

3 SEQUENCES & SERIES

1VSAQ [2M]

SINGLE PAGE SYNOPSIS

SS-1 (AP)

1. **Sequence:** It is an arrangement of numbers in a definite order according to some rule.

Ex 1: 1,2,3,4,...; 2,4,6,8,...; 1,3,5,7,...; **Ex 2:** 1,2,2²,2³,...; 1,3,9,27,....

2. **Series:** Let $a_1, a_2, a_3, \dots$ be a sequence then it's corresponding series is $a_1 + a_2 + a_3 + \dots$

3.1. Arithmetic Progression (AP):

- A sequence of the form $a, a+d, a+2d, \dots$ is called Arithmetic Progression.
- Here, **a** is called the **first term** and **d** is called the **common difference**. **Ex:** 1,3,5,7,...
- The n^{th} term of AP is $t_n = a + (n-1)d$
- Sum of n terms is $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$, l is the last term

3.2. Arithmetic Mean (AM):

- If a, b, c are in A.P then b is called Arithmetic Mean between a and c ; $b = \frac{a+c}{2}$
- The AM of two numbers a, b is $\frac{a+b}{2}$ **Ex:** AM of 2,6 is $\frac{2+6}{2} = 4$

SS-2 (GP)

4.1. Geometric Progression(GP):

- A sequence of the form $a, ar, ar^2, \dots$ is called Geometric Progression.
- Here **a** is called the **first term**, **r** is called the **common ratio**. $\left(r = \frac{ar}{a} = \frac{ar^2}{ar}, \dots\right)$
- Ex:** 1,2,4,8,16,...
- The n^{th} term of the GP is $a_n = ar^{n-1}$. Here $a_1 = a$; $a_2 = ar$; $a_3 = ar^2$; $a_4 = ar^3$; $a_5 = ar^4, \dots$
- Sum of n terms is $S_n = \frac{a(r^n - 1)}{r - 1}$, when $r > 1$; $S_n = \frac{a(1 - r^n)}{1 - r}$, when $r < 1$; $S_n = na$, when $r = 1$
- Sum to infinity is $S_\infty = \frac{a}{1-r}$, where $|r| < 1$

4.2. Geometric Mean (G.M):

- If a, b, c are in G.P then b is called the Geometric mean between a and c ; $b = \sqrt{ac}$
- The GM of a, b is $\sqrt{ab}$. GM of 2 & 6 is $\sqrt{ab} = \sqrt{2 \times 6} = \sqrt{12}$
- 3 numbers in GP can be taken as $\frac{a}{r}, a, ar$ • 4 numbers in G.P can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

SS-3 (AM & GM)

- If A and G denote the AM and GM of positive reals a, b then $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$
- $\Sigma n = \sum_{k=1}^n k = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$
- $\Sigma n^2 = \sum_{k=1}^n k^2 = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ • $\Sigma n^3 = \sum_{k=1}^n k^3 = 1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$