

2. TRANSFORMATION OF AXES

1 x 4 = 4 Marks

 IMP FORMULAS, KEY CONCEPTS 

I) TRANSLATION OF AXES (shifting of origin):

- 1.1)** To find (i) Transformed (new) equation (ii) Old coordinates of a given point, use the relations $x = X + h, y = Y + k$
- 1.2)** To find the (i) original equation (ii) new coordinates, use the relations $X = x - h, Y = y - k$
- 2.1)** The point to which the origin should be shifted to eliminate the first degree terms (i.e., x term & y term) in the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is
- $$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right), ab \neq h^2$$
- 2.2)** The point to which the axes be translated to eliminate x,y terms in the equation $ax^2 + by^2 + 2gx + 2fy + c = 0$ is $(-g/a, -f/b)$
- 2.3)** The point to which the axes be translated to eliminate x,y terms in $2hxy + 2gx + 2fy + c = 0$ is $(-f/h, -g/h)$
- 2.4)** The point to which the axes be translated to eliminate x, y terms in $(x - a)^2 + (y - b)^2 = k$ is (a, b)

II. ROTATION OF AXES:

- 1)** The formulae in the rotation of axes are given in the tabular form:

	X	Y
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

- 1.1)** To find (i) Transformed (new) equation (ii) old coordinates of a given point, use the relations $x = X\cos\theta - Y\sin\theta; y = Y\cos\theta + X\sin\theta$
- 1.2)** To find (i) the original equation (ii) the new coordinates of a given point, use the relations $X = x\cos\theta + y\sin\theta; Y = y\cos\theta - x\sin\theta$
- 2)** In order to eliminate the xy term in $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ the axes should be rotated through an angle $\theta = \frac{1}{2} \tan^{-1} \left(\frac{2h}{a-b} \right)$