SHINING STARS'

SAQ STAR QUESTIONS PLUS

RELATIONS & FUNCTIONS

1. Show that $f : N \to N$, given $f(x) = \begin{cases} x+1, \text{if } x \text{ is odd,} \\ x-1, \text{if } x \text{ is even} \end{cases}$ is both one-one and onto.

Sol: **Case i:** When x is odd we have f(x) = x+1 . Let us take x= 1,3,5,....

$\Rightarrow f(1) = 1+1=2;\ f(3) = 3+1=4;\ f(5) = 5+1=6;$

$\therefore$ We have (1,2), (3,4), (5,6),.....$\in$ f ....(1)

Case ii: When x is even we have f(x)=x–1. Let us take x= 2,4,6,....

$\Rightarrow f(2) = 2-1=1;\ f(4) = 4-1=3;\ f(6) = 6-1=5$

$\therefore$ We have (2,1), (4,3), (6,5),.....$\in$ f ....(2)

From (1) & (2) we have f = {(1,2), (2,1), (3,4), (4,3), (5,6), (6,5),....}....(3)

From (3), it is clear that distinct elements in the domain N have the distinct images.

$\therefore$ f is one-one.

Also all the elements {1,2,3,....} in the Codomain N have pre-images in domain N.

$\therefore$ f is onto.

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2. Let $f : N \to N$ be defined by $f(n) = \begin{cases} \dfrac{n+1}{2}, \text{if } n \text{ is odd} \\ \dfrac{n}{2}, \text{if } n \text{ is even} \end{cases}$ for all $n \in N$

State whether the function f is bijective. Justify your answer.

Sol: **Case i:** For odd n=1,3,5,.... we have $f(n) = \dfrac{n+1}{2}$

$\therefore f(1) = \dfrac{1+1}{2} = 1; f(3) = \dfrac{3+1}{2} = 2; f(5) = \dfrac{5+1}{2} = 3; \Rightarrow$ f= { (1,1), (3,2), (5,3),.....}..........(1)

Case ii: For even n=2,4,6,.... we have $f(n) = \dfrac{n}{2}$

$\therefore f(2) = \dfrac{2}{2} = 1; f(4) = \dfrac{4}{2} = 2; f(6) = \dfrac{6}{2} = 3; \Rightarrow$ f= { (2,1), (4,2), (6,3),.....}..........(2)

From (1) and (2) we get f= {(1,1), (2,1),(3,2), (4,2),(5,3),(6,3),....}..........(3)

a) From (3), we have f(1) =1 and f(2)=1

Thus distinct elements 1,2 of the domain N have the same image 1. $\therefore$ f is not one-one

b) From (3) the range set = { 1,1,2,2,3,3,.....}= {1,2,3,....}

$\Rightarrow$Range of f= {1,2,3,.....}= codomain N $\therefore$ f is onto.

$\therefore$ f is onto but not one-one . Hence, f is not a bijective function.

3. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Sol: Given that $f : A \to B$ is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$ for $A = R - \{3\}, B = R - \{1\}$

a) Take $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow \cancel{x_1x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1x_2} - 3x_2 - 2x_1 + \cancel{6}$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow 3x_1 - 2x_1 = 3x_2 - 2x_2 \Rightarrow x_1 = x_2$ $\therefore$ f is one-one.

b) Given that $f(x) = \left(\frac{x-2}{x-3}\right)$.....(1). Put f(x)=y

$\Rightarrow y = \frac{x-2}{x-3} \Rightarrow x - 2 = xy - 3y \Rightarrow x(1-y) = -3y + 2 \Rightarrow x = \frac{2-3y}{1-y}$ exists $\forall$ $y \in B = R - \{1\}$

$\therefore$ f is onto.

Justification: From (1), $f(x) = f\left(\frac{2-3y}{1-y}\right) = \dfrac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$

Hence, given f(x) is one-one and onto.

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4. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b): |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1. **[BMP]**

Sol: Given that $A = \{x \in Z : 0 \le x \le 12\} \Rightarrow A = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$

$R = \{(a,b) : |a-b| \text{ is a mutiple of } 4\}$

(i) $(a,a) \in R\ \forall\ a \in A$. So, R is reflexive. $[\because |a-a| = 0$ and 0 is a multiple of 4$]$

(ii) $(a,b) \in R \Rightarrow (b,a) \in R$. So, R is symmetric.

$[\because |a-b|$ is a multiple of $4 \Rightarrow |-(a-b)| = |b-a|$ is also a multiple of 4$]$

(iii) $(a,b) \in R$ and $(b,c) \in \mathbf{R} \Rightarrow (a,c) \in R$ So, R is transitive.

[$\because$ If |a–b|, |b–c| are multiples of 4 $\Rightarrow$ (a–b),(b–c) are multiples of 4.

Now (a–c) = (a–b)+(b–c) is a multiple of 4 $\Rightarrow$ |a–c| is a multiple of 4]

Thus, R is reflexive, symmetric, transitive. Hence, R is an equivalence relation.

Further, the set of elements related to 1 is $\{1,5,9\}$

$\because |1-1| = 0$ is a multiple of 4 ; $|5-1| = 4$ is a multiple of 4. , $|9-1| = 8$ is a multiple of 4

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5. Let f : {2, 3, 4, 5} → {3, 4, 5, 9} and g : {3, 4, 5, 9} → {7, 11, 15} be functions defined as f(2) = 3, f(3) = 4, f(4) = f(5) = 5 and g (3) = g(4) = 7 and g(5) = g(9) = 11. Find gof.

Sol: We have gof (2) = g (f (2)) = g (3) = 7; gof (3) = g (f (3)) = g (4) = 7,

gof (4) = g (f (4)) = g (5) = 11; gof (5) = g (5) = 11.

$\therefore$ gof = { (2,7),(3,7), (4,11), (5,11)}

INVERSE TRIGONOMETRIC FUNCTIONS

6. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

Sol: $\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}$ (by rationalizing the dr.)

$$=\frac{(1+\cancel{\sin x})+(1-\cancel{\sin x})+2\sqrt{(1+\sin x)(1-\sin x)}}{(\cancel{1}+\sin x)-(\cancel{1}-\sin x)}$$ $\left[\because 1+\cos x=2\cos^2\frac{x}{2}\right]$

$$=\frac{\cancel{2}\left(1+\sqrt{1-\sin^2 x}\right)}{\cancel{2}\sin x}=\frac{1+\cos x}{\sin x}=\frac{\cancel{2}\cos^{\cancel{2}}\frac{x}{2}}{\cancel{2}\sin\frac{x}{2}\cancel{\cos\frac{x}{2}}}=\cot\frac{x}{2}$$ $\left[\because \sin x=2\sin\frac{x}{2}\cos\frac{x}{2}\right]$

$$\therefore \text{L.H.S}=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\cot^{-1}\left(\cot\frac{x}{2}\right)=\frac{x}{2}=\text{R.H.S}$$

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7. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$

Sol: Let $x=\cos 2\theta\Rightarrow 2\theta=\cos^{-1}x\Rightarrow\theta=\frac{1}{2}\cos^{-1}x$....(1) $\left[\begin{array}{l}\because 1+\cos 2\theta=2\cos^2\theta\\ \quad 1-\cos 2\theta=2\sin^2\theta\end{array}\right]$

Now $\text{L.H.S}=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$$=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$ [Dividing the Nr. & Dr. by $\cos\theta$]

$$=\tan^{-1}1-\tan^{-1}(\tan\theta)=\frac{\pi}{4}-\theta=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x=\text{R.H.S}$$ [$\because$ from (1)]

MATRICES

8. Find the values of a, b, c and d from the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Sol: Given $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

As the two matrices are equal, equating the corresponding elements, we get

$a-b=-1$.........(1) $2a-b=0$.......(2)

$2a+c=5$.......(3) $3c+d=13$ ......(4)

From (2), $b=2a$

Putting this value in (1), $\Rightarrow a-2a=-1 \Rightarrow a=1$ Hence, $b=2$

Putting $a=1$ in (3)$\Rightarrow 2(1)+c=5 \Rightarrow c=3$

Putting $c=3$ in (4)$\Rightarrow 3(3)+d=13 \Rightarrow d=4$

Thus, $a=1$, $b=2$, $c=3$ and $d=4$

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9. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ **and** $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$

find A(BC), (AB)C and show that (AB)C = A(BC)

Sol:

$$AB = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1+0+1 & 3+2-4 \\ 2+0-3 & 6+0+12 \\ 3+0-2 & 9-2+8 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 15 \end{bmatrix}$$

$$(AB)(C) = \begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+0 & 6-2 & -8+1 \\ -1+36 & -2+0 & -3-36 & 4+18 \\ 1+30 & 2+0 & 3-30 & -4+15 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 4 & -7 \\ 35 & -2 & -39 & 22 \\ 31 & 2 & -27 & 11 \end{bmatrix} \text{............(1)}$$

$$BC = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & 2+0 & 3-6 & -4+3 \\ 0+4 & 0+0 & 0-4 & 0+2 \\ -1+8 & -2+0 & -3-8 & 4+4 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -3 & -1 \\ 4 & 0 & -4 & 2 \\ 7 & -2 & -11 & 8 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 7 & 2 & -3 & -1 \\ 4 & 0 & -4 & 2 \\ 7 & -2 & -11 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 7+4-7 & 2+0+2 & -3-4+11 & -1+2-8 \\ 14+0+21 & 4+0-6 & -6+0-33 & -2+0+24 \\ 21-4+14 & 6+0-4 & -9+4-22 & -3-2+16 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 & -7 \\ 35 & -2 & -39 & 22 \\ 31 & 2 & -27 & 11 \end{bmatrix} \text{....(2)}$$

From (1) & (2), we have $(AB)C = A(BC)$

PROBABILITY

10. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

Sol: Let E and F denote the events that first and second ball drawn are black.

We have to find P $(E \cap F)$ or P (EF).

P (black ball in first draw)=P (E) $=\frac{10}{15}$

Also given that the first ball drawn is black, i.e., event E has occurred, now there are 9 black balls and five white balls left in the urn. So the probability that the second ball drawn is black, given that the ball in the first draw is black, is nothing but the conditional probability of F given that E has occurred. $\therefore P(F|E)=\frac{9}{14}$

$P(E \cap F) = P(E)\ P(F|E)=\frac{10}{15}\times\frac{9}{14}=\frac{3}{7}$ [By multiplication rule of probability]

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11. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1– P(A′) P(B′)

Sol: P(at least one of A and B) $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A)\,P(B)$$

$$= P(A) + P(B)\,[1 - P(A)]$$

$$= P(A) + P(B).\,P(A')$$

$$= 1 - P(A') + P(B)\,P(A')$$

$$= 1 - P(A')\,[1 - P(B)]$$

$$= 1 - P(A')\,P(B')$$

CONTINUITY & DIFFERENTIABILITY

12. Find all points of discontinuity of f, where f is defined by $f(x)=\begin{cases}2x+3, \text{ if } x\le 2\\ 2x-3, \text{ if } x>2\end{cases}$

Sol: Given function is $f(x)=\begin{cases}2x+3, \text{ if } x\le 2\\ 2x-3, \text{ if } x>2\end{cases}$

It is clear that the given function is defined at all the points of the real line.

Let c be a point on the real line. Then, three cases arise. c<2, c>2, c=2

Case i: c<2

Here f(c)=2c+3. Then, $\underset{x\to c}{\text{Lt}}\, f(x)=\underset{x\to c}{\text{Lt}}\,(2x+3)=2c+3$

$\therefore \underset{x\to c}{\text{Lt}}\, f(x)=f(c)$ $\therefore$ f is continuous at all points x, such that x <2 .

Case ii:c>2

Here f(c)=2c–3. Then, $\underset{x\to c}{\text{Lt}}\, f(x)=\underset{x\to c}{\text{Lt}}\,(2x-3)=2c-3$

$\therefore \underset{x\to c}{\text{Lt}}\, f(x)=f(c)$ $\therefore$ f is continuous at all points x, such that x >2 .

Case iii:c=2

$\text{L.H.L}=\underset{x\to 2-}{\text{Lt}}\, f(x)=\underset{x\to 2-}{\text{Lt}}\,(2x+3)=2(2)+3=7$;

$\text{R.H.L}=\underset{x\to 2+}{\text{Lt}}\, f(x)=\underset{x\to 2+}{\text{Lt}}\,(2x-3)=2(2)-3=1$. Here L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at x=2.

Thus, x=2 is the only point of discontinuity of f.

13. Is the function defined by $f(x) = \begin{cases} x+5, & \text{if } x \le 1 \\ x-5, & \text{if } x > 1 \end{cases}$ a continuous function?

Sol: Given function is $f(x) = \begin{cases} x+5, \text{if } x \le 1 \\ x-5, \text{if } x > 1 \end{cases}$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case i:

If $c < 1$, then $f(c)=c+5$

$\underset{x \to c}{\text{Lt}}\ f(x) = \underset{x \to c}{\text{Lt}}\ (x+5) = c+5$ $\qquad \therefore \underset{x \to c}{\text{Lt}}\ f(x) = f(c)$

$\therefore$ f is continuous at all points x, such that $x<1$.

Case ii:

If $c=1$, then $f(1)=1+5=6$

L.H.L= $\underset{x \to 1-}{\text{Lt}}\ f(x) = \underset{x \to 1-}{\text{Lt}}\ (x+5) = 1+5 = 6$

R.H.L= $\underset{x \to 1+}{\text{Lt}}\ f(x) = \underset{x \to 1+}{\text{Lt}}\ (x-5) = 1-5 = -4$ $\qquad$ Here L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at $x=1$.

Case iii:

If $c > 1$, then $f(c)=c-5$

$\underset{x \to c}{\text{Lt}}\ f(x) = \underset{x \to c}{\text{Lt}}\ (x-5) = c-5$ $\qquad \therefore \underset{x \to c}{\text{Lt}}\ f(x) = f(c)$

$\therefore$ f is continuous at all points x, such that $x>1$.

We observe that, $x=1$ is the only point of discontinuity of f.

14. Discuss the continuity of the function f, where f is defined by $f(x)=\begin{cases}3, & \text{if } 0\le x\le 1\\ 4, & \text{if } 1<x<3\\ 5, & \text{if } 3\le x\le 10\end{cases}$

Sol: The given function is $f(x)=\begin{cases}3, & \text{if } 0\le x\le 1\\ 4, & \text{if } 1<x<3\\ 5, & \text{if } 3\le x\le 10\end{cases}$

The given function is defined at all the points of the interval $[0,10]$.

Let c be a point in the interval [0,10].

Case i:

If 0≤ c <1, then f(c)= 3

$\underset{x\to c}{\text{Lt}}\ f(x)=\underset{x\to c}{\text{Lt}}\ (3)=3$ $\quad\therefore \underset{x\to c}{\text{Lt}}\ f(x)=f(c)$

$\therefore$ f is continuous in the interval $[0,1)$

Case ii:

If c=1, then f(3)=3

L.H.L= $\underset{x\to 1-}{\text{Lt}}\ f(x)=\underset{x\to 1-}{\text{Lt}}\ (3)=3$

R.H.L= $\underset{x\to 1+}{\text{Lt}}\ f(x)=\underset{x\to 1+}{\text{Lt}}\ (4)=4$ Here L.H.L $\ne$ R.H.L

$\therefore$ f is not continuous at x=1.

Case iii:

If 1<c<3, then f(c)= 4

$\underset{x\to c}{\text{Lt}}\ f(x)=\underset{x\to c}{\text{Lt}}\ (4)=4$ $\quad\therefore \underset{x\to c}{\text{Lt}}\ f(x)=f(c)$

$\therefore$ f is continuous in the interval $(1,3)$.

Case iv:

If c=3, then f(c)=5

L.H.L= $\underset{x\to 3-}{\text{Lt}}\ f(x)=\underset{x\to 3-}{\text{Lt}}\ (4)=4$

R.H.L= $\underset{x\to 3+}{\text{Lt}}\ f(x)=\underset{x\to 3+}{\text{Lt}}\ (5)=5$ Here L.H.L $\ne$ R.H.L

$\therefore$ f is discontinuous at x=3.

Case v:

If 3<c≤ 10, then f(c)= 5

$\underset{x\to c}{\text{Lt}}\ f(x)=\underset{x\to c}{\text{Lt}}\ (5)=5$ $\quad\therefore \underset{x\to c}{\text{Lt}}\ f(x)=f(c)$

$\therefore$ f is continuous at all points of the interval $(3,10]$

Also f is discontinuous at x=1 and x=3 .

15. Determine if f defined by $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ is a continuous function?

Sol: The given function is $f(x)=\begin{cases} x^2\sin\frac{1}{x}, \text{if } x\neq 0 \\ 0, \text{if } x=0 \end{cases}$

The given function is defined at all the points of the real line.

Let c be a point on the real line.

Case i:

If $c\neq 0$, then $f(c)=c^2\sin\frac{1}{c}$

$$\underset{x\to c}{\text{Lt}}\, f(x)=\underset{x\to c}{\text{Lt}}\left(x^2\sin\frac{1}{x}\right)=\left(\underset{x\to c}{\text{Lt}}\, x^2\right)\left(\underset{x\to c}{\text{Lt}}\,\sin\frac{1}{x}\right)=c^2\sin\frac{1}{c}$$

$$\therefore \underset{x\to c}{\text{Lt}}\, f(x)=f(c)$$

$\therefore$ f is continuous at all points x , such that $x\neq 0$.

Case ii:

If c=0, then f(0)=0

We know that $-1\le\sin\frac{1}{x}\le 1, x\neq 0 \Rightarrow -x^2\le x^2\sin\frac{1}{x}\le x^2$

$$\Rightarrow \underset{x\to 0}{\text{Lt}}\,(-x^2)\le \underset{x\to 0}{\text{Lt}}\left(x^2\sin\frac{1}{x}\right)\le \underset{x\to 0}{\text{Lt}}\, x^2$$

$$\Rightarrow 0\le \underset{x\to 0}{\text{Lt}}\left(x^2\sin\frac{1}{x}\right)\le 0 \Rightarrow \underset{x\to 0}{\text{Lt}}\left(x^2\sin\frac{1}{x}\right)=0 \quad \therefore \underset{x\to 0}{\text{Lt}}\, f(x)=0$$

$\therefore$ f is continuous at x=0

Hence f is continuous at every point of the real line.

Thus, f is a continuous function on R.

16. Find the values of k, so that the function f is continuous at the indicated point

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases} \quad \text{at } x=\frac{\pi}{2}$$

Sol: The given function is $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, \text{if } x\neq\dfrac{\pi}{2}\\ 3, \text{if } x=\dfrac{\pi}{2}\end{cases}$

It is clear that f is defined at $x=\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=3$

From the given f(x) to be continuous at $x=\frac{\pi}{2}$, we have $\underset{x\to\frac{\pi}{2}}{\text{Lt}} f(x)=\underset{x\to\frac{\pi}{2}}{\text{Lt}} \frac{k\cos x}{\pi-2x}$

Put $x=\frac{\pi}{2}+h$ Then $x\to\frac{\pi}{2}\Rightarrow h\to 0$

$$\therefore \underset{x\to\frac{\pi}{2}}{\text{Lt}} f(x)=\underset{x\to\frac{\pi}{2}}{\text{Lt}} \frac{k\cos x}{\pi-2x}=\underset{h\to 0}{\text{Lt}} \frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$

$$=k\underset{h\to 0}{\text{Lt}} \frac{-\sinh}{-2h}=\frac{k}{2}\underset{h\to 0}{\text{Lt}} \frac{\sinh}{h}=\frac{k}{2}.1=\frac{k}{2}$$

$$\therefore \underset{x\to\frac{\pi}{2}}{\text{Lt}} f(x)=f\left(\frac{\pi}{2}\right)\Rightarrow\frac{k}{2}=3\Rightarrow k=6$$

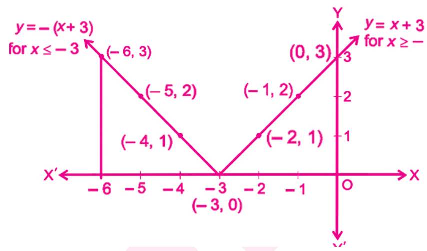
$\therefore$ the value of k=6.

APPLICATION OF INTEGRALS

17. **Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3| dx$.** **[BMP]**

Sol:

X	−6	−5	−4	−3	−2	−1	0
Y	3	2	1	0	1	2	3



$(x+3) \le 0$ for $-6 \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$

$$\therefore \int_{-6}^{0} |(x+3)|\, dx = -\int_{-6}^{-3} (x+3) dx + \int_{-3}^{0} (x+3) dx$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right] = -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right] = 9 \text{ sq.units}$$