6 APPLICATION OF DERIVATIVES

1 OMQ + 1 VSAQ + 1 LAQ [1 M + 2M +8M = 11 M]

CONCEPTS & FORMULAS

Application of Derivatives -I (Rate of change)

1) The rate of change of y=f(x) w.r.to t is given by $\frac{dy}{dt} = f'(x)\frac{dx}{dt}$

Application of Derivatives -II(Increasing & Decreasing Functions)

2) **A function f is said to be**

(a) increasing on an interval (a, b) if $x_1 < x_2$ in (a, b)$\Rightarrow f(x_1)<f(x_2)$ for all $x_1, x_2 \in (a, b)$.
Alternatively, if $f'(x) \geq 0$ for each x in (a, b)

(b) decreasing on (a,b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1)>f(x_2)$ for all $x_1, x_2 \in (a, b)$.

(c) constant in (a, b), if f (x) = c for all x ∈ (a, b), where c is a constant.

3) A point c in the domain of a function f at which either f'(c)= 0 or f is not differentiable is called a **critical point** of f.

Application of Derivatives -III(Maxima & Minima)

4) **First Derivative Test (Local Maxima or Minima) :** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

(i) If f '(x) changes sign from positive to negative as x increases through c, i.e., if f '(x) > 0 at every point sufficiently close to and to the left of c, and f '(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of **local maxima.**

(ii) If f '(x) changes sign from negative to positive as x increases through c, i.e., if f '(x) < 0 at every point sufficiently close to and to the left of c, andf '(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of **local minima.**

(iii) If f '(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called **point of inflexion.**

5) **Second Derivative Test (Local Maxima or Minima):** Let f be a function defined on an interval I and c∈ I. Let f be twice differentiable at c. Then

(i) x = c is a point of local maxima if f '(c) = 0 and f "(c) < 0
The values f (c) is **local maximum** value of f .

(ii) x = c is a point of local minima if f '(c) = 0 and f "(c) > 0
In this case, f (c) is **local minimum** value of f .

(iii) The test fails if f '(c) = 0 and f "(c) = 0.
In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a **point of inflexion.**

6) **Absolute maxima, absolute minima - Working steps:**

Step 1 : Find points x where either f '(x) = 0 or f is not differentiable.
Step 2 : Take the end points of the interval.
Step 3 : At all these points (listed in Step 1 and 2), calculate the values of f .
Step 4 : Identify the maximum and minimum values of f out of the values calculated in Step3.
This **maximum value** will be the **absolute maximum value** of f and the **minimum value** will be the **absolute minimum value** of f .