5 CONTINUITY & DIFFERENTIABILITY

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

5(a) CONTINUITY

CONCEPTS & FORMULAS

Continuity of a function at a point:

1) A real valued function is continuous at a point in its domain if the limit of the function at that point equals the value of the function at that point.

Thus, a real function f(x) is continuous at a $\Leftrightarrow f(a) = \underset{x\to a}{\mathrm{Lt}}\, f(x)$

2) **3 STEPS to check the continuity of f(x) at x=a:**

STEP 1 : Find f(a); **STEP 2 :** Find $\lim_{x\to a} f(x)$; **STEP 3 :** Check for $f(a) = \lim_{x\to a} f(x)$

HINT 1: In step (2), incase of split function like |x|, [x], $\sqrt{x}$ find both LHL& RHL.

Here if LHL= RHL = l then we have $\underset{x\to a}{\mathrm{Lt}}\, f(x) = l$ (or)

if LHL$\neq$ RHL then we say f(x) is discontinuous at x=a (or)

if any one of LHL or RHL does not exist then also f(x) is discontinuous at x=a

HINT 2: In step (3), if $\underset{x\to a}{\mathrm{Lt}}\, f(x) = f(a)$ then conclude that f(x) is continuous at x=a.

If $\underset{x\to a}{\mathrm{Lt}}\, f(x) \neq f(a)$ then conclude that f(x) is discontinuous at x=a.

3) **Result:** Graphically, if the graph of f(x) is a continuous curve on its defined domain then f(x) is continuous on its domain.

Examples: Polynomial function, constant function(k), identity function(x), |x|,sinx,cosx...... are continuous on R. log(x) is continuous on $(0,\infty)$.

[x] is discontinuous on R. But [x] is continuous on R–Z.

4) **Result:** If f is continuous at a and g is continuous at f(a), then gof is continuous at a.

5) Suppose f and g be two real functions continuous at a real number c. Then

(1) f + g is continuous at x = c. (2) f – g is continuous at x = c.

(3) f . g is continuous at x = c. (4) $\left(\frac{f}{g}\right)$ is continuous at x = c, (provided g (c)$\neq$0).

BMP BOX

VSAQ: 1. Find $\frac{dy}{dx}$ in $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **[BMP1]** 2. Find $\frac{dy}{dx}$, if x=a (θ + sin θ), y=a (1– cos θ). **[BMP2]**

SAQ: 3. Differentiate the function $(\log x)^{\cos x}$ w.r.t. x. **[BMP1]**

4. Find the relationship between a and b so that the function f defined by

$$f(x)=\begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x>3 \end{cases}$$ is continuous at x = 3. **[BMP2]**

5(b) DIFFERENTIABILITY

CONCEPTS & FORMULAS

1) **Differentiability of a function :** If y=f(x) is a real function then from the first principle its derivatives is given by $f'(x)=\underset{h\to0}{Lt}\dfrac{f(x+h)-f(x)}{h}=\dfrac{dy}{dx}=y'$

2) If a function f is differentiable at a point c then $f'(c)=\underset{x\to c}{Lt}\dfrac{f(x)-f(c)}{x-c}$

3) **Chain Rule:** $y=f(u), u=g(x)\Rightarrow\dfrac{dy}{dx}=\dfrac{dy}{du}.\dfrac{du}{dx}$ i.e., $y=f(g(x))\Rightarrow\dfrac{dy}{dx}=f'(g(x)).g'(x)$

4) **Derivatives of Exponential Functions:**

$\dfrac{d}{dx}(e^x)=e^x$; $\dfrac{d}{dx}(e^{f(x)})=e^{f(x)}f'(x)$; $\dfrac{d}{dx}(a^x)=a^x\log_e a$;

5) **Derivatives of Logarithmic Functions:**

$\dfrac{d}{dx}(\log x)=\dfrac{1}{x}$; $\dfrac{d}{dx}[\log f(x)]=\dfrac{1}{f(x)}.f'(x)$; $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x}\log_a e$

6) **Derivatives of Inverse Trigonometric Functions:**

$\dfrac{d}{dx}(\sin^{-1}x)=\dfrac{1}{\sqrt{1-x^2}}$ $\dfrac{d}{dx}(\cos^{-1}x)=\dfrac{-1}{\sqrt{1-x^2}}$ $\dfrac{d}{dx}(\tan^{-1}x)=\dfrac{1}{1+x^2}$

7) **Logarithmic Differentiaton:** If $y=f(x)^{g(x)}$, then $\log y=g(x)[\log f(x)]\Rightarrow\frac{1}{y}\frac{dy}{dx}=g(x)\frac{1}{f(x)}f'(x)+\log f(x)g'(x)$

8) **Parametric differentiation:** If $x=f(t), y=g(t)$, then $\dfrac{dy}{dx}=\dfrac{g'(t)}{f'(t)}$ and $\dfrac{d^2y}{dx^2}=\left[\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)\right]\left(\dfrac{dt}{dx}\right)$

9) **Derivative of a function w.r.t another function:** If $y=f(x), z=g(x)$ then $\dfrac{dy}{dz}=\dfrac{df}{dg}=\dfrac{f'(x)}{g'(x)}$

10) **Second order derivative:** If f is a differentiable function of x and its derivative f'(x) is also a function of x, then f'(x) is again a differentiable function of x and the derivative of f' is called the second order derivative of f and it is denoted by f". Thus $f''(x)=\underset{h\to0}{Lt}\dfrac{f'(x+h)-f'(x)}{h}$

The second order derivative is also denoted by $\dfrac{d^2y}{dx^2}$, y", $\dfrac{d^2}{dx^2}f(x)$, $D^2f(x)$, $f^{(2)}(x)$

Remark: Every differential function is continuous, but the converse is not true.