

VSAQ PQ QUESTIONS

SETS

1. List all the subsets of the set $\{-1, 0, 1\}$.
2. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Find $V - B$ and $B - V$.

RELATIONS AND FUNCTIONS

3. If $A = \{-1, 1\}$, find $A \times A \times A$.

TRIGONOMETRIC FUNCTIONS

4. Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

COMPLEX NUMBERS

5. Express $(5 - 3i)^3$ in the form $a + ib$.

PERMUTATIONS AND COMBINATIONS

6. Find the value of n such that ${}^n P_5 = 42 {}^n P_3$, $n > 4$ [Ans: $n = 10$]
7. If $(n+1)P_5 : nP_6 = 2:7$, find n .
8. Find the no. of ways of arranging the letters of the word COMBINATION
9. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$ then find r

BINOMIAL THEOREM

10. Expand the expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

SEQUENCES & SERIES

11. Which term of the $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729?

CONIC SETCTIONS

12. Find the equation of the parabola with vertex $(0, 0)$; focus $(0, 2)$
13. Find the equation of the parabola with focus $(6, 0)$; directrix $x = -6$
14. Find the equation of the parabola with Focus $(0, -3)$; directrix $y = 3$
15. Find the coordinates of the focus, axis of the parabola $y^2 = 8x$.
16. Find the coordinates of the focus and the length of the latus rectum for $x^2 = 6y$
17. Find the length of transverse axis of hyperbola $x^2 - 4y^2 = 4$

3D-GEOMETRY

18. Find the equation of the set of the points P such that its distances from the points A (3, 4, -5) and B (-2, 1, 4) are equal.

LIMITS AND CONTINUITY

19. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x}$

STATISTICS

20. Find the mean deviation about the mean for 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5
21. Find the mean and variance for first 10 multiples of 3. **[BMP2]**

VSAQ PQ SOLUTIONS

SETS

1. List all the subsets of the set $\{-1, 0, 1\}$.

Sol: Let $A = \{-1, 0, 1\}$.

i) The subset of A having 'no element' is the empty set $\emptyset$.

ii) The subsets of A having 'one element' are $\{-1\}, \{0\}, \{1\}$.

iii) The subsets of A having 'two elements' are $\{-1, 0\}, \{-1, 1\}, \{0, 1\}$.

iv) The subset of A having 'three elements' of A is A itself.

$\therefore$ List of all the subsets of $A = \{\emptyset, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}, \{0, 1\}, \{-1, 0, 1\}\}$.

2. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Find $V - B$ and $B - V$.

Sol: Given $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$

$$V - B = \{\cancel{a}, e, i, \cancel{o}, u\} - \{a, i, k, u\} = \{e, o\};$$

$$B - V = \{\cancel{a}, \cancel{i}, k, \cancel{u}\} - \{a, e, i, o, u\} = \{k\}$$

RELATIONS AND FUNCTIONS

3. If $A = \{-1, 1\}$, find $A \times A \times A$.

[MAY-26]

Sol: $A \times A = \{-1, 1\} \times \{-1, 1\} = \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

$$\therefore A \times A \times A = \{-1, 1\} \times \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$$

$$= \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}.$$

TRIGONOMETRIC FUNCTIONS

4. Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Sol: L.H.S = $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2 \sin\left(\frac{5x+3x}{2}\right) \cos\left(\frac{5x-3x}{2}\right)}{2 \cos\left(\frac{5x+3x}{2}\right) \cos\left(\frac{5x-3x}{2}\right)} = \frac{\cancel{2} \sin 4x \cdot \cancel{\cos x}}{\cancel{2} \cos 4x \cdot \cancel{\cos x}} = \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S}$$

COMPLEX NUMBERS

5. Express $(5 - 3i)^3$ in the form $a + ib$.

Sol: $(5 - 3i)^3 = 5^3 - 3(5^2)(3i) + 3(5)(3i)^2 - (3i)^3$
 $= 125 - 225i - 135 + 27i = -10 - 198i.$

PERMUTATIONS AND COMBINATIONS

6. Find the value of n such that (i) ${}^nP_5 = 42$ nP_3 , $n > 4$ (ii) $\frac{{}^nP_4}{{}^{n-1}P_4} = \frac{5}{3}$, $n > 4$

Sol: (i) Given that ${}^nP_5 = 42$ nP_3
 $\Rightarrow n \cancel{(n-1)} \cancel{(n-2)} (n-3)(n-4) = 42 n \cancel{(n-1)} \cancel{(n-2)}$
 $\Rightarrow (n-3)(n-4) = 42 \Rightarrow n^2 - 7n - 30 = 0 \Rightarrow n^2 - 10n + 3n - 30 \Rightarrow (n-10)(n+3) = 0$
 $\Rightarrow n-10 = 0 \Rightarrow n+3 = 0 \Rightarrow n = 10$ or $n = -3 \Rightarrow n = 10$ (As n cannot be negative)

7. If $(n+1)P_5 : {}^nP_6 = 2:7$, find n .

Sol: Given $(n+1)P_5 : {}^nP_6 = 2 : 7 \Rightarrow 2 {}^nP_6 = 7(n+1)P_5$
 $2n(n-1)(n-2)(n-3)(n-4)(n-5) = 7(n+1)n(n-1)(n-2)(n-3)$
 $\Rightarrow 2(n-4)(n-5) = 7(n+1) \Rightarrow 2(n^2 - 9n + 20) = 7n + 7 \Rightarrow 2n^2 - 18n + 40 = 7n + 7$
 $\Rightarrow 2n^2 - 25n + 33 = 0 \Rightarrow 2n^2 - 22n - 3n + 33 = 0 \Rightarrow 2n(n-11) - 3(n-11) = 0$
 $\Rightarrow (n-11)(2n-3) = 0 \Rightarrow n = 11$ (As n cannot be a fraction)

8. Find the number of ways of arranging the letters of the word COMBINATION

Sol: The given word COMBINATION contains 11 letters in which 2 'O's are alike,
 2 'I's are alike, 2 'N's are alike and rest are different.

$$\therefore \text{The required number of arrangements} = \frac{n!}{p!q!r!} = \frac{11!}{2!2!2!}$$

9. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$ then find r

Sol: ${}^{15}C_{2r-1} = {}^{15}C_{2r+4} \Rightarrow (2r-1) + (2r+4) = 15$ ($\because {}^nC_r = {}^nC_s \Rightarrow r+s=n$ (or) $r=s$)
 $\Rightarrow 4r+3 = 15 \Rightarrow 4r = 12 \Rightarrow r = 3$

BINOMIAL THEOREM

10. Expand the expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Sol: From the Binomial theorem we have,

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_n b^n$$

$$\begin{aligned} \left(\frac{2}{x} - \frac{x}{2}\right)^5 &= {}^5C_0 \left(\frac{2}{x}\right)^5 - {}^5C_1 \left(\frac{2}{x}\right)^4 \left(\frac{x}{2}\right) + {}^5C_2 \left(\frac{2}{x}\right)^3 \left(\frac{x}{2}\right)^2 - {}^5C_3 \left(\frac{2}{x}\right)^2 \left(\frac{x}{2}\right)^3 + {}^5C_4 \left(\frac{2}{x}\right) \left(\frac{x}{2}\right)^4 - {}^5C_5 \left(\frac{x}{2}\right)^5 \\ &= \frac{32}{x^5} - 5 \left(\frac{16}{x^4}\right) \left(\frac{x}{2}\right) + 10 \left(\frac{8}{x^3}\right) \left(\frac{x^2}{4}\right) - 10 \left(\frac{4}{x^2}\right) \left(\frac{x^3}{8}\right) + 5 \left(\frac{2}{x}\right) \left(\frac{x^4}{16}\right) - \frac{x^5}{32} \\ &= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32} \end{aligned}$$

SEQUENCES & SERIES

11. Which term of the $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729?

Sol: The given GP is $\sqrt{3}, 3, 3\sqrt{3}, \dots$ Here $a = \sqrt{3}$ and $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Also n^{th} term is $a_n = 729$

$$a_n = ar^{n-1} \Rightarrow (\sqrt{3})(\sqrt{3})^{n-1} = 729 \Rightarrow (3)^{\frac{1}{2}} (3)^{\frac{n-1}{2}} = 3^6 \Rightarrow (3)^{\frac{1}{2} + \frac{n-1}{2}} = 3^6$$

$$\Rightarrow \frac{1}{2} + \frac{n-1}{2} = 6 \Rightarrow \frac{1+n-1}{2} = 6 \Rightarrow \frac{n}{2} = 6 \Rightarrow n = 12$$

$\therefore$ the required one is the 12th term.

CONIC SECTIONS

12. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).

Sol: Given vertex is at (0,0) and the focus is (0,a) = (0,2). Hence $a=2$.

Focus lies on the y-axis. Hence the parabola is upward vertical.

$$\therefore \text{Equation of the parabola is } x^2 = 4ay = 4(2)y \Rightarrow x^2 = 8y.$$

13. Find the equation of the parabola that satisfies the following conditions:

Focus (6, 0); directrix $x = -6$

Sol: Given focus $F(a, 0) = (6, 0)$ and directrix is $x = -6$. Hence we get $a = 6$

Focus (6, 0) lies on the positive x-axis and the

directrix $x = -6$ is to the left of y-axis.

Hence the parabola is rightside horizontal .

$$\therefore \text{ the equation of the parabola is } y^2 = 4ax = 4(6)x = 24x \Rightarrow y^2 = 24x$$

14. Find the equation of the parabola that satisfies the following conditions:

Focus (0, -3); directrix $y = 3$

Sol: Given focus $F(0, -a) = (0, -3)$ and directrix is $y = 3$. Hence we get $a = 3$

Focus (0, -3) lies on the negative y-axis and

the directrix $y = 3$ is to the right of the y-axis.

Hence the parabola is downward parabola.

$$\therefore \text{ the equation of the parabola is } x^2 = -4ay = -4(3)y = -12y \Rightarrow x^2 = -12y$$

15. Find the coordinates of the focus, axis of the parabola $y^2 = 8x$.

Sol: Given parabola is $y^2 = 8x$. It is a right side horizontal parabola.

Comparing $y^2 = 8x$ with $y^2 = 4ax$, we get $4a = 8 \Rightarrow a = 2$

(i) Focus $F(a, 0) = (2, 0)$

(ii) Axis of the parabola is X-axis. Its equation is $y = 0$.

16. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$

Sol: Given parabola is $x^2 = 6y$. It is a vertical upward parabola.

Comparing $x^2 = 6y$ with $x^2 = 4ay$, we get $4a = 6 \Rightarrow a = \frac{3}{2}$

(i) Coordinates of Focus $F(0, a) = \left(0, \frac{3}{2}\right)$

(ii) Length of latus rectum $= 4a = 4\left(\frac{3}{2}\right) = 6$

17. Find the length of transverse axis of hyperbola $x^2 - 4y^2 = 4$ [MAR-26]

Sol: Given equation $x^2 - 4y^2 = 4 \Rightarrow \frac{x^2}{4} - \frac{4y^2}{1} = 1 \Rightarrow \frac{x^2}{4} - \frac{y^2}{1/4} = 1$.

This is in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the hyperbola with foci on x-axis.

Here $a^2 = 4 \Rightarrow a = 2$, $b^2 = \frac{1}{4} \Rightarrow b = \frac{1}{2}$

Length of transverse axis $2a = 2(2) = 4$

3D-GEOMETRY

18. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, -1).

Sol: Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, -1).

$$\Rightarrow PA = PB \Rightarrow PA^2 = PB^2 \Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

$$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14 \Rightarrow -2x - 6z + 6x - 2z = 0 \Rightarrow 4x - 8z = 0 \Rightarrow x - 2z = 0$$

Thus, the required equation is $x - 2z = 0$.

LIMITS AND CONTINUITY

19. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x}$

Sol:
$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x} = \lim_{x \rightarrow 0} \left(\frac{\frac{\sin 4x}{x}}{\frac{\sin 2x}{x}} \right) = \frac{\lim_{x \rightarrow 0} \frac{\sin 4x}{x}}{\lim_{x \rightarrow 0} \frac{\sin 2x}{x}} = \frac{4}{2} = 2 \quad \left(\because \lim_{x \rightarrow 0} \frac{\sin kx}{x} = k \right)$$

STATISTICS

20. Find the mean deviation about the mean for 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5

Sol: • The given data is 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5. Here $n=20$

• Mean $\bar{x} = \frac{12+3+18+17+4+9+17+19+20+15+8+17+2+3+16+11+3+1+0+5}{20} = \frac{200}{20} = 10$

• The deviations $(x_i - \bar{x})$ of the observations from the mean $(\bar{x})$ are

$$12-10 = -2; 3-10 = -7; 18-10 = 8; 17-10 = 7; 4-10 = -6; 9-10 = -1; 17-10 = 7; 19-10 = 9;$$

$$20-10 = 10; 15-10 = 5; 8-10 = -2; 17-10 = 7; 2-10 = -8; 3-10 = -7; 16-10 = 6; 11-10 = 1;$$

$$3-10=-7; 1-10=-9; 0-10=-10; 5-10=-5$$

- The absolute values of the deviations are 2, 7, 8, 7, 6, 1, 7, 9, 10, 5, 2, 7, 8, 7, 6, 1, 7, 9, 10, 5

$$M.D(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{20} = \frac{2+7+8+7+6+1+7+9+10+5+2+7+8+7+6+1+7+9+10+5}{20} = \frac{124}{20} = 6.2$$

21. Find the mean and variance for first 10 multiples of 3. [BMP2]

Sol: • The first 10 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

- Here, number of observations, $n=10$

- Mean $\bar{x} = \frac{\sum x_i}{n} = \frac{3+6+9+12+15+18+21+24+27+30}{10} = \frac{165}{10} = 16.5$

- The deviations $(x_i - \bar{x})$ of the observations from the mean $(\bar{x})$ are

$$3-16.5=-13.5; 6-16.5=-10.5; 9-16.5=-7.5; 12-16.5=-4.5; 15-16.5=-1.5;$$

$$18-16.5=1.5; 21-16.5=4.5; 24-16.5=7.5; 27-16.5=10.5; 30-16.5=13.5$$

- The absolute values of the deviations are 13.5, 10.5, 7.5, 4.5, 1.5, 1.5, 1.5, 7.5, 10.5, 13.5

- Variance $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$$= \frac{(13.5)^2 + (10.5)^2 + (7.5)^2 + (4.5)^2 + (1.5)^2 + (1.5)^2 + (4.5)^2 + (7.5)^2 + (10.5)^2 + (13.5)^2}{10}$$

$$= \frac{182.25 + 110.25 + 56.25 + 20.25 + 2.25 + 2.25 + 20.25 + 56.25 + 110.25 + 182.25}{10} = \frac{742.5}{10} = 74.25$$