac{\sqrt{170}}{2}$$

Hence, the four sides AB, BC, CD, DA are equal.



$$AC=\sqrt{(4+2)^2+(-3-7)^2}=\sqrt{36+100}=\sqrt{136}$$

$$BD=\sqrt{\left(\frac{7}{2}+\frac{3}{2}\right)^2+\left(\frac{7}{2}-\frac{1}{2}\right)^2}=\sqrt{\frac{100}{4}+\frac{36}{4}}=\frac{\sqrt{136}}{2}$$

The two diagonals AC, BD are unequal.

$\therefore$ A, B, C, D form a Rhombus.

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26. Solve the inequalities and represent the solution graphically on number line: $3x-7>2(x-6)$, $6-x>11-2x$

Sol: Given that $3x-7>2(x-6) \Rightarrow 3x-7>2x-12 \Rightarrow 3x-2x>-12+7 \Rightarrow x>-5$ ...(i)

Now $6-x>11-2x \Rightarrow -x+2x>11-6 \Rightarrow x>5$...(ii)

From (i) and (ii), we have $x \in (5, \infty)$.

Number Line graph of $(5, \infty)$:

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27. Simplify $^{34}C_5+\sum_{r=0}^{4}{}^{(38-r)}C_4$

Sol: $G.E={}^{34}C_5+\sum_{r=0}^{4}{}^{(38-r)}C_4={}^{34}C_5+\left({}^{38}C_4+{}^{37}C_4+{}^{36}C_4+{}^{35}C_4+{}^{34}C_4\right)$

$={}^{34}C_5+\left({}^{34}C_4+{}^{35}C_4+{}^{36}C_4+{}^{37}C_4+{}^{38}C_4\right)$ [On rewriting the terms in the bracket]

$=\left({}^{34}C_5+{}^{34}C_4\right)+\left({}^{35}C_4+{}^{36}C_4+{}^{37}C_4+{}^{38}C_4\right)$ (Now applying ${}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$)

$=\left({}^{35}C_5+{}^{35}C_4\right)+\left({}^{36}C_4+{}^{37}C_4+{}^{38}C_4\right)=\left({}^{36}C_5+{}^{36}C_4\right)+\left({}^{37}C_4+{}^{38}C_4\right)$

$=\left({}^{37}C_5+{}^{37}C_4\right)+{}^{38}C_4={}^{38}C_5+{}^{38}C_4={}^{39}C_5$

28. If n is a positive integer, then prove that $C_0+\frac{C_1}{2}+\frac{C_2}{3}+..........+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

Sol: $\text{L.H.S}=C_0+\frac{C_1}{2}++\frac{C_2}{3}+....+\frac{C_n}{n+1}=1+\frac{n}{2}+\frac{[n(n-1)]}{2(3)}+......+\frac{1}{n+1}$

$$=\frac{1}{n+1}\left[(n+1)+\frac{(n+1)n}{2}+\frac{(n+1)n(n-1)}{3.2.1}+......+(n+1)\frac{1}{n+1}\right]$$

$$=\frac{1}{n+1}\left[{}^{(n+1)}C_1+{}^{(n+1)}C_2+{}^{(n+1)}C_3+......+{}^{(n+1)}C_{n+1}\right]$$

$$=\frac{1}{n+1}\left[\left({}^{(n+1)}C_0+{}^{(n+1)}C_1+{}^{(n+1)}C_2+{}^{(n+1)}C_3+......+{}^{(n+1)}C_{(n+1)}\right)-{}^{(n+1)}C_0\right]$$

$$=\frac{1}{n+1}[2^{n+1}-1]=\frac{2^{n+1}-1}{n+1}=\text{R.H.S}\qquad [\because {}^nC_1x+{}^nC_2x^2+....+{}^nC_nx^n=(1+x)^n-1]$$

---

29. A(–1,1), B(5,3) are opposite vertices of a square in the XY-plane. Find the equation of the other diagonal (not passing through A, B) of the square.

Sol: Midpoint AB is (2, 2). Slope of $\overleftrightarrow{AB}$ is $\frac{3-1}{5+1}=\frac{1}{3}\Rightarrow$ Slope of other diagonal is –3.

Equation of the other diagonal passing through (2, 2) with slope –3 is $y-2=-3(x-2)=-3x+6$

$\Rightarrow 3x+y-8=0$

---

30. Find the equation of the ellipse, with major axis along the x-axis and passing through the points (4, 3) and (– 1,4).

Sol: The major axis along the x-axis . So the ellipse is horizontal

$\therefore$ Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

$(4,3)$ lies on the ellipse $\Rightarrow\frac{16}{a^2}+\frac{9}{b^2}=1\Rightarrow\frac{16b^2+9a^2}{a^2b^2}=1\Rightarrow 9a^2+16b^2=a^2b^2$......(i)

$(-1,4)$ lies on the ellipse $\Rightarrow\frac{(-1)^2}{a^2}+\frac{4^2}{b^2}=1\Rightarrow\frac{1}{a^2}+\frac{16}{b^2}=1\Rightarrow\frac{b^2+16a^2}{a^2b^2}=1\Rightarrow 16a^2+b^2=a^2b^2$......(ii)

From (i) & (ii) $9a^2+16b^2=16a^2+b^2\Rightarrow 7a^2=15b^2\Rightarrow a^2=\frac{15}{7}b^2$

From (i) $9\left(\frac{15}{7}\right)\cancel{b^2}+16\cancel{b^2}=\left(\frac{15}{7}b^2\right)\cancel{b^2}\Rightarrow\frac{135}{7}+16=\frac{15b^2}{7}\Rightarrow\frac{247}{7}=\frac{15b^2}{7}\Rightarrow b^2=\frac{247}{15}$

Now $a^2=\frac{15}{7}b^2=\frac{15}{7}\left(\frac{247}{15}\right)=\frac{247}{7}$

$\therefore$ the equation of the ellipse is $\frac{x^2}{\left(\frac{247}{7}\right)}+\frac{y^2}{\frac{247}{15}}=1\Rightarrow 7x^2+15y^2=247.$

31. Find thd derivative of the function (x + 1)(x – 2), from first principle.

Sol: Let f(x) = (x – 1)(x – 2). From the first principle, we have

$$f'(x) = \underset{h\to 0}{Lt} \frac{f(x+h)-f(x)}{h} = \underset{h\to 0}{Lt} \frac{(x+h-1)(x+h-2)-(x-1)(x-2)}{h}$$

$$= \underset{h\to 0}{Lt} \frac{(x^2+xh-2x+hx+h^2-2h-x-h+2)-(x^2-2x-x+2)}{h}$$

$$= \underset{h\to 0}{Lt} \frac{(hx+hx+h^2-2h-h)}{h} = \underset{h\to 0}{Lt} \frac{2hx+h^2-3h}{h} = \underset{h\to 0}{Lt} \frac{\cancel{h}(2x+h-3)}{\cancel{h}}$$

$$= \underset{h\to 0}{Lt} (2x+h-3) = (2x+0-3) = 2x-3$$

---

32. A die is thrown, find the probability of following events:

i) A prime number will appear,

ii) A number greater than or equal to 3 will appear,

iii) A number less than or equal to one will appear,

iv) A number more than 6 will appear,

v) A number less than 6 will appear.

Sol: When a die is thrown the Sample space S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ n(S) = 6

i) Let E_1 be the event of getting a prime number $\Rightarrow$ $E_1 = \{2, 3, 5\}$ $\therefore n(E_1) = 3$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

ii) Let E_2 be the event of number greater than or equal to 3 $\Rightarrow E_2 = \{3, 4, 5, 6\} \therefore n(E_2) = 4$

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{6} = \frac{2}{3}.$$

iii) Let E_3 be the event of a number less than or equal to 1 $\Rightarrow E_3 = \{1\}$ $\therefore n(E_3) = 1$

$$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{6}$$

iv) Let E_4 be the event of a number more than 6 .

Since there is no such number in S, $E_4 = \Phi$ $\therefore n(E_4) = 0 \Rightarrow P(E_4) = 0$[E_4 is impossible]

v) Let E_5 be the event of a number less than 6 $\Rightarrow E_5 = \{1, 2, 3, 4, 5\}$ $\therefore n(E_5) = 5$

$$\therefore \quad P(E_5) = \frac{n(E_5)}{n(S)} = \frac{5}{6}$$

SECTION-D

33. If f = {(4, 5), (5, 6), (6, –4)}; g = {(4, –4), (6, 5), (8, 5)}

find (i) f + g (ii) f – g (iii) 2f + 4g (iv) f + 4 (v) fg (vi) $\frac{f}{g}$

Sol: Here domain of f is A = {4, 5, 6} and domain of g is B={4, 6, 8}

Domain of $f \pm g = A \cap B = \{4, 6\}$. So, we have to consider only the 2 preimages 4, 6.

i) $f + g = \{(4, 5 + (-4)), (6, -4 + 5)\} = \{(4, 1), (6, 1)\}$

ii) $f - g = \{(4, 5-(-4)), (6, -4 -5)\} = \{(4, 9), (6, -9)\}$

iii) $2f = \{(4, 2(5)), (5, 2(6)), (6, 2(-4))\} = \{(4, 10), (5, 12), (6, -8)\}$

$4g = \{(4, 4(-4)), (6, 4(5)), (8, 4(5))\} = \{(4, -16), (6, 20), (8, 20)\}$

$\therefore\ 2f + 4g = \{(4, 10-16), (6, -8 + 20)\} = \{(4, -6), (6, 12)\}$ [$\because$ Domain of $f \pm g = A \cap B = \{4,6\}$]

iv) $f + 4 = \{(4, 5 + 4), (5, 6 + 4), (6, -4 + 4)\} = \{(4, 9), (5, 10), (6, 0)\}$

v) $f\,g = \{(4, 5(-4)), (6, -4(5))\} = \{(4, -20), (6, -20,)\}$

vi) $\frac{f}{g} = \left\{\left(4, -\frac{5}{4}\right), \left(6, -\frac{4}{5}\right)\right\}$

---

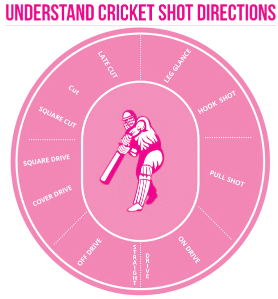
34. Prove that $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Sol: $$\text{L.H.S} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$= \frac{[2\sin 6x \cdot \cos x] + [2\sin 6x \cdot \cos 3x]}{[2\cos 6x \cdot \cos x] + [2\cos 6x \cdot \cos 3x]} = \frac{\cancel{2}\sin 6x[\cancel{\cos x + \cos 3x}]}{\cancel{2}\cos 6x[\cancel{\cos x + \cos 3x}]} = \frac{\sin 6x}{\cos 6x} = \tan 6x = \text{R.H.S}$$

UNDERSTAND CRICKET SHOT DIRECTIONS



35. Find the sum of the series .6 +. 66 +. 666+... up to n terms:

Sol: Let Sn = 0.6 +0.66 + 0.666 + ….. to n terms

$$=6[0.1+0.11+0.111+.....\text{to n terms}]=\frac{6}{9}[0.9+0.99+0.999+.....\text{to n terms}]$$

$$=\frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+......\text{to n terms}\right]=\frac{2}{3}\left[(1+1+.....\text{nterms})-\frac{1}{10}\left(1+\frac{1}{10}+\frac{1}{10^2}+.....\text{nterms}\right)\right]$$

$$=\frac{2}{3}\left[\mathbf{n}-\frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right)\right]=\frac{2}{3}n-\frac{2}{30}\times\frac{10}{9}\left(1-\frac{1}{10^n}\right)=\frac{2n}{3}-\frac{2}{27}\left(\frac{10^n-1}{10^n}\right)$$

---

36. Show that the straight lines x + y = 0, 3x + y – 4 = 0 and x + 3y – 4 = 0 form an isosceles triangle.

Sol: Given lines are $x + y = 0$........ (1), $3x + y - 4 = 0$....... (2), $x + 3y - 4 = 0$ ......(3)

If 'A' is the angle between (1),(2) then

$$\cos A=\frac{|a_1a_2+b_1b_2|}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}}=\frac{|1(3)+1(1)|}{\sqrt{(1^2+1^2)(3^2+1^2)}}=\frac{4}{\sqrt{(2)(10)}}=\frac{4}{2\sqrt{5}}=\frac{2}{\sqrt{5}}\Rightarrow A=\mathrm{Cos}^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

If 'B' is the angle between (2),(3) then

$$\cos B=\frac{|a_1a_2+b_1b_2|}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}}=\frac{|3(1)+1(3)|}{\sqrt{(3^2+1^2)(1^2+3^2)}}=\frac{6}{\sqrt{10}\sqrt{10}}=\frac{6}{10}=\frac{3}{5}\Rightarrow B=\mathrm{Cos}^{-1}\left(\frac{3}{5}\right)$$

If 'C' is the angle between (1),(3) then

$$\cos C=\frac{|a_1a_2+b_1b_2|}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}}=\frac{|1(1)+1(3)|}{\sqrt{(1^2+1^2)1^2+3^2}}=\frac{4}{\sqrt{2}\sqrt{10}}=\frac{4}{2\sqrt{5}}=\frac{2}{\sqrt{5}}\Rightarrow C=\mathrm{Cos}^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

Here A = C. So the given lines form an Isosceles triangle.

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37. Find the equation of the circle passing through points (0,0), (2,0) (0,2)

Sol: The circle passes through the origin O(0,0).

The circle passes through A(2,0) and B(0,2)

Let the equation of the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.....(i)

(i) passes through the origin O(0,0) $\Rightarrow 0 + 0 + 2g(0) + 2f(0) + c = 0 \Rightarrow c=0$

(i) passes through A(2,0) $\Rightarrow 2^2 + 0 + 2g(2) + 2f(0) + 0 = 0$

$\Rightarrow 2^2 + 2g(2) = 0 \quad \Rightarrow 2(2+2g)=0 \Rightarrow 2g = -2$

(i) passes through B(0,2) $\Rightarrow 0 + 2^2 + 2g(0) + 2f(2) + 0 = 0$

$\Rightarrow 2^2 + 2f(2) = 0 \quad \Rightarrow 2(2+2f)=0 \Rightarrow 2f = -2$

Putting c=0, 2g = –2, 2f = –2 in (i)

the equation of the circle is $x^2 + y^2 - 2x - 2y = 0$

38. a) If $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$. For what values of a does $\underset{x\to a}{Lt}\, f(x)$ exists?

b) Find the derivative of 5sinx+ e^x logx.

Sol: a) The given function is $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$

At a = 0

$$L.H.L=\underset{x\to 0^-}{Lt} f(x)=\underset{x\to 0^-}{Lt}(|x|+1)=\underset{x\to 0^-}{Lt}(-x+1)=-0+1=1 \quad [\text{When } x<0, |x|=-x]$$

$$R.H.L=\underset{x\to 0^+}{Lt} f(x)=\underset{x\to 0^+}{Lt}(|x|-1)=\underset{x\to 0^+}{Lt}(x-1)=0-1=-1 \quad [\text{When f } x>0, |x|=x]$$

$\Rightarrow$ L.H.L $\neq$ R.H.L $\quad \therefore \underset{x\to 0}{Lt}\, f(x)$ does not exist

When a < 0

$$L.H.L=\underset{x\to a^-}{Lt} f(x)=\underset{x\to a^-}{Lt}(|x|+1)=\underset{x\to a^-}{Lt}(-x+1)=-a+1 \quad [\because a<0\Rightarrow x<0\Rightarrow |x|=-x]$$

$$R.H.L=\underset{x\to a^+}{Lt} f(x)=\underset{x\to a^+}{Lt}(|x|+1)=\underset{x\to a^+}{Lt}(-x+1)=-a+1 \quad [\because a<0\Rightarrow x<0\Rightarrow |x|=-x]$$

Here L.H.L = R.H.L= –a+1

$\therefore$ Limit of f(x) exists at x = a, when a < 0.

When a > 0

$$L.H.L=\underset{x\to a^-}{Lt} f(x)=\underset{x\to a^-}{Lt}(|x|-1)=\underset{x\to a^-}{Lt}(x-1)=a-1 \quad [\because a>0\Rightarrow x>0\Rightarrow |x|=x]$$

$$R.H.L=\underset{x\to a^+}{Lt} f(x)=\underset{x\to a^+}{Lt}(|x|-1)=\underset{x\to a^+}{Lt}(x-1)=a-1 \quad [\because a>0\Rightarrow x>0\Rightarrow |x|=x]$$

Here, L.H.L = R.H.L= a–1

$\therefore$Lt of f(x) exists at x = a, when a>0 $\qquad \therefore \underset{x\to a}{Lt}\, f(x)$ exists for all a≠0

b) $$\frac{d}{dx}(5\sin x+e^x\log x)=5\frac{d}{dx}\sin x+\frac{d}{dx}(e^x\log x)=5\cos x+\left[e^x\left(\frac{1}{x}\right)+\log x(e^x)\right]$$

39. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Sol: [Converting discontinuous frequency table into continuous frequency table.

- The gap between first class interval (16 - 20) and second class interval (21 - 25) is 21-20=1.
- Adjustment factor $= \dfrac{\text{interval gap}}{2} = \dfrac{1}{2} = 0.5$
- This adjustment factor 0.5 should be subtracted from every lower limits and added to the every upper limits.]
- The continuous mean deviation table is as follows.

Class x_i	Frequency f_i	Mid point x_i	$y_i = \dfrac{x_i - 42.5}{4}$	y_i^2	$f_i y_i$	$f_i y_i^2$
32.5-36.5	15	34.5	–2	4	–30	60
36.5-40.5	17	38.5	–1	1	–17	17
40.5-44.5	21	**42.5(A)**	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				**25**	**199**

- Let us take the assumed mean as A = 42.5 (Any one of the middle values of x_i)
- h = Width of the class interval = 36.5–32.5=4 • Here, N = 100
- $\text{Mean}, \bar{x} = A + \dfrac{\sum f_i y_i}{N} \times h = 42.5 + \dfrac{25}{100} \times 4 = 43.5$
- $\text{Variance}(\sigma^2) = \dfrac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right] = \dfrac{16}{10000}\left[100 \times 199 - (25)^2\right]$

$$= \frac{16}{10000}[19900 - 625] = \frac{16}{10000} \times 19275 = 30.84$$

40. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Sol: Each of the four wheels is labelled with 10 digits.

Number of sequences with distinct digits = ${}^{10}P_4 = 10 \times 9 \times 8 \times 7 = 5040$
There is only one right sequence to open.

$\therefore$ Required probability $= \dfrac{1}{5040}$