

1 VSAQ + 1 SAQ = 2M + 4M = 6 MARKS

6. SYSTEM OF PARTICLES & ROTATIONAL MOTION

- Centre of Mass & Centre of Gravity • Angular Velocity (ω) & acceleration (α)
- Torque (τ) • Moment of Inertia (I) • Angular Momentum (L)
- Parallel axes theorem, Perpendicular axes theorem

SMART SYNOPSIS

- **System of Particles** means treating a group of objects (two or more) as one single unit.
- 1. **Centre of Mass (CM)** is the point at which the entire mass of the body is supposed to be concentrated. Here, the body moves, as if the whole external force is acting at that point. CM is independent of g. CM may or may not lie inside the body. [SAQ]
Ex: Projectile explosion, Binary Star system, Earth-moon system.
- 2. **Centre of Gravity (CG)** is the point at which the weight of the body acts. [SAQ]
 CG depends on g. CG always lie inside the body.
- 3. **Motion of centre of mass :**
Ex : In case of a projected Indian club the particle at the centre of mass moves along a parabolic path but all other particles move in various complicated paths.
- 4. **Rotational Motion:**
- 4.1 **Angular displacement (θ)** is the angle described by the radius vector at the centre, in a given interval of time. **SI unit:** radian.
- 4.2 **Angular velocity (ω)** is the rate of angular displacement. [SAQ]
Formula: $\omega = \frac{\theta}{t}$ **SI unit:** rad s^{-1} . Linear velocity $v = r\omega$
- 4.3 **Angular acceleration (α)** is the rate of angular velocity. **Formula:** $\alpha = \frac{d\omega}{dt}$
- 5. **Moment of Inertia (I)** in rotational motion is equivalent to mass (M) in linear motion. Moment of Inertia is the inherent property of rotation, which opposes change in rotation. Moment of Inertia is the product of mass(M) of the particle and square of the distance between the particle and the axis of rotation.
Formula : $I = Mr^2$; **S.I unit :** Kg m^2



Note: In linear motion we consider motion of objects of physical quantities w.r.t a fixed point while in rotational motion we consider rotation of objects w.r.t a fixed axis.

5.1 Angular momentum (L) is the product of moment of inertia of a body and its angular velocity

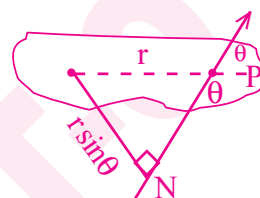
Formula : $L = I \omega = mvr$

5.2 Law of conservation of angular momentum : The angular momentum of a system always remains constant, when no external torque act on the system.

Thus, $I \omega = \text{constant}$ Hence, $I_1 \omega_1 = I_2 \omega_2$

6. Torque / Moment of force in rotational motion is equivalent to force (F) in linear motion. “Torque is the turning effect or twisting effect of the force about a point”.

It is the product of magnitude of the force and the perpendicular distance of the point from the line action of force.



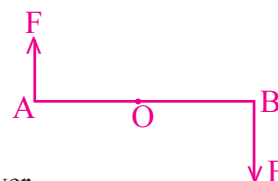
Torque $\tau = \text{Force} \times \text{perpendicular distance}$
 $= F \times r \sin \theta$

Vectorially, $\vec{\tau} = \vec{r} \times \vec{F}$

SI unit : **Nm**

Ex: Opening a bottle cap, turning a steering wheel, tightening or loosening a bolt using a wrench, opening a door.

7. Couple: Two equal and opposite parallel forces acting on a body at two different points in opposite directions constitute a couple. Couple produces rotation.



Ex: Turning a steering wheel, twisting water tap, turning a screw driver.

Note: Couple is a particular case of Torque.

8.1 Parallel axes theorem: The **moment of inertia of a rigid body about an axis** passing through a point is equal to the sum of moment of inertia about a parallel axis passing through its centre of mass and product of the mass of the body and the square of the perpendicular distance between the axes. [SAQ]

Formula : $I = I_G + Mx^2$

8.2 Perpendicular axes theorem: The **moment of inertia of a plane lamina about an axis** perpendicular to its plane and passing through a point is equal to the sum of its moment of inertia about any two mutually perpendicular axes in that plane passing through that point. [SAQ]

Formula : $I_z = I_x + I_y$

9. Comparison between Linear and Rotational Motion:

Linear Motion	Rotational Motion
1. Displacement x	1. Angular displacement θ
2. Velocity $v = \frac{ds}{dt}$	2. Angular velocity $\omega = \frac{d\theta}{dt}$
3. Acceleration $a = \frac{dv}{dt}$	3. Angular acceleration $\alpha = \frac{d\omega}{dt}$
4. Mass: $m = F/a$	4. Moment of inertia: $I = MK^2$
5. Force $F = ma = \frac{dp}{dt}$	5. Torque $\tau = I\alpha = \frac{dL}{dt}$
6. Work $W = FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	6. Work $W = \tau\theta = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$
7. Kinetic energy $K.E_T = \frac{1}{2}mv^2$	7. Kinetic energy $K.E_R = \frac{1}{2}I\omega^2$
8. Power $P = Fv$	8. Power $P = \tau\omega$
9. Linear momentum $p = mv$	9. Angular momentum $L = I\omega$
• We consider quantities w.r.t a fixed point .	• We consider quantities w.r.t a fixed axis .

Imp. Formulae

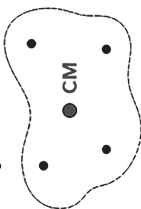
- In a plane, centre of mass of two points is $P(X_{cm}, Y_{cm}) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2} \right)$
- In a space, centre of mass of n points is $P(X_{cm}, Y_{cm}, Z_{cm}) = \left(\sum_{i=1}^n \frac{m_i x_i}{M}, \sum_{i=1}^n \frac{m_i y_i}{M}, \sum_{i=1}^n \frac{m_i z_i}{M} \right)$
- Angular velocity (ω): $\omega = \frac{d\theta}{dt}$
- Angular acceleration $\alpha = \frac{d\omega}{dt}$
- Moment of Inertia $I = Mk^2$
- Torque (or) Moment of Force: $\tau = \vec{r} \times \vec{F} = Fr \sin \theta = I\alpha$
- Angular Momentum $L = mvr = mr^2\omega = I\omega = \vec{r} \times \vec{p}$
- Law of Conservation of Angular Momentum : $I_1\omega_1 = I_2\omega_2$
- Relation between Torque and Angular Momentum $\tau = \frac{dL}{dt}$
- Rotational Kinetic Energy $K.E_R = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$
- Parallel axes theorem: $I = I_G + Mx^2$
- Perpendicular axes theorem : $I_z = I_x + I_y$

10. Moment of Inertia of different bodies about different Axes :

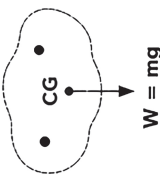
	Name of the body	Axis of rotation	Moment of Inertia
1.	Circular ring of radius R [Mass: M; Radius:R]	(i) Axis passing through its centre and perpendicular to its plane (ii) About any diameter (iii) A tangent perpendicular to its plane (iv) A tangent in its plane	MR^2 $\frac{MR^2}{2}$ $2MR^2$ $\frac{3MR^2}{2}$
2.	Thin circular disc of radius R [Mass: M; Radius:R]	(i) Axis passing through its centre and perpendicular to its plane (ii) About any diameter (iii) A tangent perpendicular to its plane (iv) A tangent in its plane	$\frac{MR^2}{2}$ $\frac{MR^2}{4}$ $\frac{3MR^2}{2}$ $\frac{5MR^2}{4}$
3.	Hollow sphere [Mass: M; Radius:R]	About its diameter	$\frac{2}{3}MR^2$
4.	Solid Sphere [Mass: M; Radius:R]	(i) About its diameter (ii) About a tangent to the surface	$\frac{2}{5}MR^2$ $\frac{7}{5}MR^2$
5.	Hollow cylinder (tube) [Mass:M,Radius:R,Length:L]	About its geometrical axis	MR^2
6.	Solid cylinder [Mass: M; Radius:R]	(i) About its geometrical axis (ii) Axis passing through its centre and perpendicular to its own axis	$\frac{MR^2}{2}$ $M\left[\frac{l^2}{12} + \frac{R^2}{2}\right]$
7.	Thin uniform rod of length L [Mass: M; Radius:R, Length :L]	(i) Axis passing through centre and perpendicular to the rod (ii) About its end and perpendicular to the rod	$\frac{ML^2}{12}$ $\frac{ML^2}{3}$
8.	Rectangular plate or bar [Mass: M; Length :L, Breadth:b]	Axis passing through its centre and perpendicular to its plane	$M\left[\frac{l^2 + b^2}{12}\right]$

1. CENTRE OF MASS (CM)

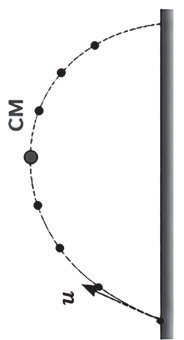
- Point at which the entire mass of the body is supposed to be concentrated.
- Centre of mass is independent of g .
- CM may or may not lie inside the body.

**2. CENTRE OF GRAVITY (CG)**

- Point at which the weight of the body acts.
- Centre of gravity depends on g .
- CG always lies inside the body.

**3. MOTION OF CENTRE OF MASS**

- In an exploded projectile, the centre of mass of all fragments continues along the same parabolic path as if no explosion had occurred.

**4. ANGULAR DISPLACEMENT (θ)**

- Angle described by the radius vector at the centre in a given interval of time.



SI Unit

radian (rad)

5. ANGULAR VELOCITY (ω)

- Rate of angular displacement.

$$\omega = \frac{\theta}{t}$$

SI Unit

rad s⁻¹**6. ANGULAR ACCELERATION (α)**

- Rate of change of angular velocity.

$$\alpha = \frac{d\omega}{dt}$$

SI Unit

rad s⁻²

6. SYSTEM OF PARTICLES & ROTATIONAL MOTION

CONCEPTS & FORMULAS

7. ANGULAR MOMENTUM (L)

- Moment of momentum of a body.
- Angular momentum = Moment of inertia $\times$ Angular velocity.

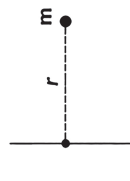
$$L = I\omega = mvr$$

9. MOMENT OF INERTIA (I)

- Product of mass (m) of a particle and square of its distance (r) from the axis of rotation.

$$I = mr^2$$

SI Unit

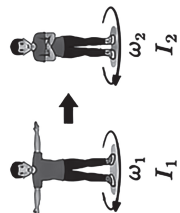
kg m²**10. LAW OF CONSERVATION OF ANGULAR MOMENTUM**

- When no external torque acts on a system, angular momentum remains constant.

$$I_1\omega_1 = I_2\omega_2$$

Example

- A skater pulls in the arms.
- Moment of inertia decreases.
- Angular velocity increases.

**8. TORQUE / MOMENT OF FORCE (τ)**

- Turning effect of a force about a point.

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

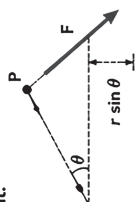
$$\tau = Fr \sin \theta$$

Vector form

$$\vec{\tau} = \vec{r} \times \vec{F}$$

SI Unit

N m

**COUPLE**

- Two equal and unlike parallel forces acting on a body at two different points in opposite directions.
- Produces rotation.

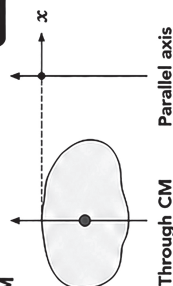
$$\text{Moment of Couple} \\ C = F \times d$$

**11.1 PARALLEL AXES THEOREM**

Moment of inertia about any axis

= Moment of inertia about a parallel axis through the centre of mass + $M \times x^2$ where x is the perpendicular distance between the two axes.

$$I = I_G + Mx^2$$

**11.2 PERPENDICULAR AXES THEOREM (For Plane Lamina)**

Moment of inertia about an axis perpendicular to the plane of a lamina

= Sum of moments of inertia about two mutually perpendicular axes in the plane through the same point.

$$I_z = I_x + I_y$$

