

# 4. PAIR OF LINES

(2 x 4) = 8 Marks

## IMP FORMULAS, KEY CONCEPTS

1) If  $h^2 \geq ab$  then  $ax^2 + 2hxy + by^2 = 0$  represents the equation of a pair of lines passing through the origin.

2.1) If  $ax^2 + 2hxy + by^2 \equiv (y - m_1x)(y - m_2x)$  then  $m_1 + m_2 = -\frac{2h}{b}$ ,  $m_1m_2 = \frac{a}{b}$

2.2) If  $ax^2 + 2hxy + by^2 \equiv (l_1x + m_1y)(l_2x + m_2y)$  then  $l_1l_2 = a$ ,  $m_1m_2 = b$ ,  $l_1m_2 + l_2m_1 = 2h$

3.1) If  $\theta$  is the angle between the pair of lines  $ax^2 + 2hxy + by^2 = 0$  then

$$(i) \cos \theta = \frac{a+b}{\sqrt{(a-b)^2 + 4h^2}} \quad (ii) \tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

3.2) The pair of lines  $ax^2 + 2hxy + by^2 = 0$  are at right angle  $\Leftrightarrow a + b = 0$   
i.e., coefficient of  $x^2$  + coefficient of  $y^2 = 0$

4) The equation of pair of lines passing through  $(x_1, y_1)$  and

(i) parallel to  $ax^2 + 2hxy + by^2 = 0$  is  $a(x - x_1)^2 + 2h(x - x_1)(y - y_1) + b(y - y_1)^2 = 0$

(ii) perpendicular to  $ax^2 + 2hxy + by^2 = 0$  is  $b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$

5) The equation of the bisectors of the angle between  $ax^2 + 2hxy + by^2 = 0$  is  
 $h(x^2 - y^2) = (a - b)xy$

6) The area of the triangle formed by  $ax^2 + 2hxy + by^2 = 0$  & the line  $lx + my + n = 0$  is  $\frac{n^2 \sqrt{h^2 - ab}}{|am^2 - 2hlm + bl^2|}$

7) The product of perpendiculars drawn from a point  $P(\alpha, \beta)$  to the pair of lines  $ax^2 + 2hxy + by^2 = 0$  is  $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$

8) If  $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents the equation of a pair of lines then

(i)  $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$  (ii)  $h^2 \geq ab$ ,  $g^2 \geq ac$ ,  $f^2 \geq bc$ .

9) The point of intersection of the pair of lines  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  is

$$\left( \frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

10) If  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents two parallel lines then (i)  $h^2 = ab$  (ii)  $af^2 = bg^2$

(iii) the distance between the parallel lines is  $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$  or  $2\sqrt{\frac{f^2 - bc}{b(a+b)}}$

11) **Homogenisation:** The equation to the pair of lines joining the origin to the points of intersection of the curve  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  and the line  $lx + my + n = 0$  is

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx + my}{-n}\right) + 2fy\left(\frac{lx + my}{-n}\right) + c\left(\frac{lx + my}{-n}\right)^2 = 0$$