

7.PRODUCT OF VECTORS

(1 × 4) + (1 × 8) = 12 Marks

1. Scalar(dot) Product Verses Vector (Cross) Product

	Scalar Product	Vector Product
1.	$\vec{a}.\vec{b} = \vec{a} \vec{b} \cos \theta$	$\vec{a} \times \vec{b} = \vec{a} \vec{b} \sin \theta \hat{n}$
2.	$\cos \theta = \frac{\vec{a}.\vec{b}}{ \vec{a} \vec{b} }$	$\sin \theta = \frac{ \vec{a} \times \vec{b} }{ \vec{a} \vec{b} }; \hat{n} = \frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} }$ $\Rightarrow$ a vector of magnitude k, $\perp$ to $\vec{a}$ & $\vec{b} = k \left(\frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } \right)$
3.	$\vec{a}.\vec{b} = \vec{b}.\vec{a}$ (commutative law)	$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ but $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
4.	$\vec{a}.\vec{a} = \vec{a} ^2 = a^2$	$\vec{a} \times \vec{a} = \vec{0}$
5.	$\vec{a}, \vec{b}$ are perpendicular $\Rightarrow \vec{a}.\vec{b} = 0$	$\vec{a}, \vec{b}$ are parallel $\Rightarrow \vec{a} \times \vec{b} = \vec{0}$
6.	$\vec{i}.\vec{j} = 0, \vec{j}.\vec{k} = 0, \vec{k}.\vec{i} = 0$ $\vec{i}.\vec{i} = 1, \vec{j}.\vec{j} = 1, \vec{k}.\vec{k} = 1$	$\vec{i} \times \vec{i} = \vec{0}, \vec{j} \times \vec{j} = \vec{0}, \vec{k} \times \vec{k} = \vec{0}$ $\vec{i} \times \vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{i}, \vec{k} \times \vec{i} = \vec{j}$
7.	If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}; \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$	$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}; \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k} \Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
8. Projections:	(i) Projection Vector of $\vec{b}$ on $\vec{a} = \frac{(\vec{a}.\vec{b})\vec{a}}{ \vec{a} ^2}$ (ii) Length of projection of $\vec{b}$ on $\vec{a} = \frac{ \vec{a}.\vec{b} }{ \vec{a} }$ (iii) Component of $\vec{b} \perp^r$ to $\vec{a} = \vec{b} - \frac{(\vec{a}.\vec{b})\vec{a}}{ \vec{a} ^2}$	(Vector) Areas: (i) Area of the parallelogram with adjacent sides $\vec{a}, \vec{b} = \vec{a} \times \vec{b} $ (ii) Area of the parallelogram with diagonals $\vec{d}_1, \vec{d}_2 = \frac{1}{2} \vec{d}_1 \times \vec{d}_2 $ (iii) Area of the quadrilateral with diagonals $\vec{AC}, \vec{BD} = \frac{1}{2} \vec{AC} \times \vec{BD} $ (iv) Area of $\triangle ABC = \frac{1}{2} \vec{AB} \times \vec{AC} $

2. The vector equation of the plane in the normal form is $\vec{r}.\hat{n} = p$, where $\hat{n}$ is the unit normal from the origin to the plane and p is the perpendicular distance from the origin to the plane.
3. The vector equation of the plane passing through the point $A(\vec{a})$ and perpendicular to $\vec{n}$ is $(\vec{r} - \vec{a}).\vec{n} = 0$ or $\vec{r}.\vec{n} = \vec{a}.\vec{n}$
4. The angle θ between the planes $\vec{r}.\vec{n}_1 = p_1$ and $\vec{r}.\vec{n}_2 = p_2$ is $\theta = \cos^{-1} \frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1| |\vec{n}_2|}$

TEACHING POINTS

TRIPLE PRODUCT OF VECTORS

I) SCALAR TRIPLE PRODUCT[STP]:

1) If $\vec{a}, \vec{b}, \vec{c}$ are any three vectors then their Scalar Triple Product is $(\vec{a} \times \vec{b}) \cdot \vec{c}$ (or) $\vec{a} \cdot (\vec{b} \times \vec{c})$
 Scalar Triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$ is denoted by $[\vec{a} \vec{b} \vec{c}]$ read as box $[\vec{a} \vec{b} \vec{c}]$.

2) $[\vec{i} \vec{j} \vec{k}] = (\vec{i} \times \vec{j}) \cdot \vec{k} = \vec{k} \cdot \vec{k} = 1$ Also $[\vec{i} \vec{k} \vec{j}] = (\vec{i} \times \vec{k}) \cdot \vec{j} = -\vec{j} \cdot \vec{j} = -1$

Hence $[\vec{i} \vec{j} \vec{k}] = 1 = [\vec{j} \vec{k} \vec{i}] = [\vec{k} \vec{i} \vec{j}] = -[\vec{i} \vec{k} \vec{j}] = -[\vec{j} \vec{i} \vec{k}] = -[\vec{k} \vec{j} \vec{i}]$.

3) $[\vec{a} \vec{b} \vec{c}] = [\vec{b} \vec{c} \vec{a}] = [\vec{c} \vec{a} \vec{b}]$ (STP is same, when $\vec{a}, \vec{b}, \vec{c}$ are in the same cyclic order).

$[\vec{a} \vec{b} \vec{c}] = -[\vec{b} \vec{a} \vec{c}] = -[\vec{a} \vec{c} \vec{b}] = -[\vec{c} \vec{b} \vec{a}]$ (STP changes its sign if the cyclic order of $\vec{a}, \vec{b}, \vec{c}$

4.1) If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}, \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}, \vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$ then $[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

4.2) If $\vec{\alpha} = x_1\vec{a} + y_1\vec{b} + z_1\vec{c}, \vec{\beta} = x_2\vec{a} + y_2\vec{b} + z_2\vec{c}, \vec{\gamma} = x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$ then $[\vec{\alpha} \vec{\beta} \vec{\gamma}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} [\vec{a} \vec{b} \vec{c}]$

5) The volume of the Parallelopiped with $\vec{a}, \vec{b}, \vec{c}$ as its coterminous edges is $V = |[\vec{a} \vec{b} \vec{c}]|$

6) The volume of the tetrahedron with $\vec{a}, \vec{b}, \vec{c}$ as its coterminous edges is $V = \frac{1}{6} |[\vec{a} \vec{b} \vec{c}]|$

7) The volume of the tetrahedron ABCD is $V = \frac{1}{6} |[\vec{AB} \vec{AC} \vec{AD}]|$

8) Three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar $\Leftrightarrow [\vec{a}, \vec{b}, \vec{c}] = 0$

9) Three vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar $\Leftrightarrow [\vec{a}, \vec{b}, \vec{c}] \neq 0$

10) If $[\vec{a}, \vec{b}, \vec{c}] = 0$ then (i) $\vec{a}, \vec{b}, \vec{c}$ are coplanar when $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors.

(ii) any two of $\vec{a}, \vec{b}, \vec{c}$ are parallel (iii) at least one of $\vec{a}, \vec{b}, \vec{c}$ is a zero vector.

II) PRODUCT OF FOUR VECTORS:

11) The scalar product of 4 vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ is

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix} = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{d})$$

12) The vector product of 4 vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ is

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \vec{c} \vec{d}] \vec{b} - [\vec{b} \vec{c} \vec{d}] \vec{a} = [\vec{a} \vec{b} \vec{d}] \vec{c} - [\vec{a} \vec{b} \vec{c}] \vec{d}$$

III) VECTOR PRODUCT OF THREE VECTORS:

13) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{c}) \vec{a}$ (This vector lies in the plane of $\vec{a}, \vec{b}$)

14) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$ (This vector lies in the plane of $\vec{b}, \vec{c}$)

BULLET MASTER'S
MATH BEATS!

PRODUCT OF VECTORS $\Rightarrow$ రెండు తలల రాశి!

In any Branch of Maths, if we find the Product of two entities, we get a Unique Answer.

But, in case of Vectors it is not like that.

Product of **Two Reals** is Unique: $2 \times 3 = 6$.

Product of **Two Functions(fg)** is Unique: $(2x)(3x) = 6x^2$.

Product of **Two Matrices (AB)** is Unique

Product (Cartesian) of **Two Sets** $A \times B$ is Unique.

Product of **Two Trigonometric Functions** is Unique: $(\tan\theta)(\cot\theta) = 1$

Derivative of Product of **Two functions (uv)** is Unique: $(uv)'$

Integral of Product of **Two functions** is Unique : $\int uv$

But, **Product of Two Vectors $\vec{a}, \vec{b}$** is **not Unique (??)**.

That too, Product of two Vectors is amazingly **a Scalar (or) a Vector**.

That is, Dot (Scalar) Product of Two Vectors gives a **Scalar Quantity**.

Ofcourse, Cross (Vector) Product of Two Vectors is a **Vector Quantity**.

A Special Case: $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \Rightarrow$ Angle between $\vec{a}, \vec{b}$ is 45°