Previous IPE

SOLVED PAPERS

MARCH -2026(TG)

PREVIOUS PAPERS

IPE: MARCH-2026 (TG)

MATHS-2B

Time : 3 Hours

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. Find the centre and radius of the circle $x^2 + y^2 + 2ax - 2by + b^2 = 0$.
2. Find the equation of the tangent at $(-1, 1)$ of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$
3. Find the equation of the radical axis of the circles $x^2 + y^2 - 2x - 4y - 1 = 0$, $x^2 + y^2 - 4x - 6y + 5 = 0$
4. Find the coordinates of the points on the parabola $y^2 = 8x$ whose focal distance is 10.
5. If e, e_1 are the eccentricities of a hyperbola and its conjugate hyperbola, prove that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$
6. Evaluate $\int \sec^2 x \operatorname{cosec}^2 x\, dx$ on $1 \subset R \setminus \left(\{n\pi : n \in Z\} \cup \left\{ (2n+1)\frac{n}{2} ; n \in Z \right\} \right)$
7. Evaluate $\int \frac{2x^3}{1+x^8} dx$ on R.
8. Evaluate $\int_0^a \frac{dx}{x^2 + a^2}$
9. Find $\int_0^{\frac{\pi}{2}} \cos^{11} x\, dx$
10. Find the order and degree of $\left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^3 \right]^{6/5} = 6y$

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. If the length of the tangent from $(2, 5)$ to the circle $x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{37}$ then find k.
12. Find the radical centre of the circles $x^2 + y^2 - 4x - 6y + 5 = 0$, $x^2 + y^2 - 2x - 4y - 1 = 0$, $x^2 + y^2 - 6x - 2y = 0$
13. Find the eccentricity of the ellipse (in standard form), if its length of the latus rectum is equal to half of its major axis.
14. Find the condition for the line $x \cos a + y \sin a = p$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
15. Find the centre, foci, eccentricity, equation of the directrices, length of the latus rectum of the hyperbola $16y^2 - 9x^2 = 144$.
16. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$
17. Solve $y(1 + x)\, dx + x(1 + y)dy = 0$

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. Find the equation of the circle which passes through (4, 1), (6, 5) and having the centre on $4x + 3y - 24 = 0$.

19. Show that $x^2 + y^2 - 6x - 9y + 13 = 0$, $x^2 + y^2 - 2x - 16y = 0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.

20. Derive the equation of the parabola in Standard form.

21. Evaluate $\int \frac{x+1}{x^2+3x+12}dx$

22. Obtain reduction formula for $\int \sin^n x\,dx$ for an integer $n \geq 2$ and hence deduce $\int \sin^4 x\,dx$

23. Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

24. Solve $(x^2+y^2)\,dx = 2xy\,dy$

IPE TG MARCH - 2026

SOLUTIONS

SECTION-A

1. Find the centre and radius of the circle $x^2+y^2+2ax-2by+b^2=0$.

Sol: Comparing with the given equation with $x^2+y^2+2gx+2fy+c=0$ we get

$g = a,\ f = -b$ and $c = b^2 \therefore$ Centre $C(-g,-f) = (-a, b)$

Radius $r=\sqrt{g^2+f^2-c}=\sqrt{a^2+b^2-b^2}=\sqrt{a^2}=a$

2: Find the equation of the tangent at (–1, 1) on $x^2+y^2-6x+4y-12=0$.

Sol: Given point $P(x_1,y_1)=(-1,1)$ and circle $S=x^2+y^2-6x+4y-12=0 \Rightarrow g=-3, f=2, c=-12$

$\therefore$ The equation of the tangent at $(-1,1)$ on $S=0$ is $S_1=0$

$\Rightarrow x_1x+y_1y+g(x_1+x)+f(y_1+y)+c=0$

$\Rightarrow -1(x)+1(y)-3(-1+x)+2(1+y)-12=0$

$\Rightarrow -4x+3y-7=0$

$\Rightarrow 4x-3y+7=0$

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3. Find the equation of the radical axis of $x^2+y^2-2x-4y-1=0$, $x^2+y^2-4x-6y+5=0$

Let $S\equiv x^2+y^2-2x-4y-1=0$ and $S'\equiv x^2+y^2-4x-6y+5=0$

Radical axis is given by $S-S'=0$

$\Rightarrow (x^2+y^2-2x-4y-1)-(x^2+y^2-4x-6y+5)=0$

$\Rightarrow -2x+4x-4y+6y-1-5=0 \Rightarrow 2x+2y-6=0$

$\Rightarrow 2(x+y-3)=0 \Rightarrow x+y-3=0.$

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4. Find the coordinates of the point on parabola $y^2=8x$, whose focal distance is 10.

Sol: Given parabola is $y^2=8x \Rightarrow 4a=8 \Rightarrow a=2$

Given focal distance SP = 10

Formula: Focal distance $SP=x_1+a$

$\Rightarrow x_1+2=10 \Rightarrow x_1=8.$

But, $y_1^2=8x_1 \Rightarrow y_1^2=8(8) \Rightarrow y_1=\pm 8$

$\therefore P(x_1,y_1)=(8,\pm 8)$

5. If e, e_1 are the eccentricities of a hyperbola and its conjugate hyperbola, then prove that $\frac{1}{e^2}+\frac{1}{e_1^2}=1$

Sol: e, e_1 are the eccentricities of the hyperbola $S=\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its conjugate hyperbola.

Then, $e=\sqrt{\frac{a^2+b^2}{a^2}}, e_1=\sqrt{\frac{a^2+b^2}{b^2}}$

$$\therefore \frac{1}{e^2}+\frac{1}{e_1^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2}=1$$

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6. Evaluate $\int \sec^2 x.\csc^2 x dx$

Sol: $\int \sec^2 x.\csc^2 x dx=\int \frac{1}{\cos^2 x \sin^2 x}dx$

$$=\int \frac{\sin^2 x+\cos^2 x}{\cos^2 x \sin^2 x}dx$$

$$=\int\left(\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}\right)dx$$

$$=\int(\sec^2 x+\csc^2 x)dx$$

$$=\int \sec^2 x dx+\int \csc^2 x dx$$

$$=\tan x-\cot x+c$$

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7. Evaluate $\int \frac{2x^3}{1+x^8}dx$.

Sol: Put $x^4=t \Rightarrow 4x^3dx=dt \Rightarrow 2x^3dx=\frac{1}{2}dt$

$$\therefore I=\int \frac{2x^3}{1+x^8}dx=\int \frac{2x^3}{1+(x^4)^2}dx=\frac{1}{2}\int \frac{dt}{1+t^2}=\frac{1}{2}\mathrm{Tan}^{-1}t+c=\frac{1}{2}\mathrm{Tan}^{-1}x^4+c$$

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8. Evaluate $\int_0^a \frac{dx}{x^2+a^2}$

Sol: $$I=\int_0^a \frac{dx}{x^2+a^2}=\left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]_0^a=\frac{1}{a}\left[\tan^{-1}(1)-\tan^{-1}(0)\right]=\frac{1}{a}\left[\frac{\pi}{4}-0\right]=\frac{\pi}{4a}$$

9. Evaluate $\int_0^{\pi/2} \cos^{11} x\, dx$

Sol : When n is odd,

$$\int_0^{\pi/2} \cos^n x dx = \frac{(n-1)(n-3)....}{n(n-2)....}\frac{2}{3}.1$$

$$\therefore \int_0^{\pi/2} \cos^{11} x\, dx = \frac{(10)(8)(6)(4)(2)}{(11)(9)(7)(5)(3)}(1) = \frac{256}{693}$$

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10. Find the order and degree of the differential equation $\left[\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right]^{\frac{6}{5}} = 6y$

Sol: Given D.E is $\left(\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right)^{6/5} = 6y \Rightarrow \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3 = (6y)^{5/6}$

Here, the highest order derivative is $\frac{d^2y}{dx^2}$ $\therefore$ order = 2

The exponent of $\frac{d^2y}{dx^2}$ is 1 $\therefore$ degree = 1

SECTION-B

11. Find the value of k if the length of the tangent from (2,5) to $x^2+y^2-5x+4y+k=0$ is $\sqrt{37}$.

Sol: Length of the tangent from (2, 5) to

$S = x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{S_{11}} = \sqrt{37}$;

On squaring both sides, we get $S_{11} = 37$

$\Rightarrow (2)^2 + 5^2 - 5(2) + 4(5) + k = 37$

$\Rightarrow 4 + 25 - 10 + 20 + k = 37$

$\Rightarrow 39 + k = 37 \Rightarrow k = -2$

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12. Find the radical centre of the circles $x^2 + y^2 - 4x - 6y + 5 = 0, x^2 + y^2 - 2x - 4y - 1 = 0$, $x^2 + y^2 - 6x - 2y = 0$.

Sol: The given circles are

$S = x^2 + y^2 - 4x - 6y + 5 = 0$,

$S' = x^2 + y^2 - 2x - 4y - 1 = 0$,

$S'' = x^2 + y^2 - 6x - 2y = 0$

One radical axis is $S - S' = 0$

$\Rightarrow (-4x + 2x) + (-6y + 4y) + (5 + 1) = 0$

$\Rightarrow -2x - 2y + 6 = 0 \Rightarrow -2(x + y - 3) = 0$

$\Rightarrow x + y - 3 = 0$ ..... (1)

Another radical axis is $S - S'' = 0$

$\Rightarrow (-4x + 6x) + (-6y + 2y) + 5 = 0$

$\Rightarrow 2x - 4y + 5 = 0$ .... (2)

Now, $(1) \times 2 \Rightarrow 2x + 2y - 6 = 0$ ....(3)

$(3) - (2) \Rightarrow 6y - 11 = 0 \Rightarrow y = 11/6$

From(1), $x = 3 - y = 3 - \frac{11}{6} = \frac{18-11}{6} = \frac{7}{6}$

$\therefore$ the radical centre is (7/6, 11/6)

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13. Find the eccentricity of the ellipse, in standard form, if its length of the latus rectum is equal to half of its major axis.

Sol: Given that Length of Latus rectum $= \frac{1}{2}(\text{length of major axis})$

$\Rightarrow \frac{2b^2}{a} = \frac{1}{2}(2a) \Rightarrow 2b^2 = a^2$

Now, $b^2 = a^2(1-e^2) = 2b^2(1-e^2) \Rightarrow 1 - e^2 = \frac{1}{2} \Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

14. Find the condition for the line xcosα+ysinα = p to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Sol: The tangential condition for the line lx+my+n = 0 and the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $n^2 = a^2l^2+b^2m^2$

The given line is x cosα + y sinα = p

$\Rightarrow l = \cos\alpha,\ m = \sin\alpha,\ n = -p$

$\therefore\ a^2l^2+b^2m^2 = n^2$

$\Rightarrow a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$

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15. Find the centre, eccentricity, foci, length of latus rectum and equations of the directrices of the Hyperbola $16y^2-9x^2=144$.

Sol: Given hyperbola is $16y^2-9x^2=144$

$$\Rightarrow\frac{16y^2}{144}-\frac{9x^2}{144}=1\Rightarrow\frac{y^2}{9}-\frac{x^2}{16}=1\Rightarrow\frac{x^2}{16}-\frac{y^2}{9}=-1$$

Here, $a^2=16,\ b^2=9$

i) Centre C= (0,0)

ii) $e=\sqrt{\frac{a^2+b^2}{b^2}}=\sqrt{\frac{16+9}{9}}=\sqrt{\frac{25}{9}}=\frac{5}{3}$

iii) Foci = (0, ±be) = (0, ±3(5/3)) = (0,±5)

iv) Equation of directrices is $y=\pm\frac{b}{e}\Rightarrow y=\pm\frac{3}{\frac{5}{3}}\Rightarrow y=\pm\frac{9}{5}$

v) Length of latusrectum $=\frac{2a^2}{b}=\frac{2(16)}{3}=\frac{32}{3}$

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16. Evaluate $\int_0^{\pi/2}\frac{\sin^2x-\cos^2x}{\sin^3x+\cos^3x}dx$

Sol: We know $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^{\pi/2}\frac{\sin^2x-\cos^2x}{\sin^3x+\cos^3x}dx \quad ....(1);$$

$$=\int_0^{\pi/2}\frac{\sin^2\left(\frac{\pi}{2}-x\right)-\cos^2\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)}dx=\int_0^{\pi/2}\frac{\cos^2x-\sin^2x}{\cos^3x+\sin^3x}dx \quad ....(2)$$

Adding (1) and (2) we get $2I=\int_0^{\pi/2}\frac{0}{\cos^3x+\sin^3x}dx=0 \qquad \therefore I=0$

17. Solve y(1 + x) dx + x(1 + y) dy = 0

Sol: Given D.E is $y(1+x)\,dx + x(1+y)\,dy = 0$

$\Rightarrow x(1+y)\,dy = -y(1+x)\,dx$

$$\Rightarrow \frac{(1+y)dy}{y} = -\frac{(1+x)dx}{x} \Rightarrow \int\frac{(1+y)dy}{y} = -\int\frac{(1+x)dx}{x} \Rightarrow \int\left(\frac{1}{y}+1\right)dy = -\int\left(\frac{1}{x}+1\right)dx$$

$\Rightarrow \log y + y = -\log x - x + c \Rightarrow y + x + \log y + \log x = c$

$\therefore$ The solution is $y + x + \log(yx) = c$

SECTION-C

18. Find the equation of the circle passing through (4,1) (6,5) and having the centre on the line 4x+3y–24=0

Sol: Let A = (4, 1), B = (6, 5).

We take $S(x_1,y_1)$ as the centre of the circle

$\Rightarrow SA = SB \Rightarrow SA^2 = SB^2$.

$\Rightarrow (x_1-4)^2+(y_1-1)^2 = (x_1-6)^2 + (y_1-5)^2$

$\Rightarrow (\cancel{x_1^2} - 8x_1 + 16) + (\cancel{y_1^2} - 2y_1 + 1) = (\cancel{x_1^2} - 12x_1 + 36) + (\cancel{y_1^2} - 10y_1 + 25)$

$\Rightarrow 17-8x_1-2y_1=61-12x_1-10y_1$

$\Rightarrow 17-8x_1-2y_1-61+12x_1+10y_1=0$

$\Rightarrow 4x_1+8y_1-44=0$..... (1)

But centre (x_1,y_1) lies on $4x+3y-24=0$

$\Rightarrow 4x_1+3y_1-24=0$ .... (2)

(2)–(1) $\Rightarrow -5y_1+20=0 \Rightarrow 5y_1=20 \Rightarrow y_1=4$

From (2), $4x_1+3(4)-24=0 \Rightarrow 4x_1-12=0$

$\Rightarrow 4x_1=12 \Rightarrow x_1=3$

$\therefore$ Centre of the circle $S(x_1,y_1) = (3,4)$. Also, we have A=(4,1)

So, radius $r=SA \Rightarrow r^2=SA^2$.

$\therefore r^2 = (3-4)^2 + (4-1)^2 = 1+9 = 10$

$\therefore$ Circle with centre (3,4) and $r^2=10$ is

$(x-3)^2+(y-4)^2=10$

$\Rightarrow (x^2+9-6x)+(y^2+16-8y)=10$

$\Rightarrow x^2+y^2-6x-8y+15=0$

19. Show that the circles $x^2+y^2-6x-9y+13=0$, $x^2+y^2-2x-16y=0$ touch each other. Find the point of contact and the equation of the common tangent at that point.

Sol: First circle is $S \equiv x^2 + y^2 - 6x - 9y + 13 = 0$

Centre $C_1 = (3, 9/2)$,

radius $r_1 = \sqrt{3^2 + \left(\frac{9}{2}\right)^2 - 13} = \sqrt{9 + \frac{81}{4} - 13}$

$= \sqrt{\frac{36+81-52}{4}} = \sqrt{\frac{65}{4}} = \frac{\sqrt{65}}{2}$

Second circle is $S' \equiv x^2+y^2-2x-16y=0$

Centre $C_2 = (1, 8)$,

radius $r_2 = \sqrt{1^2 + 8^2 - 0} = \sqrt{1+64} = \sqrt{65}$

$C_1C_2 = \sqrt{(3-1)^2 + \left(\frac{9}{2} - 8\right)^2} = \sqrt{2^2 + \left(\frac{9-16}{2}\right)^2}$

$= \sqrt{4 + \frac{49}{4}} = \sqrt{\frac{16+49}{4}} = \sqrt{\frac{65}{4}} = \frac{\sqrt{65}}{2}$

Also, $r_2 - r_1 = \sqrt{65} - \frac{\sqrt{65}}{2} = \frac{\sqrt{65}}{2} = C_1C_2$

$\therefore$ The circles touch each other **internally.**

Now, $r_1 : r_2 = \frac{\cancel{\sqrt{65}}}{2} : \cancel{\sqrt{65}} = \frac{1}{2} : 1 = 1 : 2$

So, the point of contact P divides the join of $C_1(3, 9/2)$, $C_2(1, 8)$ externally in the ratio 1:2

$\therefore\ P = \left(\frac{1(1) - 2(3)}{1-2}, \frac{1(8) - 2(\frac{9}{2})}{1-2} \right) = (5, 1)$

The equation of the common tangent to the circles $S = 0$ and $S' = 0$ at the point of contact is given by $S - S' = 0$

$\Rightarrow (\cancel{x^2} + \cancel{y^2} - 6x - 9y + 13) - (\cancel{x^2} + \cancel{y^2} - 2x - 16y) = 0$

$\Rightarrow -4x + 7y + 13 = 0$

$\Rightarrow 4x - 7y - 13 = 0$

20. Derive the standard form of the parabola.

Sol: Let S be the focus and L = 0 be the directrix of the parabola.

Let Z be the projection of S on to the directrix

Let A be the mid point of SZ

$\Rightarrow$ SA=AZ $\Rightarrow \dfrac{SA}{AZ} = 1$

$\Rightarrow$ A is a point on the parabola.

Take AS, the principle axis of the parabola as X-axis and

the line perpendicular to AS through A as the Y-axis $\Rightarrow A = (0, 0)$

Let AS = a $\Rightarrow$ S = (a, 0), Z= (–a,0)

$\Rightarrow$ the equation of the directrix is $x = -a \Rightarrow x+a=0$

Let $P(x_1, y_1)$ be any point on the parabola.

N be the projection of P on to the Y-axis.

M be the projection of P on to the directrix.

Here $PM=PN+NM=x_1+a$ $\quad (\because PN = x - \text{coordinate of P and } NM = AZ = AS = a)$

Now, by the focus directrix property of the parabola,

we have $\dfrac{SP}{PM} = 1 \Rightarrow SP = PM \Rightarrow SP^2 = PM^2$

$\Rightarrow (x_1 - a)^2 + (y_1 - 0)^2 = (x_1 + a)^2$

$\Rightarrow y_1^2 = (x_1 + a)^2 - (x_1 - a)^2$

$\Rightarrow y_1^2 = 4ax_1$ $\quad [\because (a+b)^2 - (a-b)^2 = 4ab]$

$\therefore$ The equation of locus of $P(x_1,y_1)$ is $y^2=4ax$

21. **Evaluate** $\int \frac{x+1}{x^2+3x+12}dx$

Sol: Let $x+1=A\frac{d}{dx}(x^2+3x+12)+B$ ....(i)

$\therefore x+1=A(2x+3)+B \Rightarrow x+1=2Ax+(3A+B)$

Equating the coefficients of 'x', we get $2A=1 \Rightarrow A=1/2$

Equating the constant terms, we get $3A+B=1 \Rightarrow B=1-3A=1-\frac{3}{2} \Rightarrow B=-\frac{1}{2}$

Putting $A=\frac{1}{2}, B=\frac{-1}{2}$ in (i), we get Nr. $x+1=\frac{1}{2}\frac{d}{dx}(x^2+3x+12)-\frac{1}{2}$

$$\therefore I=\int\frac{x+1}{x^2+3x+12}dx=\frac{1}{2}\int\frac{\frac{d}{dx}(x^2+3x+12)}{x^2+3x+12}dx-\frac{1}{2}\int\frac{dx}{x^2+3x+12}$$

$$=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{2}\int\frac{dx}{\left(x+\frac{3}{2}\right)^2+\left(\frac{\sqrt{39}}{2}\right)^2}$$

$$=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{\cancel{2}}\frac{\cancel{2}}{\sqrt{39}}\mathrm{Tan}^{-1}\left(\frac{x+\frac{3}{2}}{\frac{\sqrt{39}}{2}}\right)+c$$

$$=\frac{1}{2}\log\left|x^2+3x+12\right|-\frac{1}{\sqrt{39}}\mathrm{Tan}^{-1}\left(\frac{2x+3}{\sqrt{39}}\right)+c$$

$$\because x^2+3x+12=x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+12$$

$$=\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)-\frac{9}{4}+12=\left(x+\frac{3}{2}\right)^2-\frac{9+48}{4}$$

$$=\left(x+\frac{3}{2}\right)^2+\frac{39}{4}$$

$$\because \int\frac{dx}{x^2+a^2}=\frac{1}{a}\mathrm{Tan}^{-1}\frac{x}{a}$$

22. Evaluate the reduction formula for $I_n = \int \sin^n x dx$ and hence find $\int \sin^4 x dx$

Sol: Given $I_n = \int \sin^n x dx = \int \sin^{n-1} x (\sin x) dx$.

We take First function $u = \sin^{n-1} x$ and

Second function $v = \sin x \Rightarrow \int v = -\cos x$

From By parts rule, we have

$$I_n = \sin^{n-1} x(-\cos x) - \int (n-1)\sin^{n-2} x \cos x(-\cos x)dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int \sin^{n-2} x \cos^2 x dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int \sin^{n-2} x(1-\sin^2 x)dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\left[\int \sin^{n-2} x dx - \int \sin^n x dx\right]$$

$$= -\sin^{n-1} x \cos x + (n-1)[I_{n-2} - I_n]$$

$$I_n = -\sin^{n-1} x \cos x + (n-1)I_{n-2} - (n-1)I_n$$

$$= -\sin^{n-1} x \cos x + (n-1)I_{n-2} - nI_n + I_n$$

$$\Rightarrow nI_n = -\sin^{n-1} x \cos x + (n-1)I_{n-2} + \cancel{I_n} - \cancel{I_n}$$

$$\Rightarrow I_n = \frac{-\sin^{n-1} x \cos x}{n} + \left(\frac{n-1}{n}\right)I_{n-2} \quad ....(1)$$

Put n=4, 2,0 successively in (1), we get

$$I_4 = -\frac{\sin^3 x \cos x}{4} + \frac{3}{4}I_2$$

$$= -\frac{\sin^3 x \cos x}{4} + \frac{3}{4}\left[-\frac{\sin x \cos x}{2} + \frac{1}{2}I_0\right]$$

$$= \frac{-\sin^3 x \cos x}{4} - \frac{3}{8}\sin x \cos x + \frac{3}{8}I_0$$

$$= -\frac{\sin^3 x \cos x}{4} - \frac{3}{8}\sin x \cos x + \frac{3}{8}x + c$$

23. **Evaluate** $\int_0^{\pi/4} \log(1+\tan x)dx$

Sol: We know $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\therefore I = \int_0^{\pi/4} \log(1+\tan x)dx$$

$$= \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$$

$$= \int_0^{\pi/4} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} [\log 2 - \log(1+\tan x)dx$$

$$= \log 2 \int_0^{\pi/4} 1dx - \int_0^{\pi/4} \log(1+\tan x)dx$$

$$= \log 2 [x]_0^{\pi/4} - I$$

$$\Rightarrow I + I = (\log 2)\left(\frac{\pi}{4}\right) \Rightarrow 2I = \left(\frac{\pi}{4}\right)(\log 2)$$

$$\Rightarrow I = \left(\frac{\pi}{8}\right)(\log 2)$$

24. Solve $(x^2+y^2)dx=2xydy$

Sol: Given D.E is $(x^2+y^2)dx = 2xydy$.

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{2xy} \;.....(1)$$

(1) is a homogeneous D.E

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore (1) \Rightarrow v + x\frac{dv}{dx} = \frac{x^2+(vx)^2}{2x(vx)}$$

$$= \frac{x^2+v^2x^2}{2x^2v} = \frac{\cancel{x^2}(1+v^2)}{2\cancel{x^2}v} = \frac{1+v^2}{2v}$$

$$\therefore x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$= \frac{1+v^2-2v^2}{2v} = \frac{1-v^2}{2v}$$

$$\therefore x\frac{dv}{dx} = \frac{1-v^2}{2v} \Rightarrow \frac{2v}{1-v^2}dv = \frac{dx}{x}$$

$$\Rightarrow \int \frac{2v}{1-v^2}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\int \frac{-2v}{1-v^2}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\log(1-v^2) = \log x + \log c$$

$$\Rightarrow \log x + \log(1-v^2) = \log c$$

$$\Rightarrow \log(x(1-v^2)) = \log c \Rightarrow x(1-v^2) = c$$

$$\Rightarrow x\left(1-\frac{y^2}{x^2}\right) = c; \quad \left(\because v = \frac{y}{x}\right)$$

$$\Rightarrow \cancel{x}\left(\frac{x^2-y^2}{\cancel{x^2}}\right) = c \Rightarrow x^2-y^2 = cx.$$

$\therefore$ The solution is $x^2-y^2 = cx$