

SHINING STARS'LAQ STAR QUESTIONS PLUSTRIGONOMETRIC FUNCTIONS

1. Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$
2. Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}$
3. Prove that $\tan 4x = \frac{4\tan x(1 - \tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$

SEQUENCES AND SERIES

4. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.
5. Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual instalment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

STRAIGHT LINES

6. Show that the straight lines $x + y = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form an isosceles triangle.
7. Show that the straight lines $x - 7y - 22 = 0$, $3x + 4y + 9 = 0$ and $7x + y - 54 = 0$ form a right angled isosceles triangle.
8. S.T the area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$ is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$
9. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.
10. A straight line meets the coordinate axes in A and B. Find the equation of the straight line, when i) $\overline{AB}$ is divided in the ratio 2:3 at (-5, 2)
ii) $\overline{AB}$ is divided in the ratio 1:2 at (-5, 4) iii) (p, q) bisects $\overline{AB}$

PROBABILITY

11. If A, B, C are three events associated with a random experiment, prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
12. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that (i) The student opted for NCC or NSS. (ii) The student has opted neither NCC nor NSS. (iii) The student has opted NSS but not NCC.
13. Check whether the following probabilities P(A) and P(B) are consistently defined
(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$ (ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$
14. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
(i) the digits are repeated? (ii) the repetition of digits is not allowed?

TRIGONOMETRIC FUNCTIONS

1. Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Sol: L.H.S = $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2 \sin \left(\frac{5x + 3x}{2} \right) \cos \left(\frac{5x - 3x}{2} \right)}{2 \cos \left(\frac{5x + 3x}{2} \right) \cos \left(\frac{5x - 3x}{2} \right)} = \frac{\cancel{2} \sin 4x \cdot \cancel{\cos x}}{\cancel{2} \cos 4x \cdot \cancel{\cos x}} = \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S}$$

2. Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$

Sol: L.H.S = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$
 $= \cos^2 x + \cos^2 y + 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y$
 $= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$
 $= 1 + 1 + 2 \cos(x + y) \quad \left[\because \cos(x + y) = (\cos x \cos y - \sin x \sin y) \right]$
 $= 2 + 2 \cos(x + y) = 2[1 + \cos(x + y)]$
 $= 2 \left[2 \cos^2 \left(\frac{x+y}{2} \right) \right] = 4 \cos^2 \left(\frac{x+y}{2} \right) = \text{R.H.S} \quad \left[\because 1 + \cos 2x = 2 \cos^2 x \right]$

3. Prove that $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Sol: L.H.S = $\tan 4x = \tan 2(2x)$

$$= \frac{2 \tan 2x}{1 - \tan^2(2x)} = \frac{2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2} \right]} = \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]} = \frac{4 \tan x}{\cancel{(1 - \tan^2 x)}} \times \frac{(1 - \tan^2 x)^2}{(1 - \tan^2 x)^2 - 4 \tan^2 x}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{(1 - 2 \tan^2 x + \tan^4 x) - 4 \tan^2 x} = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x} = \text{R.H.S} \left[\because (a - b)^2 = a^2 - 2ab + b^2 \right]$$

SEQUENCES AND SERIES

4. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Sol: Let a be the first term and r be the common ratio of the G.P.

$$\text{Now we have } a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$$

$$\text{The third term is greater than the first term by 9} \Rightarrow a_3 = a_1 + 9$$

$$\Rightarrow ar^2 = a + 9 \Rightarrow a(r^2 - 1) = 9 \dots\dots (i)$$

$$\text{The second term is greater than the 4th by 18} \Rightarrow a_2 = a_4 + 18$$

$$\Rightarrow ar = ar^3 + 18 \Rightarrow ar(1 - r^2) = 18 \Rightarrow ar(r^2 - 1) = -18 \dots\dots (ii)$$

$$\text{Dividing (ii) by (i), we get } \frac{ar(r^2 - 1)}{a(r^2 - 1)} = -\frac{18}{9} \Rightarrow r = -2$$

$$\text{Put } r = -2 \text{ in (i), we get } a[(-2)^2 - 1] = 9 \Rightarrow a(4 - 1) = 9 \Rightarrow 3a = 9 \quad \therefore a = 3$$

$\therefore$ the first four numbers of the G.P. are 3, $3(-2)$, $3(-2)^2$, $3(-2)^3$ i.e., 3, -6, 12, -24.

5. Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual instalment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Sol: Given that Shamshad Ali buys a scooter for Rs 22000 and pays Rs 4000 in cash.

$$\therefore \text{Unpaid amount} = \text{Rs } 22000 - \text{Rs } 4000 = \text{Rs } 18000$$

According to the given condition, the interest paid annually is 10% of 18000, 10% of 17000, 10% of 16000 ... 10% of 1000

Thus, total interest to be paid

$$= 10\% \text{ of } 18000 + 10\% \text{ of } 17000 + 10\% \text{ of } 16000 + \dots + 10\% \text{ of } 1000$$

$$= 10\% \text{ of } (18000 + 17000 + 16000 + \dots + 1000)$$

$$= 10\% \text{ of } (1000 + 2000 + 3000 + \dots + 18000)$$

$$= 10\% \text{ of } \frac{18}{2}(1000 + 18000) = 10\% \text{ of } 9(19000) = 10\% \text{ of } 171000 = 17100$$

$$\therefore \text{Cost of scooter} = \text{Rs } 22000 + \text{Rs } 17100 = \text{Rs } 39100$$

STRAIGHT LINES

6. Show that the straight lines $x + y = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form an isosceles triangle.

Sol: Given lines are $x + y = 0$ (1), $3x + y - 4 = 0$ (2), $x + 3y - 4 = 0$ (3)

If 'A' is the angle between (1),(2) then

$$\cos A = \frac{|a_1a_2 + b_1b_2|}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}} = \frac{|1(3) + 1(1)|}{\sqrt{(1^2 + 1^2)(3^2 + 1^2)}} = \frac{4}{\sqrt{(2)(10)}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow A = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

If 'B' is the angle between (2),(3) then

$$\cos B = \frac{|a_1a_2 + b_1b_2|}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}} = \frac{|3(1) + 1(3)|}{\sqrt{(3^2 + 1^2)(1^2 + 3^2)}} = \frac{6}{\sqrt{10}\sqrt{10}} = \frac{6}{10} = \frac{3}{5} \Rightarrow B = \cos^{-1}\left(\frac{3}{5}\right)$$

If 'C' is the angle between (1),(3) then

$$\cos C = \frac{|a_1a_2 + b_1b_2|}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}} = \frac{|1(1) + 1(3)|}{\sqrt{(1^2 + 1^2)1^2 + 3^2}} = \frac{4}{\sqrt{2}\sqrt{10}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow C = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

Here $A=C$. So the given lines form an Isosceles triangle.

7. Show that the straight lines $x - 7y - 22 = 0$, $3x + 4y + 9 = 0$ and $7x + y - 54 = 0$ form a right angled isosceles triangle.

Sol: Given lines are $x - 7y - 22 = 0$(i), $3x + 4y + 9 = 0$(ii) and $7x + y - 54 = 0$(iii)

The slope of line (i) is $m_1 = \frac{1}{7}$, slope of line (ii) is $m_2 = -\frac{3}{4}$, slope of line (iii) is $m_3 = -\frac{7}{1}$

Clearly $m_1m_3 = \left(\frac{1}{7}\right)\left(-\frac{7}{1}\right) = -1 \Rightarrow$ Angle between lines (i) & (iii) is 90° . (right angle)

If θ is acute angle between the lines (i) & (ii) then

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1m_2} \right| \Rightarrow \tan \theta = \left| \frac{\frac{1}{7} + \frac{3}{4}}{1 + \frac{1}{7}\left(-\frac{3}{4}\right)} \right| = \left| \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{3}{28}} \right| = \left| \frac{25}{25} \right| = 1$$

$$\Rightarrow \tan \theta = 1 = \tan 45^\circ \quad \therefore \theta = 45^\circ.$$

Similarly we get angle between (ii) and (iii) is also 45° .

Hence, given lines form a right angled isosceles triangle.

8. Show that the area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$

is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$

Sol: Given lines are

$$y = m_1x + c_1 \dots (i) \quad y = m_2x + c_2 \dots (ii) \quad x = 0 \text{ (y-axis)} \dots (iii)$$

Their points of intersection P, Q, R are shown in the diagram.

From (iii), put $x=0$ in (i) then we get

$$y = m_1(0) + c_1 \Rightarrow y = c_1 \quad \therefore P(0, c_1)$$

From (iii), put $x=0$ in (ii) then we get

$$y = m_2(0) + c_2 \Rightarrow y = c_2 \quad \therefore Q(0, c_2)$$

(i) & (ii) can be re-written as $m_1x - y + c_1 = 0$, $m_2x - y + c_2 = 0$

Solving these two equations by the rule of cross multiplication

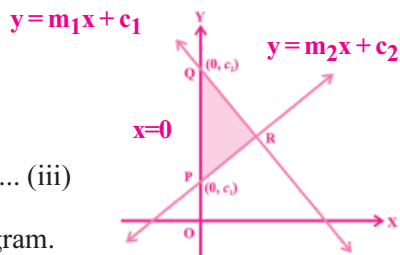
$$R = \left(\frac{(-1)c_2 + 1(c_1)}{m_1(-1) + m_2(1)}, \frac{c_1(m_2) - m_1(c_2)}{m_1(-1) + m_2(1)} \right) = R \left(\frac{c_1 - c_2}{m_2 - m_1}, \frac{c_1m_2 - c_2m_1}{m_2 - m_1} \right)$$

We know that area of $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is

$$\Delta = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\therefore \text{the area of } \triangle PQR \text{ with } P(0, c_1), Q(0, c_2), R \left(\frac{c_1 - c_2}{m_2 - m_1}, \frac{c_1m_2 - c_2m_1}{m_2 - m_1} \right)$$

$$= \frac{1}{2} \left| 0 \left(c_2 - \frac{c_1m_2 - m_1c_2}{m_2 - m_1} \right) + 0 \left(\frac{c_1m_2 - m_1c_2}{m_2 - m_1} - c_1 \right) + \frac{c_1 - c_2}{m_2 - m_1} (c_1 - c_2) \right| = \frac{(c_1 - c_2)^2}{2|m_1 - m_2|} \text{ Sq. units}$$



☺ Helicopter Shot



9. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Obtain its equation. **[BMP2]**

Sol: Given lines are $5x - y + 4 = 0 \dots (i)$ $3x + 4y - 4 = 0 \dots (ii)$

Let the required line intersects the lines (i) and (ii) at the points $A(\alpha_1, \beta_1)$ and $B(\alpha_2, \beta_2)$.

$$\Rightarrow 5\alpha_1 - \beta_1 + 4 = 0 \Rightarrow \beta_1 = 5\alpha_1 + 4 \text{ and } 3\alpha_2 + 4\beta_2 - 4 = 0 \Rightarrow \beta_2 = \frac{4 - 3\alpha_2}{4}$$

Also, the mid point of the segment of the

required line between (α_1, β_1) and (α_2, β_2) is $(1, 5)$.

$$\Rightarrow \left(\frac{\alpha_1 + \alpha_2}{2}, \frac{\beta_1 + \beta_2}{2} \right) = (1, 5) \Rightarrow \frac{\alpha_1 + \alpha_2}{2} = 1 \text{ and } \frac{\beta_1 + \beta_2}{2} = 5,$$

$$\Rightarrow \alpha_1 + \alpha_2 = 2 \text{ and } \frac{(5\alpha_1 + 4) + \left(\frac{4 - 3\alpha_2}{4} \right)}{2} = 5 \Rightarrow \frac{(20\alpha_1 + 16) + (4 - 3\alpha_2)}{4} = 10 \Rightarrow 20\alpha_1 - 3\alpha_2 = 40 - 20$$

$$\Rightarrow \alpha_1 + \alpha_2 - 2 = 0 \text{ and } 20\alpha_1 - 3\alpha_2 - 20 = 0$$

Solving these equations we get α_1 and α_2 ,

$$\left(\frac{1(-20) - 3(2)}{1(-3) - 20(1)}, \frac{(-2)(20) + 1(20)}{1(-3) - 20(1)} \right) = \left(\frac{-20 - 6}{-3 - 20}, \frac{-40 + 20}{-3 - 20} \right) = \left(\frac{-26}{-23}, \frac{-20}{-23} \right) = \left(\frac{26}{23}, \frac{20}{23} \right)$$

$$\therefore \alpha_1 = \frac{26}{23} \text{ and } \alpha_2 = \frac{20}{23} \text{ and } \because \beta_1 = 5\alpha_1 + 4 \text{ we have } \beta_1 = 5\left(\frac{26}{23}\right) + 4 = \frac{222}{23}$$

$$\text{Now slope of the line passing through } (1, 5) \text{ and } \left(\frac{26}{23}, \frac{222}{23} \right) \text{ is } \frac{\frac{222}{23} - 5}{\frac{26}{23} - 1} = \frac{222 - 115}{26 - 23} = \frac{107}{3}$$

$\therefore$ Equation of the required line passing through $(1, 5)$ with slope $\frac{107}{3}$ is

$$y - 5 = \frac{107}{3}(x - 1) \Rightarrow 3y - 15 = 107x - 107 \Rightarrow 107x - 3y - 92 = 0$$

10. A straight line meets the coordinate axes in A and B. Find the equation of the straight line, when i) $\overline{AB}$ is divided in the ratio 2:3 at $(-5, 2)$

ii) $\overline{AB}$ is divided in the ratio 1:2 at $(-5, 4)$ iii) (p, q) bisects $\overline{AB}$

Sol: i) Let the equation of the required line is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow A = (a, 0), B = (0, b)$

Let $P(-5, 2)$ divides $A(a, 0), B(0, b)$ in the ratio 2:3

$$\Rightarrow P = \left(\frac{2(0) + 3(a)}{2+3}, \frac{2(b) + 3(0)}{2+3} \right) = \left(\frac{3a}{5}, \frac{2b}{5} \right). \text{ But } P = (-5, 2)$$

$$\therefore \left(\frac{3a}{5}, \frac{2b}{5} \right) = (-5, 2) \Rightarrow a = -\frac{25}{3}, b = 5. \quad \therefore \text{The line is } \frac{x}{-25/3} + \frac{y}{5} = 1 \Rightarrow 3x - 5y + 25 = 0$$

ii) $\overline{AB}$ is divided in the ratio 1:2 at $(-5, 4)$

Let a, b be the intercepts of the line. $\therefore A = (a, 0), B = (0, b)$.

The point which divides $\overline{AB}$ in the ratio 1:2 is $\left(\frac{2a}{3}, \frac{b}{3} \right)$

$$\therefore \left(\frac{2a}{3}, \frac{b}{3} \right) = (-5, 4) \Rightarrow \frac{2a}{3} = -5, \frac{b}{3} = 4 \Rightarrow a = -\frac{15}{2}, b = 12$$

$$\text{Equation of the line is } \frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{-15/2} + \frac{y}{12} = 1 \Rightarrow -\frac{2x}{15} + \frac{y}{12} = 1$$

$$\Rightarrow -8x + 5y = 60 \Rightarrow 8x - 5y + 60 = 0$$

iii) (p, q) bisects $\overline{AB}$

Let a, b be the intercepts of the line. $\therefore A = (a, 0), B = (0, b)$

$$(p, q) \text{ bisects } \overline{AB} \Rightarrow (p, q) = \left(\frac{a}{2}, \frac{b}{2} \right) \Rightarrow \frac{a}{2} = p, \frac{b}{2} = q \Rightarrow a = 2p, b = 2q$$

$$\text{Equation of the line is } \frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{2p} + \frac{y}{2q} = 1 \Rightarrow \frac{x}{p} + \frac{y}{q} = 2$$

PROBABILITY

11. If A, B, C are three events associated with a random experiment, prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ - P(B \cap C) + P(A \cap B \cap C)$$

Sol: Put $B \cup C = E$ then

$$P(A \cup B \cup C) = P(A \cup E) = P(A) + P(E) - P(A \cap E) \dots \dots \dots (i) \text{ [From Addition Theorem]}$$

$$\text{Now } P(E) = P(B \cup C) = P(B) + P(C) - P(B \cap C) \dots \dots \dots (ii)$$

$$\text{Also } A \cap E = A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \text{[From the Laws of Sets]}$$

$$\text{Thus } P(A \cap E) = P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)] \text{ [From Addition Theorem]} \\ = P(A \cap B) + P(A \cap C) - P[A \cap B \cap C] \dots \dots \dots (iii)$$

Putting (ii) and (iii) in (i), we get

$$P[A \cup B \cup C] = P(A) + [P(B) + P(C) - P(B \cap C)] - [P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)] \\ = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

12. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that
- (i) The student opted for NCC or NSS.
 - (ii) The student has opted neither NCC nor NSS.
 - (iii) The student has opted NSS but not NCC.

Sol: Let E be the event that a student opted for NCC and F be the event of opting NSS

$$\text{Given } P(E) = \frac{30}{60}, P(F) = \frac{32}{60} \text{ and } P(E \cap F) = \frac{24}{60},$$

$$(i) P(\text{a student opted for NCC or NSS}) = P(E \text{ or } F) = P(E \cup F)$$

$$= P(E) + P(F) - P(E \cap F) = \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60} = \frac{19}{30}$$

$$(ii) P(\text{a student has opted neither NCC nor NSS}) = P(\text{neither } E \text{ nor } F) = P(E' \cap F')$$

$$= P(E \cup F)' \quad [\text{From De Morgan's Law}]$$

$$= 1 - P(E \cup F) = 1 - \frac{19}{30} = \frac{11}{30}$$

$$(iii) \quad P(\text{a student has opted NSS but not NCC}) = P(F \text{ and } E') = P(F \cap E')$$

$$= P(F) - P(E \cap F) = \frac{32}{60} - \frac{24}{60} = \frac{8}{60} = \frac{2}{15}$$

13. Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$ (ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

Sol: (i) We have to check the following:

$$A \cap B \subset A \text{ and } A \cap B \subset B \text{ and also } P(A \cap B) \leq P(A) \text{ and } P(A \cap B) \leq P(B)$$

Here $P(A) = 0.5$ and $P(A \cap B) = 0.6$ so that $P(A \cap B) > P(A)$. This is a contradiction.

$\therefore$ (i) is not consistently defined.

(ii) We check the formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \dots (i)$

$$\text{Here } P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$$

$$\text{Putting these values in (i), } 0.8 = 0.5 + 0.4 - P(A \cap B)$$

$$\Rightarrow 0.8 = 0.9 - P(A \cap B) \Rightarrow P(A \cap B) = 0.9 - 0.8 = 0.1. \text{ This is valid. } [\because 0 \leq P(E) \leq 1]$$

$\therefore$ (ii) is consistently defined.

14. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
(i) the digits are repeated? (ii) the repetition of digits is not allowed? **[BMP2]**

Sol: (i) **When the digits are repeated:**

a) **To get a 4 digit number greater than 5000.**

5 or 7

The thousands place has to be filled with either of 5 or 7 (2 ways).

The remaining 3 places can be filled by any of the 5 digits 0, 1, 3, 5, 7 .

$$\therefore \text{Required number of 4-digit numbers} = 2 \times 5 \times 5 \times 5 - 1 = 250 - 1 = 249 \Rightarrow n(S) = 249$$

[Here, 1 is subtracted because 5000 cannot be counted as it is not greater than 5000.]

b) **To get a 4 digit number greater than 5000 and divisible by 5.** 0 or 5

A number is divisible by 5 if the digit at its units place is either 0 or 5.

$$\therefore \text{Required number of 4-digit numbers} = 2 \times 5 \times 5 \times 2 - 1 = 100 - 1 = 99 \Rightarrow n(E) = 99$$

$$\therefore \text{the required probability} = \frac{n(E)}{n(S)} = \frac{99}{249} = \frac{33}{83}$$

(ii) **When repetition of digits is not allowed:**

a) **To get a 4 digit number greater than 5000 .**

5 or 7

The thousands place has to be filled with either of the two digits 5 or 7. (2 ways)

No. of ways of filling the remaining 3 places with the remaining 4 digits is ${}^4P_3 = 4 \times 3 \times 2 = 24$.

$$\therefore \text{Required number of 4-digit numbers} = 2 \times 24 = 48 \Rightarrow n(S) = 48.$$

b) **To get a 4 digit number greater than 5000 and divisible by 5:**

- When the digit at the thousands place is 5.

5

Here, the units place can be filled only with 0.

Number of ways of filling tens and hundreds places with any two of the remaining 3 digits is ${}^3P_2 = 3 \times 2 = 6$ (i)

- When the digit at the thousands place is 7.

7

Here, the units place can be filled in two ways (0 or 5) (2 ways)

Number of ways of filling tens and hundreds places with any two of the remaining 3 digits is ${}^3P_2 = 3 \times 2 = 6$.

Here, number of 4-digit numbers starting with 7 and divisible by 5 = $1 \times 6 \times 2 = 12$ (ii)

From (i) & (ii), required number of 4-digit numbers = $6 + 12 = 18 \Rightarrow n(E) = 18$

$$\therefore \text{the required probability} = \frac{n(E)}{n(S)} = \frac{18}{48} = \frac{3}{8}$$