

LAQ PQ QUESTIONS

TRIGONOMETRIC RATIOS

1. Find the values of $\sin x$ and $\tan x$ when $\cos x = -\frac{1}{2}$, x lies in 3rd quadrant. **[BMP1]**

SEQUENCE & SERIES

2. Find the sum to n terms of the sequence 8, 88, 888, 8888...
3. If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

STRAIGHT LINES

4. Find the image of $(1, 2)$ the given points with respect to $3x + 4y - 1 = 0$
5. Find the equation of the straight lines passing through the point $(-3, 2)$ and making an angle 45° with the straight line $3x - y + 4 = 0$.

CONIC SECTIONS

6. Find the equation of the circle passing through $(0,0)$, $(2,0)$ $(0,2)$ **[MAY-26]**

LIMITS AND DERIVATIVES

7. If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$ • For what values of a does $\lim_{x \rightarrow a} f(x)$ exists?

8. Find the derivative of $\frac{px^2 + qx + r}{ax + b}$

LAQ PQ SOLUTIONS

TRIGONOMETRIC RATIOS

1. Find the values of $\sin x$ and $\tan x$ when $\cos x = -\frac{1}{2}$, x lies in 3rd quadrant.

[BMP1]

Sol: Given that x lies in Q_3 and $\cos x = -\frac{1}{2}$ In Q_3 , $\sin x$ is -ve and $\tan x$ is positive.

$$\sin^2 x = 1 - \cos^2 x \Rightarrow \sin x = \pm \sqrt{1 - \cos^2 x}$$

$$\text{Now } \sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \left(-\frac{1}{2}\right)^2} = -\sqrt{1 - \frac{1}{4}} = -\sqrt{\frac{4-1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

$$\text{Also, } \tan x = \frac{\sin x}{\cos x} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}.$$

SEQUENCE & SERIES

2. Find the sum to n terms of the sequence 8, 88, 888, 8888... .

Sol: The given sequence is not a G.P.

But we can relate it to a G.P. by re-writing the terms as follows:

$$S_n = 8 + 88 + 888 + 8888 + \dots \text{ to } n \text{ terms}$$

$$= \frac{8}{9} [9 + 99 + 999 + 9999 + \dots \text{ to } n \text{ terms}]$$

$$= \frac{8}{9} [(10 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \dots \text{ to } n \text{ terms}]$$

$$= \frac{8}{9} [(10 + 10^2 + \dots \text{ to } n \text{ terms}) - (1 + 1 + 1 + \dots \text{ to } n \text{ terms})] = \frac{8}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{8}{9} \left[\frac{10(10^n - 1)}{9} - n \right] = \frac{80}{81} (10^n - 1) - \frac{8}{9} n$$

3. If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

Sol: The given positive numbers are a, b .

$$\therefore AM = \frac{a+b}{2} = 10 \Rightarrow a+b = 20 \dots (1)$$

$$GM = \sqrt{ab} = 8 \Rightarrow ab = 64 \dots (2)$$

$$\text{Now } (a-b)^2 = (a+b)^2 - 4ab = (20)^2 - 4(64) = 400 - 256 = 144 = 12^2 \Rightarrow a-b = \pm 12 \dots (3)$$

Now $a-b=12$ and from (1), $a+b=20$. Adding we get $2a=32 \Rightarrow a=16$ and hence $b=20-16=4$

When $a-b=-12$ and from (1), $a+b=20$.

Adding we get $2a=8 \Rightarrow a=4$ and hence $b=20-4=16$

Thus, the numbers a and b are 16, 4 (or) 4, 16 respectively.

STRAIGHT LINES

4. Find the image of (1, 2) the given points with respect to $3x + 4y - 1 = 0$

Sol: Let (h, k) be the image of (1, 2) w.r.to $3x + 4y - 1 = 0$

Here $(x_1, y_1) = (1, 2)$, $a = 3$, $b = 4$, $c = -1$.

$$\therefore \frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2} \Rightarrow \frac{h-1}{3} = \frac{k-2}{4} = \frac{-2[3(1)+4(2)-1]}{3^2+4^2} = \frac{-2(10)}{25} = -2\left(\frac{2}{5}\right) = -\frac{4}{5}$$

$$\text{Now, } \frac{h-1}{3} = -\frac{4}{5} \Rightarrow h-1 = -\frac{12}{5} \Rightarrow h = 1 - \frac{12}{5} = \frac{5-12}{5} = -\frac{7}{5}$$

$$\text{Also } \frac{k-2}{4} = -\frac{4}{5} \Rightarrow k-2 = -\frac{16}{5} \Rightarrow k = 2 - \frac{16}{5} = \frac{10-16}{5} = -\frac{6}{5}$$

$$\therefore \text{The image is } (h, k) = \left(-\frac{7}{5}, -\frac{6}{5}\right)$$

5. Find the equation of the straight lines passing through the point $(-3, 2)$ and making an angle 45° with the straight line $3x-y+4=0$.

Sol: Let slope of the required line be m .

Slope of the given line $3x-y+4=0$ is $-\frac{a}{b} = \frac{-3}{-1} = 3$

Angle between the lines is $\theta = 45^\circ$

$$\therefore \tan 45^\circ = \left| \frac{m-3}{1+m(3)} \right| \Rightarrow \left| \frac{m-3}{1+m(3)} \right| = 1 \Rightarrow \frac{m-3}{1+3m} = \pm 1 \Rightarrow m-3 = \pm(1+3m)$$

$$\text{Now, } m-3 = 1+3m \Rightarrow 2m = -4 \Rightarrow m = -2$$

$$\text{Also, } m-3 = -1-3m \Rightarrow 4m = 2 \Rightarrow m = \frac{1}{2}$$

The line passing through $(-3, 2)$ with slope -2 is $y-2 = -2(x+3) = -2x-6 \Rightarrow 2x+y+4=0$

The line passing through $(-3, 2)$ with slope $\frac{1}{2}$ is $y-2 = \frac{1}{2}(x+3) = \frac{x}{2} + \frac{3}{2} \Rightarrow x-2y+7=0$

CONIC SECTIONS

6. Find the equation of the circle passing through (0,0), (2,0) (0,2)

Sol: (0,0), (2,0) (0,2)

The circle passes through the origin O(0,0).

The circle passes through A(2,0) and B(0,2)

Let the equation of the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$(i)

(i) passes through the origin O(0,0) $\Rightarrow 0 + 0 + 2g(0) + 2f(0) + c = 0 \Rightarrow c = 0$

(i) passes through A(2,0) $\Rightarrow 2^2 + 0 + 2g(2) + 2f(0) + 0 = 0$

$$\Rightarrow 2^2 + 2g(2) = 0 \Rightarrow 2(2+2g) = 0 \Rightarrow 2g = -2$$

(i) passes through B(0,2) $\Rightarrow 0 + 2^2 + 2g(0) + 2f(2) + 0 = 0$

$$\Rightarrow 2^2 + 2f(2) = 0 \quad \Rightarrow 2(2+2f) = 0 \Rightarrow 2f = -2$$

Putting $c=0$, $2g = -2$, $2f = -2$ in (i)

the equation of the circle is $x^2 + y^2 - 2x - 2y = 0$

LIMITS AND CONTINUITY

7. If $f(x) = \begin{cases} |x|+1, & x < 0 \\ 0, & x = 0 \\ |x|-1, & x > 0 \end{cases}$. For what values of a does $\lim_{x \rightarrow a} f(x)$ exists?

Sol: The given function is $f(x) = \begin{cases} |x|+1, & x < 0 \\ 0, & x = 0 \\ |x|-1, & x > 0 \end{cases}$.

At $a=0$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (|x|+1) = \lim_{x \rightarrow 0^-} (-x+1) = -0+1=1 \quad [\text{When } x < 0, |x| = -x]$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (|x|-1) = \lim_{x \rightarrow 0^+} (x-1) = 0-1=-1 \quad [\text{When } x > 0, |x| = x]$$

$$\Rightarrow \text{L.H.L} \neq \text{R.H.L} \quad \therefore \lim_{x \rightarrow 0} f(x) \text{ does not exist}$$

When $a < 0$

$$\text{L.H.L} = \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^-} (|x|+1) = \lim_{x \rightarrow a^-} (-x+1) = -a+1 \quad [\because a < 0 \Rightarrow x < 0 \Rightarrow |x| = -x]$$

$$\text{R.H.L} = \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^+} (|x|+1) = \lim_{x \rightarrow a^+} (-x+1) = -a+1 \quad [\because a < 0 \Rightarrow x < 0 \Rightarrow |x| = -x]$$

$$\text{Here L.H.L} = \text{R.H.L} = -a+1$$

$\therefore$ Limit of $f(x)$ exists at $x=a$, when $a < 0$.

When $a > 0$

$$\text{L.H.L} = \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^-} (|x|-1) = \lim_{x \rightarrow a^-} (x-1) = a-1 \quad [\because a > 0 \Rightarrow x > 0 \Rightarrow |x| = x]$$

$$\text{R.H.L} = \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^+} (|x|-1) = \lim_{x \rightarrow a^+} (x-1) = a-1 \quad [\because a > 0 \Rightarrow x > 0 \Rightarrow |x| = x]$$

$$\text{Here L.H.L} = \text{R.H.L} = a-1$$

$\therefore$ Lt of $f(x)$ exists at $x=a$, when $a > 0$

$\therefore \lim_{x \rightarrow a} f(x)$ exists for all $a \neq 0$

8. Find the derivative of $\frac{px^2 + qx + r}{ax + b}$

Sol: $\frac{d}{dx} \left(\frac{px^2 + qx + r}{ax + b} \right) = \frac{(px^2 + qx + r)'(ax + b) - (px^2 + qx + r)(ax + b)'}{(ax + b)^2} \quad \left[\because \frac{d}{dx} \frac{u}{v} = \frac{u'v - uv'}{v^2} \right]$

$$= \frac{(2px + q)(ax + b) - (px^2 + qx + r) \cdot a}{(ax + b)^2} = \frac{2apx^2 + 2bpx + aqx + bq - apx^2 - aqx - ar}{(ax + b)^2}$$

$$= \frac{apx^2 + 2bpx + (bq - ar)}{(ax + b)^2}$$