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MATHEMATICAL INDUCTION

1SAQ [4M]

SINGLE PAGE SYNOPSIS

1. Principle of finite Mathematical Induction:

Let $S(n)$ be a statement involved with the natural number 'n' such that

(a) $S(1)$ is true

(b) for any $k \in \mathbb{N}$; if $S(k)$ is true then $S(k+1)$ is true.

Then, $S(n)$ is true $\forall n \in \mathbb{N}$

2. Steps to prove a given result using the principle of finite Mathematical Induction:

To prove a given statement $S(n)$, $\forall n \in \mathbb{N}$ proceed as follows:

Step 1: Show that $S(1)$ is true (i.e., LHS of $S(1)$ = RHS of $S(1)$) [**Basis of induction**]

Step 2: Assume that $S(k)$ is true, for $k \in \mathbb{N}$ [**Inductive hypothesis**]

Step 3: Hence deduce that $S(k+1)$ is true [**Inductive step**]

Note: There are three ways of proving the truthness of $S(k+1)$

(i) Write the entire statement of $S(k+1)$ and obtain R.H.S from L.H.S

(ii) Simplify L.H.S and R.H.S separately, to the extent that they both become equal

(iii) Add T_{k+1} to both sides of $S(k)$ and hence verify the truthness of $S(k+1)$.

☞ **Vide Solved Examples-1, 2, 3 to understand these three ways.**

3. Useful Formulae:

(i) **The n^{th} term of the A.P:** $a, a+d, a+2d, \dots$ is $t_n = a + (n-1)d$

(ii) **The sum of n terms of the A.P:** $a, a+d, a+2d, \dots$ is $S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a + l)$, (l is the last term)

(iii) **The n^{th} term of the G.P:** $a, ar, ar^2, \dots$ is $t_n = ar^{n-1}$.

(iv) $1+2+\dots+n = \frac{n(n+1)}{2}$

(v) $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(vi) $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

(vii) **Sum of first n odd natural numbers:** $1+3+5+\dots+(2n-1)=n^2$.

(viii) **Sum of first n even natural numbers:** $2+4+6+\dots+(2n)=n(n+1)$.