

1 SAQ = 4 MARKS

3. MOTION IN A PLANE

- Scalars & Vectors • Types of Vectors • Resolution of a vector
- Scalar & vector product • Triangle & Parallelogram law
- Relative velocity • Projectile Motion • Circular motion

SMART SYNOPSIS

1.1 Scalar : A physical quantity which has only magnitude but no direction, is called a scalar.

Ex : Distance, Mass, Speed, Time, Work etc.

[VSAQ]

1.2 Vector : A physical quantity which has both magnitude and direction and obeys law of vector addition is called a vector.

[VSAQ]

Ex : Displacement, Weight, Velocity, Acceleration, Force, Momentum, Torque etc

2. Geometrical representation of vectors :

Geometrically, a vector is represented by a **directed line segment** :

If the vector $\overline{AB}$ is denoted by $\vec{a}$ then

(i) A is called the initial point of the vector $\vec{a}$

(ii) B is called the terminal point of the vector $\vec{a}$

(iii) The magnitude of $\vec{a}$ is the length AB. It is written as $|\overline{AB}|$ or AB or $|\vec{a}|$

(iv) The direction of $\vec{a}$ is from the initial point A to the terminal point B.



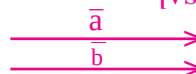
3. Types of vectors :

3.1 Equality of vectors:

Two vectors are said to be equal if they have the same magnitude & direction.

The units of equal vectors are also equal.

[VSAQ]

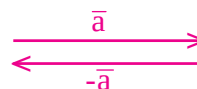


3.2 Negative of a vector :

Let $\vec{a}$ be a vector then the vector having

magnitude as that of $\vec{a}$ and direction opposite to that of $\vec{a}$ is called negative of $\vec{a}$ and it is denoted by $-\vec{a}$.

[VSAQ]



3.3 Zero vector (Null vector):

A vector of zero magnitude and arbitrary direction is called a zero vector, it is denoted by $\vec{0}$ or $\mathbf{O}$.

Significance: A null vector represents a point at rest.

[VSAQ]

3.4 Unit vector:

A vector of magnitude one unit is called a **unit vector**.

[VSAQ]

If $\vec{a}$ is a non-zero vector then the unit vector in the direction of $\vec{a}$ is denoted by $\hat{a}$.

The unit vector in the direction of the non-zero vector $\vec{a}$ is $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

3.5 Position vector :

If O is a point of reference and P is a point in space then the vector $\overline{OP}$ is called the position vector of P with reference to O.



4.1 Addition of vectors :

To add two vectors $\vec{a}$, $\vec{b}$ keep the initial point of the second vector $\vec{b}$ at the final point of the first vector $\vec{a}$. Then draw an arrow from the initial point of $\vec{a}$ to the final point of the second vector $\vec{b}$. This new arrow represents the resultant vector $\vec{c}$, which is the sum of the two vectors $\vec{a}$, $\vec{b}$.

**4.2 Laws of vector addition :**

- (i) Vector addition obeys Commutative Law : $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
 (ii) Vector addition obeys Associative Law : $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
 (iii) Vector addition obeys Distributive Law : $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$, where k is scalar.

5.1 Resolution of a vector in a plane into two rectangular components:

[VSAQ]

Consider a vector $\vec{A} = A_x \hat{i} + A_y \hat{j}$ making an angle θ with the X-axis.

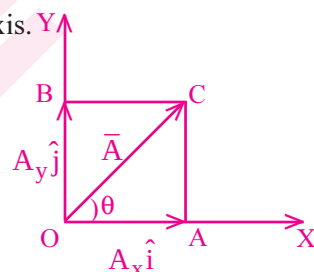
This vector $\vec{A}$ can be resolved into two rectangular components.

Horizontal component of $\vec{A}$ along X- axis is $A_x = A \cos \theta$.

Vertical component of $\vec{A}$ along Y- axis is $A_y = A \sin \theta$.

Magnitude of the vector $\vec{A}$ is $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Direction of the vector $\vec{A}$ is given by $\tan \theta = \frac{A_y}{A_x}$

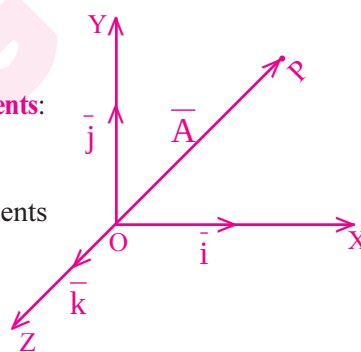
**5.2 Resolution of a vector in space into three rectangular components:**

Consider a vector $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$.

This vector $\vec{A}$ can be resolved into three rectangular components

A_x, A_y, A_z along X, Y, Z axes.

Magnitude of the vector $\vec{A}$ is $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



- 6. Scalar product of vectors:** If $\vec{a}$ and $\vec{b}$ are two vectors which include an angle θ with each other then their scalar product is given by $\vec{a} \cdot \vec{b} = ab \cos \theta$. It is a **scalar quantity**. [SAQ]

If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ then $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

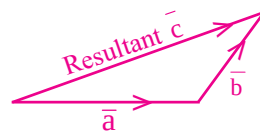
Properties of the Scalar Product: (i) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (ii) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

- 7. Vector product of vectors:** If $\vec{a}$ and $\vec{b}$ are two vectors which include an angle θ with each other then their vector product is given by $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$, $\hat{n}$ is the unit normal vector

Properties of the Vector Product: (i) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ but $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$ [SAQ]

(ii) $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$

8. **Triangle law of vectors** : If two vectors are represented in magnitude and direction by the two sides of a triangle taken in an order, then their resultant is represented by the third side taken in the reverse order. [VSAQ]

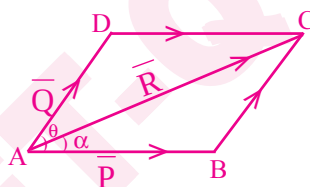


9. **Parallelogram law of vectors** : If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented by the diagonal passing through the same point. [SAQ]

Let θ be the angle between the $\vec{P}, \vec{Q}$ vectors and $\vec{R}$ be the resultant vector.

Let the resultant vector $\vec{R}$ makes an angle α with $\vec{P}$ then

$$(i) R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad (ii) \alpha = \tan^{-1} \left(\frac{Q \sin\theta}{P + Q \cos\theta} \right)$$



10. **Relative velocity in two dimensions:**

Suppose two objects A and B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$.

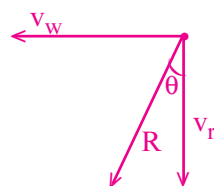
Then, velocity of object A relative to B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Velocity of object B relative to A is $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Here $\vec{v}_{AB} = -\vec{v}_{BA}$, but $|\vec{v}_{AB}| = |\vec{v}_{BA}|$

Note: Rain-umbrella : If velocity of rain is $\vec{v}_r$ and velocity of wind $\vec{v}_w$ then

$$\text{Resultant } R = \sqrt{v_r^2 + v_w^2} \quad \text{and} \quad \tan\theta = \frac{v_w}{v_r}$$



Projectile Motion:

11. **Projectile and Trajectory** : A 'body projected in to air' with some initial velocity making an oblique angle (θ) to the horizontal, is called a projectile and its path is called Trajectory.

Note : The path of a projectile is a Parabola. [SAQ]

Note: Equation of trajectory is $y = (\tan\theta)x - \left(\frac{g}{2u^2 \cos^2\theta} \right)x^2$, (a parabola) [VSAQ]

Ex: The 'sixer' hit by a batsman in the cricket game, a **Javelin** throw by an athlete.

Circular Motion (the following concepts comes again in chapter 6):

- 12.1 **Uniform circular motion** : If a particle moves along a circle with constant speed, then it is said to be in uniform circular motion.
- 12.2 **Period of revolution (T)** is the time taken for **one revolution**.
- 12.3 **Frequency of revolution (n)** is the number of revolutions per second.
13. **Angular displacement (θ)** is the angle described by the radius vector at the centre, in a given interval of time. **SI unit** : radian.

14. Angular velocity (ω) is the rate of angular displacement. **SI unit :** rad s^{-1} .

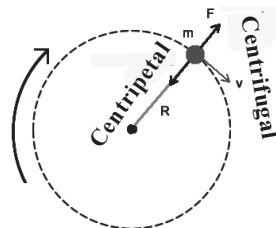
Formula: $\omega = \frac{\theta}{t}$ (or) $\omega = \frac{v}{r}$ [$\because v = r\omega$] (or) $\omega = 2\pi n$, n is frequency.

15. Angular acceleration (α) is the rate of angular velocity.

SI unit : rad s^{-2}

16.1 Centrifugal force(CFF) is the outward tangential force experienced by a particle in circular motion.

Ex: Sparks of Bhoochakra/ Vishnuchakra (Diwali crackers)



16.2 Centripetal force (CPF) is the inward force directed towards the centre along the radius. It maintains the body to move in a circular path.

Ex: Force towards centre in whirlpool of seas.

16.3 Centripetal acceleration (a_c) is the linear acceleration directed towards the centre along

the radius of the circle. **Formulas:** $a_c = \frac{v^2}{r}$ (or) $a_c = \omega^2 r$ [$\because v = r\omega$]

For better concept clarity refer **PHYSI BEATS P.No: 261**

Imp. Formulae

- If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ then $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
- If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ then $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
- Resultant vector $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$; $\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$
- If $\vec{P}, \vec{Q}$ are perpendicular vectors then $R = \sqrt{P^2 + Q^2}$
- Velocity of projectile $v = \sqrt{v_x^2 + v_y^2}$
- Rain-umbrella :** $R = \sqrt{v_r^2 + v_w^2}$ and $\tan \theta = \frac{v_w}{v_r}$
- Time of ascent $t = \frac{u \sin \theta}{g}$ = Time of descent
- Time of flight $T = 2t = \frac{2u \sin \theta}{g}$ [SAQ] 9. Maximum height $h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$ [SAQ]
- Range $R = \frac{u^2 \sin 2\theta}{g}$; 10. $R_{\max} = \frac{u^2}{g}$, ($\because \theta = 45^\circ$)
- Velocity $v = \frac{dr}{dt}$; acceleration $a = \frac{dv}{dt}$ 12. $\omega = \frac{\theta}{t}$ (or) $\omega = \frac{v}{r}$ (or) $\omega = 2\pi n$
- Linear Velocity $v = r\omega$ 14. $a_c = \frac{v^2}{r}$ (or) $a_c = r\omega^2$ [$\because \omega = \frac{v}{r}$]

1. SCALAR

- A physical quantity which has only magnitude but no direction.
- Ex : Distance, Mass, Speed, Time, Work etc.

[VSAQ]

2. VECTOR

- A physical quantity which has both magnitude and direction and obeys law of vector addition.
- Ex : Displacement, Weight, Velocity, Acceleration, Force, Momentum, Torque etc

3. ZERO VECTOR (NULL VECTOR)

- A vector of zero magnitude and arbitrary direction is called a zero vector, denoted by 0 or O.
- Significance: A null vector represents a point at rest.

[VSAQ]

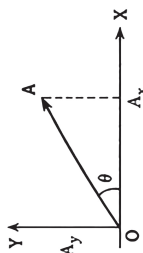
4. UNIT VECTOR

- A vector of magnitude one unit is called a unit vector.
- If a is a non-zero vector, then the unit vector in the direction of a is denoted by $\hat{a}$.

$$\hat{a} = \frac{a}{|a|}$$

5. RESOLUTION OF A VECTOR IN A PLANE INTO TWO RECTANGULAR COMPONENTS

- Consider $A = A\hat{i} + A\hat{j}$ making an angle θ with the X-axis.
- This vector $\hat{A}$ can be resolved into two rectangular components.
- Horizontal component (along X-axis): $A_x = A \cos \theta$
- Vertical component (along Y-axis): $A_y = A \sin \theta$



6. SCALAR PRODUCT OF VECTORS

- If a and b are two vectors which include an angle θ with each other, then their scalar product is given by $a \cdot b = |a||b| \cos \theta$
- It is a scalar quantity.
- If $A = A\hat{i} + A\hat{j} + A\hat{k}$ and $B = B\hat{i} + B\hat{j} + B\hat{k}$, then $A \cdot B = A_x B_x + A_y B_y + A_z B_z$

[SAQ]

Properties of the Scalar Product:

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

7. VECTOR PRODUCT OF VECTORS

- If a and b are two vectors which include an angle θ with each other, then their vector product is given by $a \times b = |\vec{a}||\vec{b}| \sin \theta \hat{n}$, where $\hat{n}$ is the unit normal vector.

[Properties of the Vector Product:]

- $a \times b \neq b \times a$ but $a \times b = -(b \times a)$
- $a \times (b + c) = a \times b + a \times c$

8. TRIANGLE LAW OF VECTORS

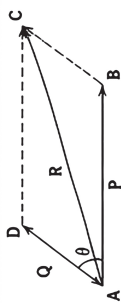
- If two vectors are represented in magnitude and direction by the two sides of a triangle taken in an order, then their resultant is represented by the third side taken in the reverse order.

[VSAQ]

9. PARALLELOGRAM LAW OF VECTORS

- If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented by the diagonal passing through the same point.

$$(i) R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \quad (ii) \alpha = \tan^{-1} \left[\frac{Q \sin \theta}{P + Q \cos \theta} \right]$$



3. MOTION IN A PLANE

CONCEPT MAP

CONCEPTS AND FORMULAS

10. RELATIVE VELOCITY IN TWO DIMENSIONS

- Velocity of object A relative to B is $v_{A/B} = v_A - v_B$
- Velocity of object B relative to A is $v_{B/A} = v_B - v_A$
- Note: Rain-umbrella : If velocity of rain is v_r and velocity of wind is v_w , then

$$\text{Resultant } R = \sqrt{v_r^2 + v_w^2} \quad \text{and} \quad \tan \theta = \frac{v_w}{v_r}$$

11. PROJECTILE MOTION

- Projectile and Trajectory : A body projected in air with some initial velocity making an oblique angle (θ) to the horizontal is called a projectile and its path is called Trajectory.
- Note : The path of a projectile is a Parabola.

$$\text{Equation of trajectory is } y = (\tan \theta)x - \frac{g}{2u^2 \cos^2 \theta} x^2, \text{ (a parabola)}$$

Equations of Projectile Motion:

- Time of ascent $t = \frac{u \sin \theta}{g}$ = Time of descent
- Time of flight $T = 2t = \frac{2u \sin \theta}{g}$
- Maximum height $h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$
- Range $R = \frac{u^2 \sin 2\theta}{g}$

[SAQ]