3 MATRICES

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

3(a) MATRICES-I

[FORMATION OF MATRIX, CONSTRUCTION OF A MATRIX, ORDER OF A MATRIX, EQUALITY OF MATRICES & TYPES OF MATRICES]

CONCEPTS & FORMULAS

1) Matrix : An **ordered rectangular array** of elements (numbers) arranged in **m** rows and **n** columns is called a matrix of order **m×n.**

Ex: $A=\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{2\times3}$, $B=\begin{bmatrix}1&0\\0&1\end{bmatrix}_{2\times2}$, $C=\begin{bmatrix}1&2\\-3&4\\5&-6\end{bmatrix}_{3\times2}$, $D=\begin{bmatrix}2&5&0&2\\-1&\omega&1/3&1\\\sqrt{7}&4&i&1\end{bmatrix}_{3\times4}$

Here, matrix A has 2 rows and 3 columns. Hence order of A is 2×3

Further in the matrix A, the element in the first row and second column is 2.

Also in A 6 is the element in the second row and third column.

2) General representation of a matrix is as follows:

☛ The element in the i^{th} row and j^{th} column is denoted by a_{ij}

$$A=[a_{ij}]_{m\times n}=\begin{bmatrix}a_{11}&a_{12}&\cdots\cdots&a_{1j}\cdots\cdots&a_{1n}\\a_{21}&a_{22}&\cdots\cdots&a_{2j}\cdots\cdots&a_{2n}\\\vdots&\vdots&&\vdots&\vdots\\a_{i1}&a_{i2}\cdots\cdots&&a_{ij}\cdots\cdots&a_{in}\\\vdots&\vdots&&\vdots&\vdots\\a_{m1}&a_{m2}\cdots\cdots&&a_{mj}\cdots\cdots&a_{mn}\end{bmatrix}_{m\times n};1\le i\le m,1\le j\le n$$

3) Equality of matrices: Matrices A,B are said to be equal, written as A=B if

(i) A, B are of same order (ii) the corresponding elements in A,B are equal

Thus, the matrices $A=[a_{ij}]_{mxn}$, $B=[b_{ij}]_{mxn}$ are equal if $a_{ij}=b_{ij}\ \forall\ i,j$.

Ex: $\begin{bmatrix}1&2\\3&4\end{bmatrix}_{2\times2}=\begin{bmatrix}1&2\\3&4\end{bmatrix}_{2\times2}$ but $\begin{bmatrix}1&2\\3&4\end{bmatrix}_{2\times2}\ne\begin{bmatrix}1&3\\2&4\end{bmatrix}_{2\times2}\ne\begin{bmatrix}1&2&5\\3&4&6\end{bmatrix}_{2\times3}$

CONCEPTS & FORMULAS

Types of Matrices

4) **Row matrix:** A matrix with only one row is called a row matrix.
Ex: A=[1 2 3] is a row matrix.

5) **Column matrix:** A matrix with only one column is called a column matrix.

Ex: $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a column matrix.

6) **Square matrix:** A matrix in which the number of rows is equal to number of columns, is called a square matrix.A square matrix of order n×n is also called a square matrix of order n.

Ex1: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is a square matrix of order 2 **Ex2:** $B = \begin{bmatrix} 2 & 5 & 0 \\ 1 & 2 & 4 \\ 0 & 8 & 7 \end{bmatrix}$ is a square matrix of order 3

☛ **Diagonal of a square matrix:** If A is a square matrix then the diagonal in A, from the first element of first row to the last element of the last row is called the principal diagonal of A.

Ex: In the above matrix A, the diagonal elements of A are 1,4

and in the matrix B, the diagonal elements are 2,2,7.

7) **Diagonal matrix:** If each non-diagonal element of a square matrix is zero then the matrix is called a diagonal matrix. i.e., a matrix $A=[a_{ij}]_{n\times n}$ where $a_{ij}=0$ for $i\neq j$ is called a diagonal matrix

Ex: $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8) **Scalar matrix:** If all the elements in a diagonal matrix are equal to a same number then the matrix is called a Scalar matrix.

Ex: $[2]$ $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

9) **Identity Matrix (Unit Matrix) :** A square matrix in which elements in the diagonal all are 1 and rest are all zero is called an Identity matrix and it is denoted by I.

The identity matrix of order n is denoted by I_n.

$[a_{ij}]_{n\times n}$ is a unit matrix if $a_{ij} = 1$ for $i = j$ and $a_{ij} = 0$ for $i \neq j$

Ex: $I_1 = [1]$, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10) **Zero matrix or Null matrix :** If each element of a matrix is zero then it is called a null matrix and it is denoted by O

Ex: $[0]$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

3(b) MATRICES-II

[OPERATIONS ON MATRICES: ADDITION OF MATRICES, SCALAR MULTIPLICATION, MULTIPLICATION OF MATRICES]

CONCEPTS & FORMULAS

I) ADDITION OF MATRICES

1) ADDITION OF MATRICES [A+B] : If A and B are two matrices of same order then their sum denoted by A+B, is defined to be the matrix obtained by adding the corresponding elements of A and B.

If $A=[a_{ij}]_{mxn}$, $B=[b_{ij}]_{mxn}$ then $A+B=[c_{ij}]_{mxn}$ where $c_{ij}=a_{ij}+b_{ij}$.

Ex: If $A=\begin{bmatrix}3 & 2 & -1\\ 1 & 2 & 5\end{bmatrix}, B=\begin{bmatrix}4 & -3 & 1\\ 3 & 2 & -2\end{bmatrix}$ then $A+B=\begin{bmatrix}3+4 & 2-3 & -1+1\\ 1+3 & 2+2 & 5-2\end{bmatrix}=\begin{bmatrix}7 & -1 & 0\\ 4 & 4 & 3\end{bmatrix}$

2) PROPERTIES OF ADDITION OF MATRICES:

i) Addition of matrices satisfies the Associative law:

If A,B,C are matrices of same order then (A+B)+C=A+(B+C).

ii) Addition of matrices satisfies the commutative law:

If A,B are two matrices of same order then A+B=B+A.

iii) If A is a an mxn matrix and O be an mxn zero matrix such that A+O=O+A=A, then O is called additive identity w.r.to matrix addition.

iv) Additive inverse of A is –A. This is called negative of matrix A.

v) A–B= A+(–1)B

II) SCALAR MULTIPLICATION OF A MATRIX

3) Let A be a matrix of order mxn and k be a scalar, then the mxn matrix obtained by multiplying each element of A by k is called a scalar multiple of A and is denoted by kA.

Thus, if $A=[a_{ij}]_{mxn}$, then $kA=[ka_{ij}]_{mxn}$.

Ex: If $A=\begin{bmatrix}2 & 5 & 0\\ 1 & 2 & 4\end{bmatrix}$ then $2A=2\begin{bmatrix}2 & 5 & 0\\ 1 & 2 & 4\end{bmatrix}=\begin{bmatrix}2(2) & 2(5) & 2(0)\\ 2(1) & 2(2) & 2(4)\end{bmatrix}=\begin{bmatrix}4 & 10 & 0\\ 2 & 4 & 8\end{bmatrix}$

4) PROPERTIES OF SCALAR MULTIPLICATION:

Result: Let A and B be matrices of the same order and k,*l* be scalars, then

(i) k(A+B)=kA+kB
(ii) (k+*l*)A=kA+*l*A
(iii) k(*l*A)=(k*l*)A=*l*(kA)
(iv) 0(A)=O
(v) aO=O

III) MULTIPLICATION OF MATRICES

5) Multiplication of Matrices: Two Matrices A and B are said to be **Conformable for Multiplication** in that order, if the number of columns of A is equal to the number of rows of B.

The product of A,B in this order is denoted by AB.

Let $A=[a_{ik}]_{mxn}$ & $B=[b_{kj}]_{nxp}$ be two matrices which are conformable for multiplication, then product of A, B is $C=[c_{ij}]_{mxp}$, where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ... + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$.

Thus, the $(i,j)^{th}$ element c_{ij} of C is the sum of the product of elements of A in the i^{th} row with the corresponding elements of B in the j^{th} column. **(This is Row - by - Column multiplication)**

☛ If the orders of A and B are mxn and nxp, the order of the product matrix AB is mxp.

Ex1: $$\begin{bmatrix}1 & 2\end{bmatrix}_{1x2}\begin{bmatrix}3\\4\end{bmatrix}_{2x1} = \begin{bmatrix}1x3 + 2x4\end{bmatrix}_{1x1} = [3+8] = [11]$$

Ex2: $$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}_{2x2}\begin{bmatrix}5\\6\end{bmatrix}_{2x1} = \begin{bmatrix}1x5+2x6\\3x5+4x6\end{bmatrix}_{2x1} = \begin{bmatrix}5+12\\15+24\end{bmatrix}_{2x1} = \begin{bmatrix}17\\39\end{bmatrix}_{2x1}$$

Ex3: $$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}_{2x2}\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix}_{2x2} = \begin{bmatrix}1x5+2x7 & 1x6+2x8\\3x5+4x7 & 3x6+4x8\end{bmatrix}_{2x2} = \begin{bmatrix}19 & 22\\43 & 50\end{bmatrix}_{2x2}$$

Ex4: $$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}_{2x3}\begin{bmatrix}7 & 8\\9 & 10\\11 & 12\end{bmatrix}_{3x2} = \begin{bmatrix}1(7)+2(9)+3(11) & 1(8)+2(10)+3(12)\\4(7)+5(9)+6(11) & 4(8)+5(10)+6(12)\end{bmatrix}_{2x2} = \begin{bmatrix}58 & 64\\139 & 154\end{bmatrix}_{2x2}$$

Note1: If the orders of A,B are mxn and kx*l* respectively then both AB & BA are defined iff n=k and *l*=m. In particular, both AB, BA are defined for square matrices of same order.

Note2: Even if both AB and BA are defined then it is not necessary that AB = BA.

Note3: But matrix multiplication is commutative (AB=BA) when A,B are diagonal matrices of same order.

$$A = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}, B = \begin{bmatrix}3 & 0\\0 & 4\end{bmatrix} \text{ then } AB = BA = \begin{bmatrix}3 & 0\\0 & 8\end{bmatrix}$$

Note 4: The product of two non-zero matrices may be a zero matrix.

$$A = \begin{bmatrix}0 & -1\\0 & 2\end{bmatrix}, B = \begin{bmatrix}3 & 5\\0 & 0\end{bmatrix} \text{ then } AB = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Note 5: Associative law is satisfied in Matrix multiplication. Thus (AB)C=A(BC)

Note 6: Distributive law is satisfied in Matrix multiplication.

(i) A(B+C)=AB+AC (ii) (A+B)C=AC+BC

3(c)MATRICES-III

[TRANSPOSE OF A MATRIX, SYMMETRIC MATRIX & SKEW SYMMETRIC MATRIX]

CONCEPTS & FORMULAS

1) TRANSPOSE OF A MATRIX: The matrix obtained by interchanging the rows and columns of a given matrix is called the Transpose of the given matrix. Transpose of A is denoted by A' or A^T

Ex 1: If $A=\begin{bmatrix}1&2\\4&5\\7&8\end{bmatrix}_{3\times2}$ then $A'=\begin{bmatrix}1&4&7\\2&5&8\end{bmatrix}_{2\times3}$ **Ex2:** If $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}_{2\times2} \Rightarrow A^T=\begin{bmatrix}1&3\\2&4\end{bmatrix}_{2\times2}$

2) SYMMETRIC MATRIX: A square matrix A is said to be a Symmetric matrix if $A'=A$.

Ex: $A=\begin{bmatrix}1&2\\2&3\end{bmatrix}$, $B=\begin{bmatrix}1&2&0\\2&-3&-1\\0&-1&4\end{bmatrix}$, $C=\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}$

3) SKEW-SYMMETRIC MATRIX:

A square matrix A is said to be a skew-symmetric matrix if $A'=-A$.

Ex: $A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$, $B=\begin{bmatrix}0&1&-2\\-1&0&4\\2&-4&0\end{bmatrix}$

4) Properties of transpose of matrices:

For any matrices A and B of suitable orders, we have

(i) $(A')'=A$,

(ii) $(kA)'=kA'$ (where k is any constant)

(iii) $(A+B)'=A'+B'$

(iv) $(AB)'=B'A'$

5) Theorem: For any square matrix A with real number entries, A + A' is a symmetric matrix and A –A' is a skew symmetric matrix.

Proof : Let $B=A+A'$ then

$B'=(A+A')'=A'+(A')'$ as $(A+B)'=A'+B'$ $=A'+A$ (as $(A')'=A$)

$=A+A'$ (as $A+B=B+A$) $=B$

$\therefore B=A+A'$ is a symmetric matrix

Now let $C=A-A'$

$C'=(A-A')'=A'-(A')'=A'-A=-(A-A')=-C$

$\therefore C=A-A'$ is a skew symmetric matrix.

6) Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

If A is a square matrix, then $A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$