

8. LIMITS & CONTINUITY

(2 x 2) + (1 x 4) = 8 Marks

 IMP FORMULAS, KEY CONCEPTS 

1) Standard Limits:

$$1.1) \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}; \quad \lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$$

$$1.2) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1; \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1; \quad \lim_{x \rightarrow 0} \frac{\sin(kx)}{x} = k; \quad \lim_{x \rightarrow 0} \frac{\tan kx}{x} = k$$

$$1.3) \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$1.4) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e; \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$2) \text{ Indeterminate forms: } \frac{0}{0}, \frac{\infty}{\infty}, 1^\infty, 0 \times \infty, \infty - \infty, 0^\infty, \infty^0, 0^0$$

3) **Evaluation of $\lim_{x \rightarrow a} f(x)$:** First, check whether $f(a)$ exists as a real number or not. If $f(a)$ exists as a real then $\lim_{x \rightarrow a} f(x) = f(a)$. For if, $f(a)$ takes the indeterminate form such as $\frac{0}{0}, \frac{\infty}{\infty}, 1^\infty$ etc.,

then reduce the given limit into standard limit forms (or) rationalise the numerator / denominator or both accordingly (or) factorise; (or) take proper substitutions and proceed accordingly.

4.1) **Left handed Limit(L.H.L):** If x approaches a from the left i.e., through values just smaller than a then the limit of f defined in the usual sense is called the left limit of $f(x)$ and it is written as $\lim_{x \rightarrow a-} f(x)$

 L.H.L: $\lim_{x \rightarrow a-} f(x) = \lim_{h \rightarrow 0+} f(a-h) = \lim_{x \rightarrow 0} f(a-x), (x \rightarrow a- \Rightarrow x < a)$

4.2) **Right handed Limit(R.H.L):** If x approaches a from the right i.e., through values just larger than a then the limit of f defined in the usual sense is called the right limit of $f(x)$ and it is written as $\lim_{x \rightarrow a+} f(x)$

 R.H.L: $\lim_{x \rightarrow a+} f(x) = \lim_{h \rightarrow 0+} f(a+h) = \lim_{x \rightarrow 0} f(a+x), (x \rightarrow a+ \Rightarrow x > a)$

Note: If L.H.L=R.H.L then we say the limit exists, otherwise the limit does not exist.

5) A function $f(x)$ is said to be **continuous** at $x=a$ if

(i) $f(a)$ exists (naturally or by attribution) (ii) $f(a) = \lim_{x \rightarrow a+} f(x) = \lim_{x \rightarrow a-} f(x)$

If any of the above conditions is not satisfied then $f(x)$ is said to be discontinuous at $x = a$.