

10. INVERSE TRIGONOMETRIC FUNCTIONS

1 x 4 = 4 Marks

 IMP FORMULAS, KEY CONCEPTS 

- 1) $\sin^{-1}(\sin \theta) = \theta$ for $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
 $\cos^{-1}(\cos \theta) = \theta$ for $\theta \in [0, \pi]$
 $\tan^{-1}(\tan \theta) = \theta$ for $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sin(\sin^{-1}x) = x$ for $x \in [-1, 1]$
 $\cos(\cos^{-1}x) = x$ for $x \in [-1, 1]$
 $\tan(\tan^{-1}x) = x$ for $x \in \mathbb{R}$
- 2) $\sin^{-1}(-x) = -\sin^{-1}x$; $\cos^{-1}(-x) = \pi - \cos^{-1}x$; $\tan^{-1}(-x) = -\tan^{-1}x$; $\cot^{-1}(-x) = \pi - \cot^{-1}x$
- 3) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$; $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$; $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$
- 4) (i) $\sin^{-1}(x) = \operatorname{Cosec}^{-1}\frac{1}{x}$ for $x \in [-1, 0) \cup (0, 1]$ (ii) $\cos^{-1}(x) = \operatorname{Sec}^{-1}\frac{1}{x}$ for $x \in [-1, 0) \cup (0, 1]$
 (iii) $\cot^{-1}x = \tan^{-1}\frac{1}{x}$ if $x > 0$ (iv) $\cot^{-1}x = \pi + \tan^{-1}\frac{1}{x}$ if $x < 0$
- 5) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ if $x \geq 0, y \geq 0$ and $x^2 + y^2 \leq 1$
- 6) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$ if $x, y \in [0, 1]$
- 7) $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \text{if } xy < 1 \\ \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \pi, & \text{if } xy > 1, x > 0, y > 0 \\ \tan^{-1}\left(\frac{x+y}{1-xy}\right) - \pi, & \text{if } xy > 1, x < 0, y < 0 \\ \frac{\pi}{2}, & \text{if } xy = 1 \end{cases}$
- 8) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$
- 9) $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$
 $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$ $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$
 $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$