

10. APPLICATIONS OF DERIVATIVES

(2 x 2) + (2 x 4) = 12 Marks

 **IMP FORMULAS, KEY CONCEPTS** 

- 1) If $y = f(x)$ is a differentiable function of x and Δx is a small change in x then (i) the actual **change** in y is $\Delta y = f(x + \Delta x) - f(x)$ (ii) the **differential** of y is $dy = f'(x)\Delta x$.
- 2) **Approximations:** The approximate value of $f(x)$ in Δx neighbourhood of known x , is $f(x + \Delta x) \cong f(x) + f'(x)\Delta x$. Here, $\Delta x = \text{given value of } x - \text{known value of } x$.
- 3) If an error Δx occurs in x of $y = f(x)$ then (i) Δy is called **error** in y
 (ii) $\frac{\Delta y}{y}$ is called **relative error** in y (iii) $\frac{\Delta y}{y} \times 100$ is called **percentage error** in y .
- 4) If $P(x_1, y_1)$ is a point on $y = f(x)$ then the slope of the tangent at P is $m = \left(\frac{dy}{dx} \right)_{P(x_1, y_1)}$
- 5) If $P(x_1, y_1)$ is a point of intersection of the curves $f(x)$, $g(x)$ and θ is the angle between the 2 curves then $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ where $m_1 = (f'(x))$ at (x_1, y_1) , $m_2 = (g'(x))$ at (x_1, y_1)

Note: $m_1 = m_2 \Rightarrow$ the two curves touch each other at (x_1, y_1) and $m_1 m_2 = -1 \Rightarrow$ the 2 curves cut orthogonally.

- 6) If $P(x_1, y_1)$ is a point on the curve $y = f(x)$ and $m = \left(\frac{dy}{dx} \right)_{P(x_1, y_1)}$, then

(i) the **length of the tangent** to the curve at P is $\left| \frac{y_1 \sqrt{1 + m^2}}{m} \right|$

(ii) the **length of the normal** to the curve at P is $|y_1 \sqrt{1 + m^2}|$

(iii) the **length of subtangent** to the curve at P is $\left| \frac{y_1}{m} \right|$

(iv) the **length of subnormal** to the curve at P is $|y_1 m|$

- 7.1) The rate of change of $y = f(x)$ w.r.to t is given by $\frac{dy}{dt} = f'(x) \frac{dx}{dt}$

- 7.2) If $s = f(t)$ denotes the distance travelled by a body in time t then

(i) the velocity of the body at time t is $v = \frac{ds}{dt}$

(ii) the acceleration of the body at time t is $a = \frac{d^2 s}{dt^2} = \frac{d}{dt} \left(\frac{ds}{dt} \right) = \frac{dv}{dt}$

- 8.1) A differentiable function f is increasing at $a \Leftrightarrow f'(a) > 0$

- 8.2) A differentiable function f is decreasing at $a \Leftrightarrow f'(a) < 0$

- 8.3) A differentiable function f is said to be stationary at $x = a$ if $f'(a) = 0$

Here $f(a)$ is the stationary value & $(a, f(a))$ is the stationary point of $f(x)$ at $x = a$

- 9) Let $f(x)$ be a differentiable function in a given interval I , $a \in I$, $f(x)$, $f'(x)$ exist at a and if

(i) $f'(a) = 0$, $f''(a) < 0$ then $f(a)$ is a local maxima (ii) $f'(a) = 0$, $f''(a) > 0$ then $f(a)$ is a local minima

**BULLET MASTER'S
MATH BEATS!****APPLICATIONS OF DERIVATIVES**

పార్కులు పార్కులుగా Practice చేస్తే **Applications of Derivatives** అన్నీ Easy గానే ఉంటాయి!

IPE Weightage for Errors & Approximations: 2Marks

IPE Weightage for Rate Measure: 2Marks

IPE Weightage for Tangents & Normals: 4Marks

IPE Weightage for Maxima & Minima: 4Marks