

SAQ PQ QUESTIONS

SETS

- Find the intersection of each pair of sets.
(i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$ (ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$
- Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$. **[MAR-26]**
Find A' , B' , $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$

RELATIONS AND FUNCTIONS

- Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$.

COMPLEX NUMBERS

- If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that
 $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

LINEAR INEQUALITIES

- Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

BINOMIAL THEOREM

- Prove that $C_0 + 3.C_1 + 5.C_2 + \dots + (2n+1)C_n = (2n+2).2^{n-1}$
- Using Binomial theorem, evaluate $(102)^5$
- Using Binomial theorem compute $(98)^5$.
- Using Binomial Theorem, evaluate $(99)^5$
- Which is larger $(1.01)^{1000000}$ or 10,000?

STRAIGHT LINES

- Find the value of p so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

CONIC SECTIONS

- Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$
- Find the equation of the ellipse whose vertices are $(\pm 13, 0)$ and foci are $(\pm 5, 0)$.
- Find the equation of the ellipse in the standard form whose distance between foci is 2 and length of latus rectum is $15/2$.
- Find the equation for the ellipse that satisfies the given conditions:
Length of major axis 26, foci $(\pm 5, 0)$

SAQ PQ SOLUTIONS

SETS

1. Find the intersection of each pair of sets.

(i) $X = \{1, 3, 5\}, Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}, B = \{a, b, c\}$

Sol: i) $X \cap Y = \{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$ ii) $A \cap B = \{a, e, i, o, u\} \cap \{a, b, c\} = \{a\}$

2. Let $U = \{1, 2, 3, 4, 5, 6\}, A = \{2, 3\}$ and $B = \{3, 4, 5\}$.

[MAR-26]

Find $A', B', A' \cap B', A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

Sol: Given $U = \{1, 2, 3, 4, 5, 6\}, A = \{2, 3\}$ and $B = \{3, 4, 5\}$.

$A' = \{1, 4, 5, 6\}, B' = \{1, 2, 6\}$. Hence, $A' \cap B' = \{1, 6\}$ (i)

Now, $A \cup B = \{2, 3\} \cup \{3, 4, 5\} = \{2, 3, 4, 5\}$;

$(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4, 5\} = \{1, 6\}$(ii)

$\therefore$ From (i) & (ii), $(A \cup B)' = \{1, 6\} = A' \cap B'$

RELATIONS AND FUNCTIONS

3. Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined, respectively by $f(x) = x + 1, g(x) = 2x - 3$. Find $f + g, f - g$ and $\frac{f}{g}$.

Sol: i) $(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$.

ii) $(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = -x + 4 = 4 - x$.

iii) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 = \frac{x + 1}{2x - 3}, 2x - 3 \neq 0 \Rightarrow 2x \neq 3 \Rightarrow x \neq \frac{3}{2}$

COMPLEX NUMBERS

4. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

Sol: Given that $(a + ib)(c + id)(e + if)(g + ih) = A + iB$

$$\therefore |(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$$

$$\Rightarrow |(a + ib)| |(c + id)| |(e + if)| |(g + ih)| = |A + iB| \quad [\because |z_1 z_2| = |z_1| |z_2|]$$

$$\Rightarrow \left(\sqrt{a^2 + b^2} \right) \left(\sqrt{c^2 + d^2} \right) \left(\sqrt{e^2 + f^2} \right) \left(\sqrt{g^2 + h^2} \right) = \sqrt{A^2 + B^2} \quad (\text{On squaring both sides})$$

$$\text{we get } (a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2.$$

Hence, the given result is proved.

LINEAR INEQUALITIES

5. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Sol: Of the two consecutive odd positive integers, let x be the smaller one and then the other is $x + 2$.

Given that the both the integers are smaller than 10,

$$x < 10 \text{ and } x + 2 < 10 \Rightarrow x < 10 \text{ and } x < 8 \dots (i)$$

Also, the sum of the two integers is more than 11.

$$\therefore x + (x + 2) > 11 \Rightarrow 2x + 2 > 11 \Rightarrow 2x > 11 - 2 \Rightarrow 2x > 9 \Rightarrow x > \frac{9}{2} \Rightarrow x > 4.5 \dots (ii)$$

From (i) and (ii), we obtain $4.5 < x < 8$

Since x is an odd number, x takes the values 5 and 7.

$\therefore$ the required possible pairs whose sum is more than 11 are (5, 7) and (7, 9).

BINOMIAL THEOREM

6. Prove that $C_0 + 3.C_1 + 5.C_2 + \dots + (2n+1)C_n = (2n+2).2^{n-1}$

Sol: Let $S = C_0 + 3.C_1 + 5.C_2 + \dots + (2n+1)C_n \dots (1)$

By writing the terms in (1) in the reverse order we get

$$S = (2n+1).C_n + (2n-1).C_{n-1} + (2n-3).C_{n-2} + \dots + 3.C_1 + 1.C_0$$

$$= (2n+1).C_0 + (2n-1).C_1 + (2n-3).C_2 + \dots + 3.C_{n-1} + 1.C_n \dots (2) \quad (\because C_{n-r} = C_r)$$

On adding (1) and (2) we get

$$\begin{aligned} 2S &= (2n+2).C_0 + (2n+2).C_1 + (2n+2).C_{n-1} + \dots + (2n+2).C_n \\ &= (2n+2)(C_0 + C_1 + C_2 + \dots + C_n) = (2n+2).2^n \Rightarrow 2S = (2n+2).2^n \\ \therefore S &= (2n+2).2^{n-1} \end{aligned}$$

7. Using Binomial theorem, evaluate $(102)^5$

Sol: Let $102 = 100 + 2$

$$\begin{aligned} \therefore (102)^5 &= (100 + 2)^5 \\ &= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5 \\ &= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5 \\ &= 100000000000 + 10000000000 + 40000000 + 800000 + 8000 + 32 = 11040808032 \end{aligned}$$

8. Compute $(98)^5$.

Sol: Let $98 = 100 - 2$

$$\begin{aligned} \therefore (98)^5 &= (100 - 2)^5 \\ &= {}^5C_0(100)^5 - {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 - {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 - {}^5C_5(2)^5 \\ &= 100000000000 - 5 \times 100000000 \times 2 + 10 \times 1000000 \times 4 - 10 \times 10000 \times 8 + 5 \times 100 \times 16 - 32 \\ &= 10040008000 - 1000800032 = 9039207968. \end{aligned}$$

9. Using Binomial Theorem, evaluate $(99)^5$

Sol: Let $99 = 100 - 1$

$$\begin{aligned} \therefore (99)^5 &= (100 - 1)^5 \\ &= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3 + {}^5C_4(100)(1)^4 - {}^5C_5(1)^5 \\ &= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1 \\ &= 100000000000 - 5000000000 + 100000000 - 100000 + 500 - 1 \\ &= 10010000500 - 500100001 = 9509900499 \end{aligned}$$

10. Which is larger $(1.01)^{1000000}$ or 10,000?

Sol: Let $1.01 = 1 + 0.01$

$$\begin{aligned} \therefore (1.01)^{1000000} &= (1 + 0.01)^{1000000} = 1000000C_0 + 1000000C_1(0.01) + \text{remaining positive terms} \\ &= 1 + 1000000 \times 0.01 + \text{remaining positive terms} = 1 + 10000 + \text{remaining positive terms} > 10000 \end{aligned}$$

Thus $(1.01)^{1000000} > 10000$

STRAIGHT LINES

11. Find the value of p so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Sol: First we solve the equations $3x + y - 2 = 0$ and $2x - y - 3 = 0$

By adding the above equations we get $5x - 5 = 0 \Rightarrow 5x = 5 \Rightarrow x = 1$

Put $x = 1$ in $y = 2x - 3 \Rightarrow y = 2 - 3 \Rightarrow y = -1$

$\therefore$ the point of intersection is $(1, -1)$

This point $(1, -1)$ should satisfy $px + 2y - 3 = 0 \Rightarrow p(1) + 2(-1) - 3 = 0 \Rightarrow p - 2 - 3 = 0 \Rightarrow p = 5$

CONIC SECTIONS

12. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Sol: From the equation of the given ellipse we have

Here, $a^2 = 16 \Rightarrow a = 4$ and $b^2 = 9 \Rightarrow b = 3$

Now, $c^2 = a^2 - b^2 = 16 - 9 = 7 \Rightarrow c = \sqrt{7}$

(i) Foci $= (\pm c, 0) = (\pm\sqrt{7}, 0)$

(ii) Vertices $= (\pm a, 0) = (\pm 4, 0)$

(iii) Length of Major axis $= 2a = 2(4) = 8$

(iv) Length of Minor axis $= 2b = 2(3) = 6$

(v) Eccentricity $e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

(vi) Length of Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$

- 13. Find the equation of the ellipse whose vertices are $(\pm 13, 0)$ and foci are $(\pm 5, 0)$.**

[BMP2]

Sol: Given vertices are $(\pm a, 0) = (\pm 13, 0) \Rightarrow a=13$ and Foci $(\pm c, 0) = (\pm 5, 0) \Rightarrow c=5$

$$\text{Now } c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 169 - 25 = 144 \quad \therefore b^2 = 144$$

Given vertices & foci are on the x-axis. $\therefore$ the ellipse is horizontal.

$$\therefore \text{the equation of the ellipse is } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{169} + \frac{y^2}{144} = 1$$

- 14. Find the equation of the ellipse in the standard form whose distance between foci is 2 and length of latus rectum is $15/2$.**

Sol: Distance between foci $S(ae, 0)$ and $S'(-ae, 0)$ is 2 $\Rightarrow 2ae=2 \Rightarrow ae=1$ (1)

$$\text{Length of latus rectum} = \frac{15}{2} \Rightarrow \frac{2b^2}{a} = \frac{15}{2} \Rightarrow b^2 = \frac{15}{4}a \quad \dots\dots(2)$$

$$\text{Now, } b^2 = a^2(1 - e^2) = a^2 - a^2e^2 = a^2 - (ae)^2 = a^2 - 1 \quad \dots\dots(3)$$

$$\text{From (2) \& (3), } \frac{15}{4}a = a^2 - 1 \Rightarrow 15a = 4a^2 - 4 \Rightarrow 4a^2 - 15a - 4 = 0$$

$$\Rightarrow (a - 4)(4a + 1) = 0 \Rightarrow a = 4 \text{ (or) } -\frac{1}{4}$$

$$\text{If } a=4 \text{ then } b^2 = a^2 - 1 = 16 - 1 = 15$$

$$\therefore \text{Equation of the ellipse is } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{15} = 1$$

- 15. Find the equation of the ellipse that satisfies the given conditions:**

Length of major axis 26, foci $(\pm 5, 0)$

Sol: Length of major axis $2a = 26 \Rightarrow a=13$ and foci $(\pm c, 0) = (\pm 5, 0) \Rightarrow c=5$

$$\text{Now } c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 169 - 25 = 144 \quad \therefore b^2 = 144$$

Also the foci are on the x-axis, the major axis is along the x-axis.

$\therefore$ the ellipse is horizontal.

$$\therefore \text{the equation of the ellipse is } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{169} + \frac{y^2}{144} = 1$$