

# SHINING STARS'

## VSAQ STAR QUESTIONS PLUS

### SETS

1. Write the roster form of the set of all letters in the word TRIGONOMETRY
2. Make correct statements by filling with the symbols or in the blank spaces : **[BMP2]**  
 $\{x : x \text{ is a student of class XI of your school}\} \dots \{x : x \text{ student of your school}\}$
3. Let  $A = \{1, 2, \{3, 4\}, 5\}$ . Which of the following statements are incorrect and why?  
 \* (i)  $\{3, 4\} \subset A$     \* (ii)  $\{3, 4\} \in A$  **[MAR-26]**    \* (iii)  $\{\{3, 4\}\} \subset A$     (iv)  $1 \in A$   
 \* (v)  $1 \subset A$     (vi)  $\{1, 2, 5\} \subset A$     (vii)  $\{1, 2, 5\} \in A$     (viii)  $\{1, 2, 3\} \subset A$   
 \* (ix)  $\emptyset \in A$     (x)  $\emptyset \subset A$     (xi)  $\{\emptyset\} \subset A$
4. Find the union of each of the following pairs of sets :  
 \* (i)  $X = \{1, 3, 5\}$ ,  $Y = \{1, 2, 3\}$     (ii)  $A = \{a, e, i, o, u\}$ ,  $B = \{a, b, c\}$   
 (iii)  $A = \{x : x \text{ is a natural number and multiple of } 3\}$ ,  $B = \{x : x \text{ is a natural number less than } 6\}$   
 \* (iv)  $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$ ,  $B = \{x : x \text{ is a natural number and } 6 < x < 10\}$   
 (v)  $A = \{1, 2, 3\}$ ,  $B = \emptyset$

### RELATIONS AND FUNCTIONS

5. Define and find the domain and range of the Signum function.

### TRIGONOMETRIC FUNCTIONS

6. Find the radian  $-47^\circ 30'$
7. Find the degree measures corresponding to the following radian measures  $\left(\text{use } \pi = \frac{22}{7}\right)$   
 \* (i)  $11/16$     (ii)  $-4$

### PERMUTATION AND COMBINATION

8. Evaluate  $\frac{n!}{(n-r)!}$ , when  $n = 6, r = 2$
9. Find the value of  $n$  such that  $\frac{{}^n P_4}{{}^{n-1} P_4} = \frac{5}{3}$ ,  $n > 4$
10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

### STRAIGHT LINES

11. Find the condition for the points  $(a, 0)$ ,  $(h, k)$  and  $(0, b)$  where  $a, b \neq 0$  to be collinear.

**[BMP1]**

### CONIC SECTIONS

12. Find the equation of the circle with centre at  $(0, 0)$  and radius  $r$ .

### LIMITS & DERIVATIVES

13. Compute  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$

SETS

1. Write the roster form of the set of all letters in the word TRIGONOMETRY

**Sol:** In the word TRIGONOMETRY, the letters T, R, O are repeated.

Dropping these repetitions, in roster form,  $E = \{ T, R, I, G, O, N, M, E, Y \}$

2. Make correct statements by filling with the symbols or in the blank spaces : **[BMP2]**  
 $\{x : x \text{ is a student of class XI of your school}\} \dots \{x : x \text{ student of your school}\}$

**Sol:**  $\{x : x \text{ is a student of class XI of your school}\} \subset \{x : x \text{ student of your school}\}$

3. Let  $A = \{ 1, 2, \{ 3, 4 \}, 5 \}$ . Which of the following statements are incorrect and why?

\* (i)  $\{3, 4\} \subset A$  \* (ii)  $\{3, 4\} \in A$  **[MAR-26]** \* (iii)  $\{\{3, 4\}\} \subset A$  (iv)  $1 \in A$

\* (v)  $1 \subset A$  (vi)  $\{1, 2, 5\} \subset A$  (vii)  $\{1, 2, 5\} \in A$  (viii)  $\{1, 2, 3\} \subset A$

\* (ix)  $\emptyset \in A$  (x)  $\emptyset \subset A$  (xi)  $\{\emptyset\} \subset A$

**Sol:** i) **False.** [Reason: We treat  $\{3, 4\}$  as an element of  $A$ .  $\therefore$  We write  $\{3, 4\} \in A$  but not  $\{3, 4\} \subset A$ ]

ii) **True.** [Reason:  $\{3, 4\}$  is an element of  $A$ .]

iii) **True.** [Reason:  $\{3, 4\} \in A \therefore \{\{3, 4\}\} \subset A$ .]

iv) **True.** [Reason: 1 is an element of  $A$ .]

v) **False.** [Reason: 1 is not a set. Only a set can be a subset of another set.]

vi) **True.** [Reason: 1, 2, 5 are elements of  $A$ .]

vii) **False.** [Reason:  $\{1, 2, 5\}$  is not an element of  $A$ .]

viii) **False.** [Reason:  $3 \notin A$ .]

ix) **False.** [Reason:  $\emptyset$  is not an element of  $A$ .]

x) **True.** [Reason:  $\emptyset$  is a subset of every set.]

xi) **False.** [Reason:  $\emptyset \subset A$  is true but  $\{\emptyset\} \subset A$  is false.  $\therefore \{\emptyset\}$  is set element with  $\emptyset$  as its member.

☞ Observe that in the given question it was asked to find the incorrect statements but not the correct statements. So, here the answer **true** is equivalent to **not incorrect**.

4. Find the union of each of the following pairs of sets :

\* (i)  $X = \{1, 3, 5\}$ ,  $Y = \{1, 2, 3\}$  (ii)  $A = \{a, e, i, o, u\}$ ,  $B = \{a, b, c\}$

(iii)  $A = \{x : x \text{ is a natural number and multiple of } 3\}$ ,  $B = \{x : x \text{ is a natural number less than } 6\}$

\* (iv)  $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$ ,  $B = \{x : x \text{ is a natural number and } 6 < x < 10\}$

(v)  $A = \{1, 2, 3\}$ ,  $B = \emptyset$

**Sol:** i)  $X \cup Y = \{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$ .

ii)  $A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\} = \{a, b, c, e, i, o, u\}$

- iii)  $A = \text{Set of natural number and multiple of } 3 = \{3, 6, 9, 12, \dots\}$   
 $B = \text{Set of natural number less than } 6 = \{1, 2, 3, 4, 5\}$   
 $\Rightarrow A \cup B = \{3, 6, 9, 12, \dots\} \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6, 9, 12, \dots\}$
- iv)  $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\} \Rightarrow A = \{2, 3, 4, 5, 6\}$   
 $B = \{x : x \text{ is a natural number and } 6 < x < 10\} \Rightarrow B = \{7, 8, 9\}$   
 $\Rightarrow A \cup B = \{2, 3, 4, 5, 6\} \cup \{7, 8, 9\} = \{2, 3, 4, 5, 6, 7, 8, 9\}$
- v)  $A \cup B = \{1, 2, 3\} \cup \emptyset = \{1, 2, 3\} = A$

## RELATIONS AND FUNCTIONS

5. Define and find the domain and range of the Signum function.

**Sol:** The signum function  $f(x)$  is defined as follows:

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \quad (\text{or}) \quad f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$

The **domain** of signum function is  $\mathbf{R}$  and its **Range** is  $\{-1, 0, 1\}$

$$\text{Ex : } f(5) = \frac{|5|}{5} = 1; \quad f(0) = 0; \quad f(-5) = \frac{|-5|}{-5} = \frac{5}{-5} = -1$$

## TRIGONOMETRIC FUNCTIONS

6. Find the radian  $-47^\circ 30'$

**Sol:** We know  $180^\circ = \pi \text{ rad} \Rightarrow 1^\circ = \frac{\pi}{180} \text{ rad}$

$$-47^\circ 30' = -47 \frac{1^\circ}{2} = -\frac{95}{2} \times \frac{\pi}{180} \text{ rad} = -\frac{19\pi}{72} \text{ rad} \quad \left( \because 30' = \left( \frac{30}{60} \right)^\circ = \frac{1}{2}^\circ \right)$$

7. Find the degree measures corresponding to the following radian measures (use  $\pi = \frac{22}{7}$ )

**\* (i)**  $\frac{11}{16}$       **(ii)**  $-4$

**Sol:** We know that  $\pi \text{ rad} = 180^\circ \Rightarrow 1 \text{ rad} = \frac{180^\circ}{\pi}$

$$\text{i) } \frac{11}{16} \text{ rad} = \left( \frac{11}{16} \times \frac{180}{\pi} \right)^\circ = \left( \frac{45 \times 11}{4 \times \pi} \right)^\circ = \left( \frac{45 \times 11 \times 7}{22 \times 4} \right)^\circ = \left( \frac{315}{8} \right)^\circ = \left( 39 \frac{3}{8} \right)^\circ$$

$$\text{ii) } -4 \text{ rad} = \left( (-4) \times \frac{180}{\pi} \right)^\circ = \left( \frac{7(-4) \times 180}{22} \right)^\circ = \left( \frac{-2520}{11} \right)^\circ = \left( -229 \frac{1}{11} \right)^\circ$$

**PERMUTATION AND COMBINATION**

8. Evaluate  $\frac{n!}{(n-r)!}$ , when  $n = 6, r = 2$

**Sol:** When  $n = 6, r = 2$ :  $\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 30$

9. Find the value of  $n$  such that  $\frac{{}^n P_4}{{}^{n-1} P_4} = \frac{5}{3}, n > 4$

**Sol:** (ii) Given that  $\frac{{}^n P_4}{{}^{n-1} P_4} = \frac{5}{3} \Rightarrow 3 {}^n P_4 = 5 ({}^{n-1} P_4)$

$$\Rightarrow 3n \frac{(n-1)(n-2)(n-3)}{\cancel{(n-4)}} = 5 \frac{(n-1)(n-2)(n-3)}{\cancel{(n-4)}} (n-4) \quad [\text{as } (n-1)(n-2)(n-3) \neq 0, n > 4]$$

$$\Rightarrow 3n = 5(n-4) \Rightarrow 3n = 5n - 20 \Rightarrow 2n = 20 \Rightarrow n = 10.$$

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

**Sol:** The word MISSISSIPPI contains 11 letters.

Here 4 I's are alike, 4 S's are alike and 2 P's are alike and M appears just once.

$\therefore$  The number of distinct permutations of the letters in the given word

$$= \frac{11!}{4!4!2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!} \times (4 \times 3 \times 2 \times 1)(2 \times 1)} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2 \times 1} = 34,650$$

There are 4 I's in the given word. When they occur together, they are treated as a single object

IIII

This single object together with the remaining 7 objects will account for 8 objects.

Number of ways that these 8 objects in which there are 4S's and 2P's

can be arranged is  $\frac{8!}{4!2!} = \frac{8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!} \times 2} = 840$

Number of arrangements where all I's occur together = 840

$\therefore$  The required number of ways that 4 I's do not come together =  $34,650 - 840 = 33,810$

STRAIGHT LINES

11. Find the condition for the points  $(a, 0)$ ,  $(h, k)$  and  $(0, b)$  where  $a, b \neq 0$  to be collinear.

[BMP1]

**Sol:** Equation of the line passing through  $(a, 0)$  and  $(0, b)$  in the intercept lines is  $\frac{x}{a} + \frac{y}{b} = 1$

If the line passes through the point  $(h, k)$  then  $\frac{h}{a} + \frac{k}{b} = 1 \Rightarrow \frac{hb + ka}{ab} = 1 \Rightarrow hb + ka - ab = 0$

CONIC SECTIONS

12. Find an equation of the circle with centre at  $(0, 0)$  and radius  $r$ .

**Sol:** Given centre  $(h, k) = (0, 0)$  and radius  $= r$ .

The equation of the circle is  $(x - h)^2 + (y - k)^2 = r^2$

$$\Rightarrow (x - 0)^2 + (y - 0)^2 = r^2 \Rightarrow x^2 + y^2 = r^2.$$

LIMITS & DERIVATIVES

13. Compute  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$

**Sol:**  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = \lim_{3x \rightarrow 0} \frac{e^{3x} - 1}{3x} \times 3 = (1)3 = 3$

$$\left( \because \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right)$$