

8. TRIGONOMETRIC RATIOS UPTO TRANSFORMATIONS

(2 x 2) + (1 x 8) = 12 Marks

 IMP FORMULAS, KEY CONCEPTS 

1) $\sin^2\theta + \cos^2\theta \equiv 1$; $\sin^2\theta \equiv 1 - \cos^2\theta$; $\cos^2\theta \equiv 1 - \sin^2\theta$

2.1) $\sec^2\theta - \tan^2\theta = 1$; $\sec^2\theta = 1 + \tan^2\theta$; $\tan^2\theta = \sec^2\theta - 1$

2.2) If $\sec\theta + \tan\theta = p$ then $\sec\theta - \tan\theta = \frac{1}{p}$

3) $\csc^2\theta - \cot^2\theta = 1$, $\csc^2\theta = 1 + \cot^2\theta$; $\cot^2\theta = \csc^2\theta - 1$

4) (i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$ (ii) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(iii) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ (iv) $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

(v) $\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan \theta}{1 - \tan \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

5) (i) $\sin(A - B) = \sin A \cos B - \cos A \sin B$ (ii) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(iii) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ (iv) $\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

(v) $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan \theta}{1 + \tan \theta} = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$

6) $\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$

$\sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)$

$\cos^2 A - \cos^2 B = -\sin(A + B) \sin(A - B)$

$\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$

$\cos^2 A - \sin^2 B = \cos(A + B) \cos(A - B)$

$\sin^2 A - \cos^2 B = -\cos(A + B) \cos(A - B)$

7) $\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$; $\sin 75^\circ = \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$; $\tan 15^\circ = 2 - \sqrt{3}$; $\tan 75^\circ = 2 + \sqrt{3}$

8) $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1 = \cot 67\frac{1}{2}^\circ$

9) $\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$; $\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$

10) $\sin 36^\circ = \cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$; $\sin 72^\circ = \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

11.1) $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

11.2) $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$

$$12) \cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$13.1) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad 13.2) \tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

$$14.1) 1 - \cos 2A = 2\sin^2 A, 1 + \cos 2A = 2\cos^2 A$$

$$14.2) \sin^2 A = \frac{1 - \cos 2A}{2}; \cos^2 A = \frac{1 + \cos 2A}{2}; \tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A} \quad \left(\text{Power reduction formulae} \right)$$

$$14.3) \sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}, \cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}, \tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$15.1) (i) \sin 3A = 3\sin A - 4\sin^3 A \quad (ii) \cos 3A = 4\cos^3 A - 3\cos A \quad (iii) \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$15.2) \sin^3 A = \frac{1}{4}(3\sin A - \sin 3A); \cos^3 A = \frac{1}{4}(3\cos A + \cos 3A)$$

16) Sum to product formulae:

$$\begin{array}{l|l} \sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right) & \sin(A+B) + \sin(A-B) = 2 \sin A \cos B \\ \sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right) & \sin(A+B) - \sin(A-B) = 2 \cos A \sin B \\ \cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right) & \cos(A+B) + \cos(A-B) = 2 \cos A \cos B \\ \cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right) & \cos(A+B) - \cos(A-B) = -2 \sin A \sin B \\ & = 2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{D-C}{2} \right) \end{array}$$

17) Product to sum formulae:

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B); \quad 2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B); \quad 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

☞ If $A+B+C = 180^\circ$ then $\sin(A+B) = \sin C$, $\cos(A+B) = -\cos C$, $\sin\left(\frac{A+B}{2}\right) = \cos \frac{C}{2}$, $\cos\left(\frac{A+B}{2}\right) = \sin \frac{C}{2}$

$$18.1) \text{ The maximum value of } a \cos x + b \sin x \text{ is } \sqrt{a^2 + b^2} \text{ and minimum value is } -\sqrt{a^2 + b^2}$$

$$18.2) \text{ The maximum value of } a \cos x + b \sin x + c \text{ is } c + \sqrt{a^2 + b^2} \text{ and}$$

$$\text{minimum value of } a \cos x + b \sin x + c \text{ is } c - \sqrt{a^2 + b^2}$$

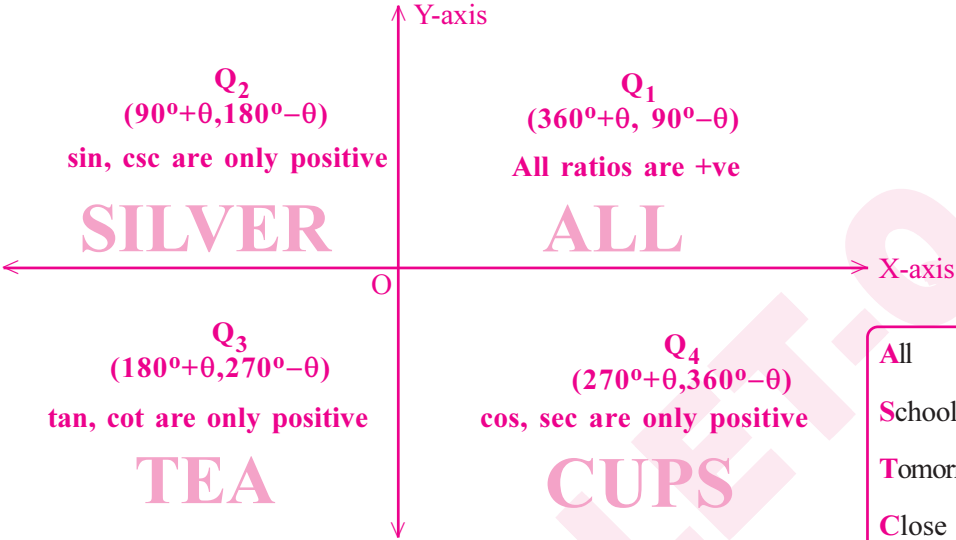
$$19.1) \text{ The period of a } \sin(bx + c) + d \text{ is } \frac{2\pi}{|b|}; \quad \text{The period of a } \cos(bx + c) + d \text{ is } \frac{2\pi}{|b|}$$

$$19.2) \text{ The period of a } \tan(bx + c) + d \text{ is } \frac{\pi}{|b|}$$

BULLET MASTER'S

MATH BEATS!

T'Ratios for Allied angles of θ using ALL SILVER TEA CUPS:



All
Students
Take
Care

All
Schools
Tomorrow
Close

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Student Teaser-1: Tell us 'two numbers' which become equal when Added & Multiplied.

Student Teaser-2: Tell us 'three numbers' which become equal when Added & Multiplied.

Student Teaser-3: Tell 'another set of 3 numbers' which become equal when Added & Multiplied.

Answer for Teaser-1: 2,2 $\therefore 2+2=2 \times 2$

Answer for Teaser-2: 1,2,3 $\therefore 1+2+3=1 \times 2 \times 3$

Answer for Teaser-3: See Q.12 in the Next Page.

i) When $A=60^\circ$, $B=60^\circ$, $C=60^\circ$ then $\tan A + \tan B + \tan C = \sqrt{3} + \sqrt{3} + \sqrt{3} = 3\sqrt{3}$

$$\tan A \tan B \tan C = [(\sqrt{3})(\sqrt{3})](\sqrt{3}) = 3\sqrt{3}$$

ii) When $A=45^\circ$, $B=60^\circ$, $C=75^\circ$ then $\tan A + \tan B + \tan C = (1) + (\sqrt{3}) + (2 + \sqrt{3}) = \sqrt{3} + \sqrt{3} + 3 = 2\sqrt{3} + 3$

$$\tan A \tan B \tan C = (1)(\sqrt{3})(2 + \sqrt{3}) = \sqrt{3}(2 + \sqrt{3}) = 2\sqrt{3} + 3$$

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Multiple Angles అంటే Angles యొక్క అన్నలు/అక్కలు

Sub-Multiple Angles అంటే Angles యొక్క తమ్ముళ్ళు/చెల్లెళ్ళు

Multiple angles of 15° : 30° , 45° , 60° , **75°** , 90° , **105°** ,(Just recall 15 Table)

Multiple angles of 18° : 36° , **54°** , **72°** , 90° , **108°** , 126° ,(Just recall 18 Table)

Sub-multiple angle of 45° : **$22\frac{1}{2}^\circ$**

Sub-multiple angle of 15° : **$7\frac{1}{2}^\circ$**

Trigonometric Ratios of Multiple angles of A: **$\sin 2A$, $\cos 3A$, $\tan 3A$,**

Trigonometric Ratios of Multiple angles of θ : **$\sin 2\theta$, $\cos 2\theta$, $\tan 3\theta$,**

Trigonometric Ratios of Sub-multiple angle of A: **$\sin \frac{A}{2}$, $\cos \frac{A}{2}$, $\tan \frac{A}{2}$,**

Trigonometric Ratios of Sub-multiple angle of θ : **$\sin \frac{\theta}{2}$, $\cos \frac{\theta}{2}$, $\tan \frac{\theta}{2}$,**