

STAR QUESTIONS PLUS

THE STRAIGHT LINE

- Find the equation of the line containing the points $(2, -3)$ and $(0, -3)$
- Find the equation of the line containing the points $(1, 2)$ and $(1, -2)$
- Write the equations of the straight lines parallel to X-axis and (i) at a distance of 3 units above the X- axis and (ii) at a distance of 4 units below the X-axis.
- Write the equations of the straight lines parallel to Y-axis and (i) at a distance of 2 units from the Y-axis to the right of it (ii) at a distance of 5 units from the Y-axis to the left of it.
- Find the equation of the straight line, which make 150° with the X-axis in the positive direction and which pass through the point $(-2, -1)$.
- Find the sum of the squares of the intercepts of the line $4x - 3y = 12$ on the axes of co-ordinates.
- If the product of the intercepts made by the straight line $x \tan \alpha + y \sec \alpha = 1$, $\left(0 \leq \alpha < \frac{\pi}{2}\right)$, on the co-ordinates axes is equal to $\sin \alpha$, find α .
- Find the points on the line $3x - 4y - 1 = 0$ which are at a distance of 5 units from the point $(3, 2)$.

PAIR OF LINES

- Find the angle between the straight lines represented by $2x^2 + 5xy + 2y^2 - 5x - 7y + 3 = 0$
- Find the point of intersection of the pair of straight lines $x^2 + 4xy + 3y^2 - 4x - 10y + 3 = 0$

3-D GEOMETRY

- Find the centroid of the tetrahedron whose vertices are $(2, 3, 4)$, $(-3, 3, -2)$, $(-1, 4, 2)$, $(3, 5, 1)$
- Find the centroid of the triangle formed by $(5, 4, 6)$, $(1, -1, 3)$, $(4, 3, 2)$

SOLUTIONS

THE STRAIGHT LINE

1. Find the equation of the line containing the points (2, -3) and (0, -3)

Sol: We observe that both the y-coordinates of the given points are equal to -3.

∴ Equation of the line containing the points (2, -3) and (0, -3) is $y = -3 \Rightarrow y + 3 = 0$

2. Find the equation of the line containing the points (1, 2) and (1, -2)

Sol: We observe that both the x-coordinates of the given points are equal to 1.

∴ Equation of the line containing the points (1, 2) and (1, -2) is $x = 1 \Rightarrow x - 1 = 0$

3. Write the equations of the straight lines parallel to X-axis and (i) at a distance of 3 units above the X-axis and (ii) at a distance of 4 units below the X-axis.

Sol: Equation of any line parallel to X-axis is of the form $y = \pm k$.

(i) Equation of the line parallel to X-axis, at a distance 3 units above the X-axis is $y = 3$

(ii) Equation of the line parallel to X-axis, at a distance 4 units below the X-axis is $y = -4$

4. Write the equations of the straight lines parallel to Y-axis and (i) at a distance of 2 units from the Y-axis to the right of it (ii) at a distance of 5 units from the Y-axis to the left of it.

Sol: Equation of any line parallel to Y-axis is of the form $x = \pm k$.

(i) Equation of the line parallel to Y-axis, at a distance 2 units from the Y-axis to the right of it is $x = 2$

(ii) Equation of the line parallel to Y-axis, at a distance 5 units from the Y-axis to the left of it is $x = -5$

5. Find the equation of the straight line, which make 150° with the X-axis in the positive direction and which pass through the point (-2, -1).

Sol: Inclination of the line is $\theta = 150^\circ \Rightarrow \text{slope } m = \tan \theta = \tan 150^\circ$

$$= \tan(180^\circ - 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$$

∴ Equation of the line through (-2, -1) having slope $-\frac{1}{\sqrt{3}}$ is $y - y_1 = m(x - x_1)$

$$\Rightarrow y + 1 = -\frac{1}{\sqrt{3}}(x + 2) \Rightarrow \sqrt{3}(y + 1) = -(x + 2) \Rightarrow \sqrt{3}y + \sqrt{3} = -x - 2$$

$$\Rightarrow x + \sqrt{3}y + (2 + \sqrt{3}) = 0$$

6. Find the sum of the squares of the intercepts of the line $4x - 3y = 12$

on the axes of co-ordinates.

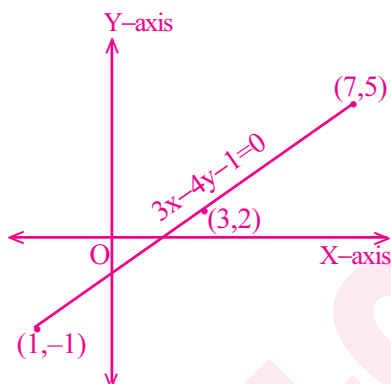
Sol: Given line is $4x - 3y = 12$

$$\Rightarrow \frac{4x}{12} - \frac{3y}{12} = 1 \Rightarrow \frac{x}{3} - \frac{y}{4} = 1$$

$\Rightarrow$ X-intercept = 3 and Y-intercept = -4

$\therefore$ Sum of the squares of the intercepts

$$= 3^2 + (-4)^2 = 9 + 16 = 25.$$



7. If the product of the intercepts made by the straight line $x \tan \alpha + y \sec \alpha = 1$, $\left(0 \leq \alpha < \frac{\pi}{2}\right)$, on the co-ordinates axes is equal to $\sin \alpha$, find α .

Sol: Given line is $x \tan \alpha + y \sec \alpha = 1 \Rightarrow \frac{x}{\cot \alpha} + \frac{y}{\cos \alpha} = 1$

Product of intercepts is $\sin \alpha \Rightarrow (\cot \alpha)(\cos \alpha) = \sin \alpha \Rightarrow \left(\frac{\cos \alpha}{\sin \alpha}\right)(\cos \alpha) = \sin \alpha$

$$\Rightarrow \cos^2 \alpha = \sin^2 \alpha \Rightarrow \tan^2 \alpha = 1 \Rightarrow \tan \alpha = \pm 1 \Rightarrow \alpha = 45^\circ, \left(0 \leq \alpha < \frac{\pi}{2}\right)$$

8. Find the points on the line $3x - 4y - 1 = 0$ which are at a distance of 5 units from the point $(3, 2)$.

Sol : Here $P(x_1, y_1) = (3, 2)$ and $|r| = 5$

Slope of the line $3x - 4y - 1 = 0$ is $m = -\frac{a}{b} = -\left(\frac{3}{-4}\right) = \frac{3}{4}$

$$\therefore m = \tan \theta = \frac{3}{4} \Rightarrow \cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}$$

Required points are given by $(x, y) = (x_1 \pm r \cos \theta, y_1 \pm r \sin \theta)$

$$= \left(3 + 5\left(\frac{4}{5}\right), 2 + 5\left(\frac{3}{5}\right)\right) = (3 + 4, 2 + 3) = (7, 5)$$

$$\text{and } \left(3 - 5\left(\frac{4}{5}\right), 2 - 5\left(\frac{3}{5}\right)\right) = (3 - 4, 2 - 3) = (-1, -1)$$

$\therefore$ The required points are $(7, 5)$ and $(-1, -1)$

PAIR OF LINES

9. Find the angle between the straight lines represented by $2x^2 + 5xy + 2y^2 - 5x - 7y + 3 = 0$

Sol: Comparing $2x^2 + 5xy + 2y^2 - 5x - 7y + 3 = 0$ with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get $a = 2$, $2h = 5$ and $b = 2$

$$\cos \theta = \frac{|a+b|}{\sqrt{(a-b)^2 + (2h)^2}} = \frac{|2+2|}{\sqrt{(2-2)^2 + (5)^2}} = \frac{|4|}{\sqrt{25}} = \frac{4}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\theta = \cos^{-1} \frac{a+b}{\sqrt{(a-b)^2 + (2h)^2}} = \cos^{-1} \frac{4}{\sqrt{0+5^2}} = \cos^{-1}\left(\frac{4}{5}\right)$$

10. Find the point of intersection of the pair of straight lines $x^2 + 4xy + 3y^2 - 4x - 10y + 3 = 0$

Sol: Comparing the given equation with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get

$$a=1, 2h=4 \Rightarrow h=4/2=2, b=3, 2g=-4 \Rightarrow g=-4/2=-2, 2f=-10 \Rightarrow f=-10/2=-5, c=3$$

$$\text{Now, the point of intersection is } \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

$$= \left(\frac{(2)(-5) - 3(-2)}{1(3) - 4}, \frac{-2(2) - 1(-5)}{1(3) - 4} \right) = \left(\frac{-10 + 6}{3 - 4}, \frac{-4 + 5}{3 - 4} \right) = \left(\frac{-4}{-1}, \frac{1}{-1} \right) = (4, -1)$$

3-D GEOMETRY

11. Find the centroid of the tetrahedron whose vertices are $(2,3,-4)$, $(-3,3,-2)$, $(-1,4,2)$, $(3,5,1)$

Sol: Let $A = (2,3,-4)$, $B = (-3,3,-2)$, $C = (-1,4,2)$, $D = (3,5,1)$

$$\text{Centroid } G = \left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4} \right)$$

$$= \left(\frac{2 - 3 - 1 + 3}{4}, \frac{3 + 3 + 4 + 5}{4}, \frac{-4 - 2 + 2 + 1}{4} \right) = \left(\frac{1}{4}, \frac{15}{4}, \frac{-3}{4} \right)$$

12. Find the centroid of the triangle formed by $(5,4,6)$, $(1,-1,3)$, $(4,3,2)$

Sol: Centroid $G = \left(\frac{5+1+4}{3}, \frac{4-1+3}{3}, \frac{6+3+2}{3} \right) = \left(\frac{10}{3}, 2, \frac{11}{3} \right)$