



MARCH -2026(AP)

PREVIOUS PAPERS

IPE: MARCH-2026 (AP)

Time : 3 Hours

JR. MATHS

Max.Marks : 100

SECTION-A

I. Answer ALL the questions. Each question carries ONE mark.

 $12 \times 1 = 12$

- The domain of the function $\frac{1}{\sqrt{x^2 - 25}}$ is
 1) $(-\infty, -5) \cup (5, \infty)$ 2) $(-\infty, -5] \cup [5, \infty)$ 3) $(-\infty, -5] \cup (5, \infty)$ 4) $(-\infty, -5) \cup [5, \infty)$
- The value of $\sqrt{-25} \times \sqrt{-9}$
 1) 15 2) -15 3) 15i 4) None of these
- If ${}^{12}C_{S+1} = {}^{12}C_{2S-5}$ then the value of s is
 1) 4 2) 8 3) 12 4) 6
- The coefficient of $x^8 y^{10}$ in the expansion of $(x + y)^{18}$ is
 1) ${}^{18}C_8$ 2) ${}^{18}P_{10}$ 3) 2^{18} 4) None of these
- XOZ-plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio
 1) 3 : 7 2) 2 : 7 3) -3 : 7 4) -2 : 4
- If $P(A) = 0.5$ $P(B) = 0.3$. (given that A and B are mutually exclusive), then $P(A' \cap B') =$
 1) 0.6 2) 0.5 3) 0.7 4) 0.2
- List all the elements of the following set $C = \{x : x \text{ is an integer, } x^2 \leq 4\}$
- Find the value of the $\sin\left(-\frac{11\pi}{3}\right)$
- Find the sum to infinity in Geometric Progression $1, \frac{1}{3}, \frac{1}{9}, \dots$
- Find the equation of the line which is passing through the point $(-4, 3)$ with slope $1/2$.
- Find the length of transverse axis of hyperbola $x^2 - 4y^2 = 4$
- Find the derivative of $x^2 - 2$ at $x = 10$.

SECTION-B

II. Answer ALL the questions. Each question carries TWO marks.

 $10 \times 2 = 20$

- Let $A = \{1, 2, \{3, 4\}, 5\}$. Is the statement $\{3, 4\} \in A$ incorrect? And why?
- If $P = \{a, b, c\}$ and $Q = \{r\}$, form the sets $P \times Q$ and $Q \times P$. Are these two products equal?
- A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

16. Express the complex number $\left(\frac{1}{3} + 3i\right)^3$ in the form $a + ib$.
17. How many 4-digit numbers can be formed by using the digits 1 to 9 if repetition of digits is not allowed?
18. Expand $\left(\frac{2p}{5} - \frac{3q}{7}\right)^6$
19. Find the equation of the parabola with vertex (0, 0) passing through (2, 3) and axis is along x-axis.
20. Find 'x' if the distance between (5, -1, 7) and (x, 5, 1) is 9 units.
21. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$
22. Find the mean deviation about the mean for the following data 4, 7, 8, 9, 10, 12, 13, 17

SECTION-C

III. Answer ANY SEVEN the questions. Each question carries FOUR marks. $7 \times 4 = 28$

23. $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$.
Find A' , B' , $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.
24. $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by $R = \{(x, y) : y = x + 1\}$
i) Depict this relation using an arrow diagram.
ii) Write down the domain, codomain and range of R.
25. Show that the four points in the Argand plane represented by the complex numbers $2+i$, $4+3i$, $2+5i$, $3i$ are the vertices of a square.
26. A solution is to be kept between 68°F and 77°F . What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$?
27. Prove that for $3 \leq r \leq n$; ${}^{(n-3)}C_r + 3 \cdot {}^{(n-3)}C_{(r-1)} + 3 \cdot {}^{(n-3)}C_{(r-2)} + {}^{(n-3)}C_{(r-3)} = {}^nC_r$
28. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} (2n)x^n$
29. Find the equation of the straight line passing through A (1, 3) and
i) parallel ii) perpendicular to the straight line passing through B (3, -5) and C (-6, 1).
30. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100} + \frac{y^2}{400} = 1$.
31. Compute the derivative of $\sin 2x$ from the first principle.
32. A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.
Find (i) $P(A \cup B)$ (ii) $P(A' \cap B')$ (iii) $P(A \cap B')$ (iv) $P(B \cap A')$

SECTION-D**IV. Answer ANY FIVE the questions. Each question carries EIGHT marks. $5 \times 8 = 40$** 33. If f and g are real -valued functions defined by $f(x) = 2x - 1$ and $g(x) = x^2$ then find

$$(i) (3f-2g)(x) \quad (ii) (fg)(x) \quad (iii) \left(\frac{f}{g}\right)(x) \quad (iv) (f+g+2)(x) \quad (v) 2f(x) \quad (vi) 2+f(x)$$

34. Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$ 35. A man deposited ₹10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15th year since he deposited the amount and also calculate the total amount after 20 years.36. A straight line through $Q(2,3)$ makes an angle of $3\pi/4$ with the negative direction of the X-axis. If the straight line intersects the line $x+y-7=0$ at P , find the distance PQ .37. Find the equation of the circle passing through the points $(3,4)$, $(3,2)$ and $(1,4)$.

$$38. \text{ If } f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1. \\ nx^3 + m, & x > 1 \end{cases}$$

For what integers m and n does both $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$ exist?

39. Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

40. a) In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

b) There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Infinite sum $S_{\infty} = \frac{a}{1-r} = \frac{1}{1-(1/3)} = \frac{1}{2/3} = \frac{3}{2}$

10. Find the equation of the line which passes through the point $(-4, 3)$ with slope $1/2$.

Sol: Equation of the line passing through the point $(-4, 3)$ with slope $1/2$ is given by $(y - y_0) = m(x - x_0)$

$$\Rightarrow (y - 3) = \frac{1}{2}(x + 4) \Rightarrow 2(y - 3) = (x + 4) \Rightarrow 2y - 6 = x + 4 \Rightarrow x - 2y + 10 = 0$$

11. Find the length of transverse axis of hyperbola $x^2 - 4y^2 = 4$

Sol: Given equation $x^2 - 4y^2 = 4 \Rightarrow \frac{x^2}{4} - \frac{4y^2}{1} = 1 \Rightarrow \frac{x^2}{4} - \frac{y^2}{1/4} = 1$.

This is in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the hyperbola with foci on x-axis.

Here, $a^2 = 4 \Rightarrow a = 2$, $b^2 = \frac{1}{4} \Rightarrow b = \frac{1}{2}$

Length of transverse axis $2a = 2(2) = 4$

12. Find the derivative of $x^2 - 2$ at $x = 10$.

Sol: Given $f(x) = x^2 - 2 \Rightarrow f'(x) = 2x$

At $x = 10$, $\therefore f'(10) = 2(10) = 20$

$\therefore$ the derivative of $x^2 - 2$ at $x = 10$ is 20.

SECTION-B

13. Let $A = \{1, 2, \{3, 4\}, 5\}$. Is the statement $\{3, 4\} \in A$ incorrect? And why?

Sol: True. [Reason: $\{3, 4\}$ is an element of A .]

14. If $P = \{a, b, c\}$ and $Q = \{r\}$, form the sets $P \times Q$ and $Q \times P$. Are these two products equal?

Sol: i) $P \times Q = \{a, b, c\} \times \{r\} = \{(a, r), (b, r), (c, r)\}$

ii) $Q \times P = \{r\} \times \{a, b, c\} = \{(r, a), (r, b), (r, c)\}$

Here, $P \times Q \neq Q \times P$ [$\because$ the order in the elements of ordered pairs is reversed]

15. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Sol: Number of revolutions made by the wheel in 1 minute = 60 seconds = 360

Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

We know 1 revolution = $360^\circ = 2\pi$ rad

$\therefore$ 6 revolutions = $6 \times 2\pi = 12\pi$ rad

Number of radians turned in one second = 12π .

16. Express the complex numbers $\left(\frac{1}{3} + 3i\right)^3$ in the form $a + ib$.

Sol: $G.E = \left(\frac{1}{3} + 3i\right)^3 = \left(\frac{1}{3}\right)^3 + \cancel{3}\left(\frac{1}{3}\right)^2(\cancel{3}i) + \cancel{3}\left(\frac{1}{3}\right)(\cancel{3}i)^2 + (3i)^3$ [$\because (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]
 $= \frac{1}{27} + i + 9i^2 + 27i^3 = \frac{1}{27} + i - 9 - 27i$ [$\because i^3 = -i; i^2 = -1$]
 $= \left(\frac{1}{27} - 9\right) - 26i = \left(\frac{1 - 243}{27}\right) - 26i = \frac{-242}{27} - 26i$

17. How many 4-digit numbers can be formed by using the digits 1 to 9 if repetition of digits is not allowed?

Sol: The required number of 4 digit number that can be formed form 1 to 9 without repetition

$$= {}^9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \times 8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}} = 9 \times 8 \times 7 \times 6 = 3,024. \quad \left[\because {}^nP_r = \frac{n!}{(n-r)!} \right]$$

18. Expand the expression $\left(\frac{2p}{5} - \frac{3q}{7}\right)^6$

Sol: $\left(\frac{2p}{5} - \frac{3q}{7}\right)^6 = {}^6C_0 \left(\frac{2p}{5}\right)^6 + {}^6C_1 \left(\frac{2p}{5}\right)^5 \left(\frac{-3q}{7}\right)^1 + \dots + (-1)^r {}^6C_r \left(\frac{2p}{5}\right)^{6-r} \left(\frac{3q}{7}\right)^r + \dots + {}^6C_6 \left(\frac{-3q}{7}\right)^6$
 $= \sum_{r=0}^6 (-1)^r {}^6C_r \left(\frac{2p}{5}\right)^{6-r} \left(\frac{3q}{7}\right)^r$

19. Find the equation of the parabola that satisfies the following conditions:

Vertex (0, 0) passing through (2, 3) and axis is along x-axis

Sol: The vertex is (0, 0) and the axis of the parabola is the x-axis.

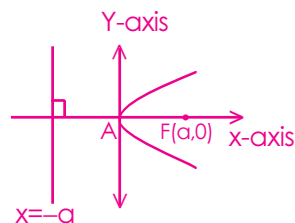
The parabola passes through point (2, 3), which lies in the first quadrant.

Hence the parabola is rightside horizontal.

∴ the equation of the parabola is of the form $y^2 = 4ax$.

The point (2, 3) should satisfy the equation $y^2 = 4ax$. ∴ $3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$

∴ the equation of the parabola is $y^2 = 4\left(\frac{9}{8}\right)x \Rightarrow y^2 = \frac{9}{2}x \Rightarrow 2y^2 = 9x$



20. Find 'x' if the distance between the points (5, -1, 7) and (x, 5, 1) is 9 units.

Sol: Let A = (5, -1, 7) and B = (x, 5, 1)

$$\text{Given that } AB = 9 \Rightarrow \sqrt{(5-x)^2 + (-1-5)^2 + (7-1)^2} = 9$$

$$\text{Squaring both sides we have } (5-x)^2 + 36 + 36 = 81 \Rightarrow (5-x)^2 + 72 = 81 \Rightarrow (5-x)^2 = 9$$

$$\Rightarrow 5-x = \pm 3 \Rightarrow x = 5 \pm 3 \Rightarrow x = 8 \text{ or } 2$$

21. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

Sol: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - 1)(\sqrt{1+x} + 1)}{x(\sqrt{1+x} + 1)}$

$$= \lim_{x \rightarrow 0} \frac{1+x-1}{x(\sqrt{1+x} + 1)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x} + 1)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1} = \frac{1}{\sqrt{1+0} + 1} = \frac{1}{2}$$

22. Find the mean deviation about the mean for 4, 7, 8, 9, 10, 12, 13, 17.

Sol: • The given data is 4, 7, 8, 9, 10, 12, 13, 17.

Here n = 8

• Mean $\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$

• The deviations ($x_i - \bar{x}$) of the observations from the mean ($\bar{x}$) are

$$4-10 = -6; 7-10 = -3; 8-10 = -2; 9-10 = -1; 10-10 = 0; 12-10 = 2; 13-10 = 3; 17-10 = 7$$

• The absolute values of the deviations are 6, 3, 2, 1, 0, 2, 3, 7

• The mean deviation about mean is

$$M.D(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{8} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$$

SECTION-C

23. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$.

Find A' , B' , $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

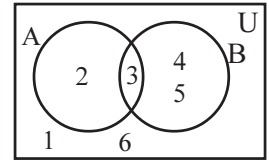
Sol: Given $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$.

$A' = \{1, 4, 5, 6\}$, $B' = \{1, 2, 6\}$. Hence, $A' \cap B' = \{1, 6\}$ (i)

Now, $A \cup B = \{2, 3\} \cup \{3, 4, 5\} = \{2, 3, 4, 5\}$;

$(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4, 5\} = \{1, 6\}$(ii)

$\therefore$ From (i) & (ii), $(A \cup B)' = \{1, 6\} = A' \cap B'$



24. Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by $R = \{(x, y) : y = x + 1\}$

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, codomain and range of R .

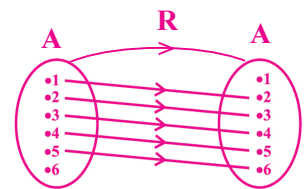
Sol: Given $R : A \rightarrow A$

i) $A = \{1, 2, 3, 4, 5, 6\}$ and elements in the range follow $y = x + 1$

$\therefore$ Range = $\{1+1, 2+1, 3+1, 4+1, 5+1, 6+1(\times)\} = \{2, 3, 4, 5, 6\}$

$\therefore R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

The arrow diagram of R is shown here.



ii) We have $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

The domain = {Set of first elements} = $\{1, 2, 3, 4, 5\}$

The range = {Set of second elements} = $\{2, 3, 4, 5, 6\}$

The codomain = $\{1, 2, 3, 4, 5, 6\}$. [$\because R : A \rightarrow A$]

25. Show that the four points in the Argand plane represented by the complex numbers $2+i$, $4+3i$, $2+5i$, $3i$ are the vertices of a square.

Sol: Let the given points are taken as $A(2, 1)$, $B(4, 3)$, $C(2, 5)$, $D(0, 3)$

$$AB = \sqrt{(2-4)^2 + (1-3)^2} = \sqrt{8}; \quad BC = \sqrt{(4-2)^2 + (3-5)^2} = \sqrt{8}$$

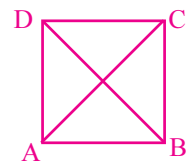
$$CD = \sqrt{(2-0)^2 + (5-3)^2} = \sqrt{8}; \quad DA = \sqrt{(2-0)^2 + (1-3)^2} = \sqrt{8}$$

Hence, the four sides AB, BC, CD, DA are equal.

$$AC = \sqrt{(2-2)^2 + (1-5)^2} = \sqrt{16} = 4; \quad BD = \sqrt{(4-0)^2 + (3-3)^2} = \sqrt{16} = 4$$

The two diagonals AC, BD are equal.

$\therefore A, B, C, D$ form a square.



26. A solution is to be kept between 68°F and 77°F . What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by

$$F = \frac{9}{5}C + 32?$$

Sol: The solution has to be kept between 68°F and $77^{\circ}\text{F} \Rightarrow 68 < F < 77$

$$\text{Also we have } F = \frac{9}{5}C + 32. \text{ Here } 68 < \left(\frac{9}{5}C + 32\right) < 77 \Rightarrow 68 - 32 < \frac{9}{5}C < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45 \Rightarrow 36\left(\frac{5}{9}\right) < C < 45\left(\frac{5}{9}\right) \Rightarrow 20 < C < 25$$

Thus, the required range of temperature in degree Celsius is between 20°C and 25°C .

27. Prove that for $3 \leq r \leq n$; ${}^{(n-3)}C_r + 3 \cdot {}^{(n-3)}C_{r-1} + 3 \cdot {}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} = {}^nC_r$

Sol: We know that ${}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$

$$\text{L.H.S} = {}^{(n-3)}C_r + 3 \cdot {}^{(n-3)}C_{r-1} + 3 \cdot {}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} \quad [\text{On rewriting the terms}]$$

$$= \left[{}^{(n-3)}C_r + {}^{(n-3)}C_{r-1} \right] + 2 \left[{}^{(n-3)}C_{r-1} + {}^{(n-3)}C_{r-2} \right] + \left[{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} \right]$$

$$= {}^{(n-3+1)}C_r + 2 \cdot {}^{(n-3+1)}C_{r-1} + {}^{(n-3+1)}C_{r-2} = {}^{(n-2)}C_r + 2 \cdot {}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}$$

$$= \left[{}^{(n-2)}C_r + {}^{(n-2)}C_{r-1} \right] + \left[{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2} \right]$$

$$= {}^{(n-2+1)}C_r + {}^{(n-2+1)}C_{r-1} = {}^{(n-1)}C_r + {}^{(n-1)}C_{r-1} = {}^{(n-1+1)}C_r = {}^nC_r = \text{R.H.S}$$

28. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} (2x)^n$. Where n is a positive integer.

Sol: Since the index $2n$ is even, the middle term of $(1+x)^{2n}$ is $T_{\frac{2n}{2}+1} = T_{n+1} = {}^{2n}C_n (x)^n = \frac{(2n)!}{n!n!} (x)^n$

$$= \frac{(2n)(2n-1)(2n-2)(2n-3) \cdots 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{n!n!} x^n = \frac{[2n(2n-2) \cdots 4 \cdot 2][(2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1]}{n!n!} x^n$$

$$= \frac{[2^n \cdot n(n-1) \cdots 2 \cdot 1][(2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1]}{n!n!} x^n$$

$$= \frac{n![(2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1]}{n!n!} x^n \cdot 2^n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-3)(2n-1)}{n!} (2x)^n$$

29. Find the equation of the straight line passing through A (1, 3) and

(i) parallel (ii) perpendicular to the straight line passing through B (3, -5) and C (-6, 1).

Sol: Slope of the line joining B(3, -5) and C(-6, 1) is $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-5 - 1}{3 + 6} = \frac{-6}{9} = \frac{-2}{3}$

i) Equation of the line parallel to BC passing through A(1, 3) with slope $-2/3$ is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 3 = \frac{-2}{3}(x - 1) \Rightarrow 3(y - 3) = -2(x - 1)$$

$$\Rightarrow 3y - 9 = -2x + 2 \Rightarrow 2x + 3y - 11 = 0$$

ii) Slope of line perpendicular to BC is $3/2$.

$$[\because m_1 m_2 = -1]$$

Equation of the line perpendicular to BC passing through A(1, 3) with slope $3/2$ is $y - 3 = \frac{3}{2}(x - 1)$

$$\Rightarrow 2(y - 3) = 3(x - 1) \Rightarrow 2y - 6 = 3x - 3$$

$$\Rightarrow 3x - 3 - 2y + 6 = 0 \Rightarrow 3x - 2y + 3 = 0$$

30. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis,

the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100} + \frac{y^2}{400} = 1$.

Sol: The denominator of $\frac{x^2}{100}$ is less than the denominator of $\frac{y^2}{400}$.

Hence, the ellipse is vertical.

$\therefore$ Equation of the ellipse is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$\text{Here, } b^2 = 100 \Rightarrow b = 10 \text{ and } a^2 = 400 \Rightarrow a = 20$$

$$\text{Now, } c^2 = a^2 - b^2 = 400 - 100 = 300 \Rightarrow c = 10\sqrt{3}$$

$$\text{i) Foci} = (0, \pm c) = (0, \pm 10\sqrt{3})$$

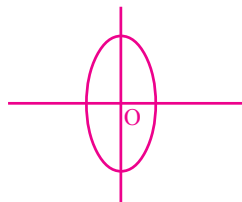
$$\text{ii) Vertices} = (0, \pm a) = (0, \pm 20)$$

$$\text{iii) Length of Major axis} = 2a = 2(20) = 40$$

$$\text{iv) Length of Minor axis} = 2b = 2(10) = 20$$

$$\text{v) Eccentricity } e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

$$\text{vi) Length of Latus rectum} = \frac{2b^2}{a} = \frac{2 \times 100}{20} = 10$$



31. Find the derivative of $\sin 2x$ from the first principle**Sol:** Let $f(x) = \sin 2x \Rightarrow f(x+h) = \sin 2(x+h) = \sin (2x+2h)$

$$\therefore f'(x) = \frac{f(x+h) - f(x)}{h}, \text{ [From the first principle]}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) - \sin 2x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(2 \cos \left(\frac{(2x+2h)+2x}{2} \right) \sin \left(\frac{(2x+2h)-2x}{2} \right) \right) \quad \left[\because \sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right) \right]$$

$$= 2 \lim_{h \rightarrow 0} \frac{1}{h} \cos \left(\frac{4x+2h}{2} \right) \sin \left(\frac{2h}{2} \right) = 2 \lim_{h \rightarrow 0} \frac{1}{h} \cos \left(\frac{2(2x+h)}{2} \right) \sin(h) = 2 \lim_{h \rightarrow 0} \frac{1}{h} \cos(2x+h) \sin(h)$$

$$= 2 \lim_{h \rightarrow 0} \cos(2x+h) \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} = 2 \cos(2x+0)(1) = 2 \cos 2x$$

 $\therefore$ derivative of $\sin 2x$ is $2 \cos 2x$ **32. A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.****Find (i) $P(A \cup B)$ (ii) $P(A' \cap B')$ (iii) $P(A \cap B')$ (iv) $P(B \cap A')$** **Sol:** It is given that $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$

$$\text{i) } P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.54 + 0.69 - 0.35 = 1.23 - 0.35 = 0.88$$

$$\text{ii) } P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.88 = 0.12$$

$$\text{iii) } P(A \cap B') = P(A) - P(A \cap B) = 0.54 - 0.35 = 0.19$$

$$\text{iv) } P(B \cap A') = P(B) - P(A \cap B) = 0.69 - 0.35 = 0.34.$$

SECTION-D

33. If the real functions f, g are defined by $f(x) = 2x - 1$ and $g(x) = x^2$ then find

i) $(3f-2g)(x)$ ii) $(fg)(x)$ iii) $\left(\frac{f}{g}\right)(x)$ iv) $(f+g+2)(x)$ v) $2f(x)$ vi) $2+f(x)$

Sol: i) $(3f-2g)(x) = 3f(x) - 2g(x) = 3(2x-1) - 2(x^2) = 6x - 3 - 2x^2.$

ii) $(fg)(x) = f(x).g(x) = (2x-1)x^2 = 2x^3 - x^2$

iii) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x-1}{x^2}, x \neq 0$

iv) $(f + g + 2)(x) = f(x) + g(x) + 2 = (2x-1) + x^2 + 2 = x^2 + 2x + 1 = (x+1)^2$

v) $2f(x) = 2(2x-1) = 4x - 2$

vi) $2 + f(x) = 2 + 2x - 1 = 2x + 1$

34. Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Sol: L.H.S = $\sin 2x + 2\sin 4x + \sin 6x = [\sin 2x + \sin 6x] + 2\sin 4x$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right) \right] + 2\sin 4x \quad \left[\because \sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \right]$$

$$= 2\sin 4x \cos(-2x) + 2\sin 4x = 2\sin 4x \cos 2x + 2\sin 4x = 2\sin 4x (\cos 2x + 1) \quad [\because \cos(-x) = \cos x]$$

$$= 2\sin 4x (2\cos^2 x) = 4\cos^2 x \sin 4x = \text{R.H.S} \quad [\because \cos 2x + 1 = 2\cos^2 x]$$

35. A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15th year since he deposited the amount and also calculate the total amount after 20 years.

Sol: Given that the man deposited Rs 10000 in a bank at the rate of 5% simple interest annually

$$= \frac{5}{100} \times \text{Rs } 10000 = \text{Rs } 500$$

$$\therefore \text{Interest in first year} = 10000 + \underbrace{500 + 500 + \dots + 500}_{14 \text{ times}}$$

$$\therefore \text{Amount in 15}^{\text{th}} \text{ year} = \text{Rs } 10000 + 14 \times \text{Rs } 500$$

$$= \text{Rs } 10000 + \text{Rs } 7000 = \text{Rs } 17000$$

$$\text{Amount after 20 years} = 10000 + \underbrace{500 + 500 + \dots + 500}_{20 \text{ times}}$$

$$= \text{Rs } 10000 + 20 \times \text{Rs } 500 = \text{Rs } 10000 + \text{Rs } 10000 = \text{Rs } 20000$$

36. A straight line through Q(2, 3) makes an angle of $3\pi/4$ with the negative direction of the X-axis. If the straight line intersects the line $x + y - 7 = 0$ at P, find the distance PQ.

Sol: Slope of the line, $m = \tan\left(\pi - \frac{3\pi}{4}\right) = 1$

Equation of the line passing through Q (2, 3) and having slope 1 is $y - 3 = 1(x - 2)$

$$\Rightarrow x - y + 1 = 0 \dots\dots\dots (1)$$

Given line is $x + y - 7 = 0 \dots\dots\dots (2)$

The point of intersection of (1) and (2) is (3, 4) $\Rightarrow P = (3, 4)$

$$\therefore \text{Distance } PQ = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}$$

37. Find the equation of the circle passing through (3,4), (3,2) and (1,4).

Sol: Let A(3,4), B(3,2), C(1,4)

Let S(h,k) be the centre of the circle $\Rightarrow SA = SB = SC$

Now, $SA = SB \Rightarrow SA^2 = SB^2$

$$\Rightarrow (\cancel{h-3})^2 + (k-4)^2 = (\cancel{h-3})^2 + (k-2)^2$$

$$\Rightarrow (k-4)^2 = (k-2)^2$$

$$\Rightarrow (k-4) = \pm(k-2) \Rightarrow (k-4) = (k-2) \text{ or } (k-4) = -(k-2)$$

Now, $k-4 = -(k-2) = -k+2 \Rightarrow 2k = 6 \Rightarrow k = 3$

Also, $SA = SC \Rightarrow SA^2 = SC^2$

$$\Rightarrow (h-3)^2 + (k-4)^2 = (h-1)^2 + (k-4)^2 \Rightarrow (h-3)^2 = (h-1)^2$$

$$\Rightarrow (h-3) = \pm(h-1) \Rightarrow (h-3) = (h-1) \text{ or } (h-3) = -(h-1)$$

Now, $h-3 = -(h-1) = -h+1 \Rightarrow 2h = 4 \Rightarrow h = 2$

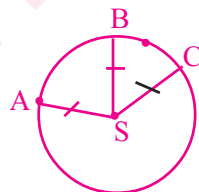
$\therefore$ Centre of circle S(h, k) = (2, 3). Also A = (3, 4)

Now, radius $r = SA \Rightarrow r^2 = SA^2 = (2-3)^2 + (3-4)^2 = 1+1 = 2$

Equation of the circle with centre (2, 3) and radius 2 is $(x-h)^2 + (y-k)^2 = r^2$

$$\Rightarrow (x-2)^2 + (y-3)^2 = 2 \Rightarrow (x^2 - 4x + 4) + (y^2 - 6y + 9) = 2$$

$$\Rightarrow x^2 + y^2 - 4x - 6y + 11 = 0$$



38. If $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1. \\ nx^3 + m, & x > 1 \end{cases}$

For what integers m and n does both $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$ exist?

Sol: The given function is $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1. \\ nx^3 + m, & x > 1 \end{cases}$

i) At $x=0$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (mx^2 + n) = m(0)^2 + n = n$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (nx + m) = n(0) + m = m$$

Thus, $\lim_{x \rightarrow 0} f(x)$ exist when $m=n$ (equal values of m, n .)

ii) At $x=1$

$$\text{L.H.L} = \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (nx + m) = n(1) + m = m+n$$

$$\text{R.H.L} = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (nx^3 + m) = n(1)^3 + m = m+n$$

Thus, $\lim_{x \rightarrow 1} f(x)$ exist when for all integral values of m, n .

39. Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Sol:

Class x_i	Frequency f_i	Mid point x_i	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	77.5	-3	9	-12	36
80-85	7	82.5	-2	4	-14	28
85-90	7	87.5	-1	1	-7	7
90-95	15	92.5(A)	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

- Let us take the assumed mean as $A = 92.5$ (Any one of the middle values of x_i)
- h = Width of the class interval = $75 - 70 = 5$. Here $N = 60$

• Mean, $\bar{x} = A + \frac{\sum f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$

• Variance(σ^2) = $\frac{h^2}{N^2} \left[N \sum f_i y_i^2 - (\sum f_i y_i)^2 \right] = \frac{(5)^2}{(60)^2} \left[60 \times 254 - (6)^2 \right]$

$$= \frac{25}{3600} (15204) = 105.58$$

40. a) In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.
- b) There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Sol: a) Let M be the event that a student studies Mathematics

Let B be the event that a student studies Biology

$$\text{Given that } P(M) = \frac{40}{100}, P(B) = \frac{30}{100}, P(M \cap B) = \frac{10}{100}$$

$$\therefore P(M \text{ or } B) = P(M \cup B) = P(M) + P(B) - P(M \cap B) = \frac{40}{100} + \frac{30}{100} - \frac{10}{100} = \frac{60}{100} = 0.6$$

b) Number of men = 4, Number of women = 6

Total number of council members = 4 + 6 = 10

$$\text{Let E be the event of getting a woman member } \therefore P(E) = \frac{{}^6C_1}{{}^{10}C_1} = \frac{6}{10} = \frac{3}{5}$$