

LAQ PQ QUESTIONS

1. Find the circumcentre of triangle whose vertices are $(-2, 3)$, $(2, -1)$, $(4, 0)$
2. Find the orthocentre of the triangle whose vertices are $(-5, -7)$, $(13, 2)$, $(-5, 6)$
3. Find the orthocentre of the triangle whose sides are $x+y+10=0$, $x-y-2=0$, $2x+y-7=0$
4. Find the circumcentre of the triangle whose sides are $3x-y-5=0$, $x+2y-4=0$, $5x+3y+1=0$
5. If $xy=yx$ then show that $\frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$

SAQ PQ QUESTIONS

6. Prove that the equation $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines.
7. Find the distances between pairs of parallel straight lines $x^2 + 2\sqrt{3}xy + 3y^2 - 3x - 3\sqrt{3}y - 4 = 0$
8. Find the equations of the tangent to the curve $y=3x^2-x^3$, where it meets the X-axis.

VSAQ PQ QUESTIONS

9. Show that the points $(5,4,2)$, $(6,2,-1)$ and $(8,-2,-7)$ are collinear.
10. S.T the points $A(3,-2,4)$, $B(1, 1, 1)$, $C(-1, 4, -2)$ are collinear

LAQ PQ SOLUTIONS

1. Find the circumcentre of triangle whose vertices are $(-2, 3)$, $(2, -1)$, $(4, 0)$

Sol: We take circumcentre as $S(x, y)$ and vertices $A=(-2, 3)$, $B=(2, -1)$, $C=(4, 0)$, then

$$SA = SB = SC \quad (\because S \text{ is Equidistant from } A, B, C)$$

$$\text{First we take, } SA = SB \Rightarrow \sqrt{(x+2)^2 + (y-3)^2} = \sqrt{(x-2)^2 + (y+1)^2}$$

$$\text{Squaring on both sides we get } (x+2)^2 + (y-3)^2 = (x-2)^2 + (y+1)^2$$

$$\Rightarrow (x^2 + 4 + 4x) + (y^2 + 9 - 6y) = (x^2 + 4 - 4x) + (y^2 + 1 + 2y)$$

$$\Rightarrow 4x - 6y + 13 = -4x + 2y + 5 \Rightarrow 8x - 8y + 8 = 0 \Rightarrow x - y + 1 = 0 \dots\dots\dots(1)$$

$$\text{Now we take } SB = SC \Rightarrow \sqrt{(x-2)^2 + (y+1)^2} = \sqrt{(x-4)^2 + (y-0)^2}$$

$$\text{Squaring on both sides we get } (x-2)^2 + (y+1)^2 = (x-4)^2 + (y-0)^2$$

$$\Rightarrow (x^2 + 4 - 4x) + (y^2 + 1 + 2y) = (x^2 + 16 - 8x) + (y^2) \Rightarrow -4x + 2y + 5 = -8x + 16$$

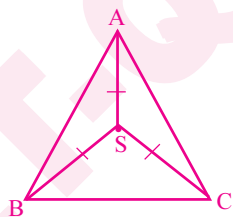
$$\Rightarrow -4x + 2y + 5 + 8x - 16 = 0 \Rightarrow 4x + 2y - 11 = 0 \dots\dots\dots(2)$$

$$\text{Solving (1) and (2), we get } S; \quad \text{From (1), } x = y - 1 \dots\dots\dots(3)$$

$$\text{From (2), } 4(y-1) + 2y - 11 = 0 \Rightarrow 4y - 4 + 2y - 11 = 0 \Rightarrow 6y - 15 = 0 \Rightarrow 6y = 15 \Rightarrow y = \frac{5}{2}$$

$$\text{From (3), } x = \frac{5}{2} - 1 = \frac{3}{2} \Rightarrow x = \frac{3}{2}$$

$$\therefore \text{Circumcentre } S(x, y) = \left(\frac{3}{2}, \frac{5}{2} \right)$$



2. Find the orthocentre of the triangle whose vertices are $(-5, -7)$, $(13, 2)$, $(-5, 6)$

Sol: We take O as orthocentre and vertices $A = (-5, -7)$, $B = (13, 2)$, $C = (-5, 6)$

To find altitude through A $(-5, -7)$:

$$\text{Slope of } \overline{BC} \text{ is } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{-5 - 13} = \frac{4}{-18} = \frac{-2}{9}$$

$$\text{So, slope of its perpendicular is } \frac{9}{2} \quad [\because m_1 m_2 = -1]$$

$$\text{Equation of altitude through A}(-5, -7) \text{ having slope } \frac{9}{2} \text{ is}$$

$$y + 7 = \frac{9}{2}(x + 5) \Rightarrow 2y + 14 = 9x + 45 \Rightarrow 9x - 2y + 31 = 0 \dots\dots (1)$$

To find altitude through B $(13, 2)$:

$$\text{Slope of } \overline{AC} \text{ is } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 + 7}{-5 + 5} = \frac{13}{0} = \infty$$

$$\text{So, slope of its perpendicular is } 0 \quad [\because m_1 m_2 = -1]$$

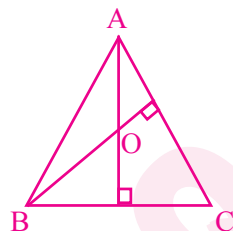
$$\text{Equation of altitude through B}(13, 2) \text{ having slope } 0 \text{ is } y - 2 = 0 \dots\dots (2)$$

Solving (1), (2), we get the orthocentre O;

$$\text{From (2), } y = 2$$

$$\text{From (1), } 9x - 2(2) + 31 = 0 \Rightarrow 9x + 27 = 0 \Rightarrow x + 3 = 0 \Rightarrow x = -3$$

$$\therefore \text{Orthocentre } O(x, y) = (-3, 2)$$



3. Find the orthocentre of the triangle whose sides are $x+y+10=0$, $x-y-2=0$, $2x+y-7=0$

Sol: Let $x+y+10=0$(1), $x-y-2=0$ (2), $2x+y-7=0$ (3)

represent the sides of $\triangle ABC$.

Let O be the orthocentre of $\triangle ABC$

First we find the equation of the altitude through A

Solving (1) and (2), we get A; $x+y+10=0$

$$x-y-2=0$$

$$\Rightarrow \frac{x}{-2+10} = \frac{y}{10+2} = \frac{1}{-1-1} \Rightarrow \frac{x}{8} = \frac{y}{12} = \frac{-1}{2}$$

$$\Rightarrow x = -4, y = -6 \quad \therefore A = (-4, -6)$$

The slope of the opposite side BC, $2x+y-7=0$ is -2

$\Rightarrow$ the slope of its perpendicular is $1/2$

The equation of the altitude passing through A(-4,-6) and with slope $1/2$ is

$$y+6=(1/2)(x+4) \Rightarrow 2y+12=x+4=0 \Rightarrow x-2y-8=0 \quad \dots (4)$$

Now, we find the equation of the altitude through B

Solving (1) & (3) we get B; $x+y+10=0$

$$2x+y-7=0$$

$$\Rightarrow \frac{x}{-7-10} = \frac{y}{20+7} = \frac{1}{1-2} \Rightarrow \frac{x}{-17} = \frac{y}{27} = -1$$

$$\Rightarrow x = 17, y = -27 \quad \therefore B = (17, -27)$$

The slope of the opposite side AC, $x-y-2=0$ is 1

$\Rightarrow$ the slope of its perpendicular is -1

The equation of the altitude through B(17,-27) and with slope -1 is $y+27=-1(x-17)$

$$\Rightarrow x+y+10=0 \quad \dots (5)$$

Solving (4) & (5) we get orthocentre O;

$$(4)-(5) \Rightarrow -3y-18=0 \Rightarrow 3y=-18 \Rightarrow y=-6$$

$$\text{From (4), } x=2y+8=2(-6)+8=-12+8=-4. \quad \therefore \text{Orthocentre} = (-4, -6)$$

4. Find the circumcentre of the triangle whose sides are $3x-y-5=0$, $x+2y-4=0$, $5x+3y+1=0$

Sol: Given lines are $3x-y-5=0$ (1), $x+2y-4=0$ (2), $5x+3y+1=0$ (3)

Solving (1) & (2), we get A; $3x-y-5=0$;
 $x+2y-4=0$

$$\Rightarrow \frac{x}{(-1)(-4) - 2(-5)} = \frac{y}{(-5)(1) - (-4)3} = \frac{1}{3(2) - 1(-1)}$$

$$\Rightarrow \frac{x}{4+10} = \frac{y}{-5+12} = \frac{1}{6+1} \Rightarrow \frac{x}{14} = \frac{y}{7} = \frac{1}{7} \Rightarrow x = \frac{14}{7} = 2, y = \frac{7}{7} = 1 \quad \therefore A = (2,1)$$

Solving (1) & (3), we get B; $3x-y-5=0$
 $5x+3y+1=0$

$$\Rightarrow \frac{x}{(-1)(1) - 3(-5)} = \frac{y}{(-5)(5) - 1(3)} = \frac{1}{3(3) - 5(-1)}$$

$$\Rightarrow \frac{x}{14} = \frac{y}{-28} = \frac{1}{14} \Rightarrow x = \frac{14}{14} = 1, y = \frac{-28}{14} = -2 \quad \therefore B = (1, -2)$$

Solving (2) & (3), we get C; $x+2y-4=0$
 $5x+3y+1=0$

$$\Rightarrow \frac{x}{2(1) - 3(-4)} = \frac{y}{(-4)(5) - 1(1)} = \frac{1}{1(3) - 5(2)} \Rightarrow \frac{x}{2+12} = \frac{y}{-20-1} = \frac{1}{3-10}$$

$$\Rightarrow \frac{x}{14} = \frac{y}{-21} = \frac{1}{-7} \Rightarrow x = -\frac{14}{7} = -2, y = \frac{-21}{-7} = 3 \quad \therefore C = (-2, 3)$$

We take S(x,y) as circumcentre and vertices $A=(1,-2)$, $B=(2,1)$, $C=(-2,3)$.

$$\text{So, } SA = SB \Rightarrow \sqrt{(x-1)^2 + (y+2)^2} = \sqrt{(x-2)^2 + (y-1)^2}$$

$$\Rightarrow (x^2 + 1 - 2x) + (y^2 + 4 + 4y) = (x^2 + 4 - 4x) + (y^2 + 1 - 2y)$$

$$\Rightarrow -2x + 4x + 4y + 2y + 1 + 4 - 4 - 1 = 0$$

$$\Rightarrow 2x + 6y = 0 \Rightarrow 2(x + 3y) = 0 \Rightarrow x + 3y = 0 \quad \dots(4)$$

$$\text{Also, } SB = SC \Rightarrow \sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x+2)^2 + (y-3)^2}$$

$$\Rightarrow (x^2 + 4 - 4x) + (y^2 + 1 - 2y) = (x^2 + 4 + 4x) + (y^2 + 9 - 6y)$$

$$\Rightarrow -4x - 4x - 2y + 6y + 4 + 1 - 4 - 9 = 0$$

$$\Rightarrow -8x + 4y - 8 = 0 \Rightarrow -4(2x - y + 2) = 0 \Rightarrow 2x - y + 2 = 0 \quad \dots(5)$$

Solving (4) & (5) we get S; $x + 3y + 0 = 0$

$$\Rightarrow \frac{x}{3(2) - (-1)(0)} = \frac{2x - y + 2 = 0}{0(2) - 2(1)} = \frac{1}{(-1) - 2(3)} \Rightarrow \frac{x}{6} = \frac{y}{-2} = \frac{1}{-7} \Rightarrow x = -\frac{6}{7}, y = \frac{2}{7}$$

$$\therefore \text{Circumcentre } S(x,y) = \left(-\frac{6}{7}, \frac{2}{7}\right)$$

5. If $x^y = y^x$ then show that $\frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$

Sol : Given that $x^y = y^x \Rightarrow \log x^y = \log y^x \Rightarrow y \log x = x \log y$

Now, differentiating w.r.t x, we get $y \left(\frac{1}{x} \right) + \log x \left(\frac{dy}{dx} \right) = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y (1)$

$$\Rightarrow \frac{y}{x} + \log x \frac{dy}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y \Rightarrow \frac{dy}{dx} \left(\log x - \frac{x}{y} \right) = \log y - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} \left(\frac{y \log x - x}{y} \right) = \frac{x \log y - y}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x} \left(\frac{x \log y - y}{y \log x - x} \right)$$

SAQ PQ SOLUTIONS

6. Prove that the equation $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines.

Sol: Comparing the given equation with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

we get $a = 2, 2h = 3 \Rightarrow h = 3/2, b = -2, 2g = 3 \Rightarrow g = 3/2, 2f = 1 \Rightarrow f = 1/2, c = 1$

(i) Consider $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$$\begin{aligned} &= 2(-2) + 2 \left(\frac{1}{2} \right) \left(\frac{3}{2} \right) \left(\frac{3}{2} \right) - 2 \left(\frac{1}{2} \right)^2 + 2 \left(\frac{3}{2} \right)^2 - 1 \left(\frac{3}{2} \right)^2 \\ &= -4 + \frac{9}{4} - \frac{1}{2} + \frac{9}{2} - \frac{9}{4} = \frac{-16 + 9 - 2 + 18 - 9}{4} = \frac{0}{4} = 0 \quad \therefore \Delta = 0 \end{aligned}$$

(ii) $h^2 - ab = \frac{9}{4} + 4 > 0, g^2 - ac = \frac{1}{4} + 2 > 0, f^2 - bc = \frac{9}{4} - 2 = \frac{1}{4} > 0 \Rightarrow h^2 > ab, g^2 > ac, f^2 > bc$

(iii) Also $a+b = 2 - 2 = 0$.

Hence the given equation represents a pair of perpendicular lines.

7. Find the distances between pairs of parallel straight lines $x^2 + 2\sqrt{3}xy + 3y^2 - 3x - 3\sqrt{3}y - 4 = 0$

Sol: Comparing the given equation with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

we get $a = 1$, $2h = 2\sqrt{3} \Rightarrow h = \sqrt{3}$, $b = 3$,

$$2g = -3 \Rightarrow g = -\frac{3}{2}, 2f = -3\sqrt{3} \Rightarrow f = -\frac{3\sqrt{3}}{2}, c = -4$$

The distance between the pair of parallel lines is given by

$$2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\sqrt{\frac{\left(-\frac{3}{2}\right)^2 - (1)(-4)}{1(1+3)}} = 2\sqrt{\frac{\frac{9}{4} + 4}{4}} = 2\sqrt{\frac{\frac{25}{4}}{4}} = 2\sqrt{\frac{25}{16}} = 2 \cdot \frac{5}{4} = \frac{5}{2}$$

8. Find the equations of the tangent to the curve $y = 3x^2 - x^3$, where it meets the X-axis.

Sol: The equation of the X-axis is $y = 0$.

Putting $y = 0$ in $y = 3x^2 - x^3$, we get the points of intersection of the curve and X-axis.

$$\therefore y = 0 \Rightarrow 3x^2 - x^3 = 0 \Rightarrow x^2(3 - x) = 0 \Rightarrow x = 0, x = 3$$

Thus the curve meets the X-axis at the points $O(0,0)$ and $A(3,0)$

$$\text{Now, } y = 3x^2 - x^3 \Rightarrow \frac{dy}{dx} = 6x - 3x^2$$

Slope of the tangent at $O(0,0)$ is $m_1 = 6(0) - 3(0) = 0$

$\therefore$ The equation of the tangent at $(0,0)$ with slope 0 is $y - 0 = 0(x - 0) \Rightarrow y = 0$

Slope of the tangent at $A(3,0)$ is $m_2 = 6(3) - 3(3)^2 = 18 - 27 = -9$

$\therefore$ Equation of the tangent at $(3,0)$ with slope -9 is $y - 0 = -9(x - 3) \Rightarrow 9x + y - 27 = 0$

VSAQ PQ SOLUTIONS

9. Show that the points (5,4,2), (6,2,-1) and (8,-2,-7) are collinear.

Sol : Let A=(5,4,2), B=(6,2,-1), C=(8,-2,-7)

$$AB = \sqrt{(5-6)^2 + (4-2)^2 + (2+1)^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$BC = \sqrt{(6-8)^2 + (2+2)^2 + (-1+7)^2} = \sqrt{4+16+36} = \sqrt{56} = 2\sqrt{14}$$

$$CA = \sqrt{(8-5)^2 + (-2-4)^2 + (-7-2)^2} = \sqrt{9+36+81} = \sqrt{126} = 3\sqrt{14}$$

$$\text{Now, } AB + BC = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = AC$$

∴ A,B,C are collinear

10. S.T the points A(3,-2,4), B(1, 1, 1), C(-1, 4, -2) are collinear

Sol : Let A = (3,-2,4), B = (1, 1, 1), C = (-1, 4, -2)

$$AB = \sqrt{(3-1)^2 + (-2-1)^2 + (4-1)^2} = \sqrt{4+9+9} = \sqrt{22}$$

$$BC = \sqrt{(1-(-1))^2 + (1-4)^2 + (1-(-2))^2} = \sqrt{4+9+9} = \sqrt{22}$$

$$CA = \sqrt{(-1-3)^2 + (4-(-2))^2 + (-2-4)^2} = \sqrt{16+36+36} = \sqrt{84} = 2\sqrt{22}$$

$$\text{Now, } AB + BC = \sqrt{22} + \sqrt{22} = 2\sqrt{22} = AC$$

∴ A,B,C are collinear