SAQ PQ QUESTIONS

1. Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$

2. Write the function $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a>0$ in the simplest form.

3. Express the matrix $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix

4. Express the matrix $B=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

5. Find area of the triangle with vertices (2, 7), (1, 1), (10, 8)

6. Find equation of line joining (3, 1) and (9, 3) using determinants.

7. Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

8. **Let $A=\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1}=B^{-1}A^{-1}$.**

9. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k},\quad \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

10. Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k})$, $B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear.

11. If A and B are two events such that $P(A)=\frac{1}{4}, P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$, find P(not A and not B).

12. Determine P(E|F) when a coin is tossed three times, where
E : head on third toss , F : heads on first two tosses

13. Maximise $Z = 4x + y$ subject to the constraints: $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$.

14. Show that the minimum of Z occurs at more than two points. Minimise and Maximise
$Z = x + 2y$ subject to $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$; $x, y \ge 0$.

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15. Find the values of k so that the function f is continuous at the indicated point $f(x)=\begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$ at $x = 5$

16. Find $\frac{dy}{dx}$ in $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0<x<1$

17. Find $\frac{dy}{dx}$ in $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

18. Differentiate the function $(\log x)^{\log x}$, $x > 1$ w.r.t x.

19. Find $\int \frac{3x-2}{(x+1)^2(x+3)}dx$

20. Find integral of $\frac{1}{1-\tan x}$

21. Find the integral of cos 2x cos 4x cos 6x

22. Find the integral of $\frac{1}{\cos(x-a)\cos(x-b)}$

23. Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

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SAQ PQ SOLUTIONS

INVERSE TRIGONOMETRIC FUNCTIONS

1. Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$

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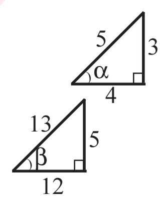
- PT_1: (3,4,5)
- PT_2: (5,12,13)

Sol: Let $\sin^{-1}\frac{3}{5}=\alpha$ and $\cos^{-1}\frac{12}{13}=\beta$.

Required To Prove (RTP): $\alpha+\beta=\cos^{-1}\frac{33}{65}\Rightarrow\cos(\alpha+\beta)=\frac{33}{65}$

Then $\sin\alpha=\frac{3}{5}$ and $\cos\beta=\frac{12}{13}$.



$\therefore\cos\alpha=\frac{4}{5},\sin\beta=\frac{5}{13}$

Now $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}=\frac{48-15}{13}=\frac{33}{65}$. Hence proved.

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2. Write the function $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a>0; \frac{-a}{\sqrt{3}}<x<\frac{a}{\sqrt{3}}$ **in the simplest form.**

Sol: Put $x=a\tan\theta\Rightarrow\tan\theta=\frac{x}{a}\Rightarrow\theta=\tan^{-1}\left(\frac{x}{a}\right)$

$$\therefore\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)=\tan^{-1}\left(\frac{3a^2\cdot(a\tan\theta)-a^3\tan^3\theta}{a^3-3a.(a^2\tan^2\theta)}\right)$$

$$=\tan^{-1}\left(\frac{3a^3\tan\theta-a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right)=\tan^{-1}(\tan 3\theta)=3\theta=3\tan^{-1}\left(\frac{x}{a}\right)$$

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MATRICES

3. Express the matrix $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix

Sol: Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Now $A + A' = \begin{bmatrix} 3+3 & 5+1 \\ 1+5 & -1-1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$

Hence, $P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$. Thus P is a symmetric matrix.

Also, $A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Hence $Q' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -Q$

Thus Q is a skew symmetric matrix.

Expressing A as the sum of P and Q:

$P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$

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4. Express the matrix $B=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Sol: Given that $B=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix} \Rightarrow B'=\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}$

$$(B+B')=\begin{bmatrix}2+2&-2-1&-4+1\\-1-2&3+3&4-2\\1-4&-2+4&-3-3\end{bmatrix}=\begin{bmatrix}4&-3&-3\\-3&6&2\\-3&2&-6\end{bmatrix}$$

Let $P=\frac{1}{2}(B+B')=\frac{1}{2}\begin{bmatrix}4&-3&-3\\-3&6&2\\-3&2&-6\end{bmatrix}=\begin{bmatrix}2&\frac{-3}{2}&\frac{-3}{2}\\\frac{-3}{2}&3&1\\\frac{-3}{2}&1&-3\end{bmatrix}$,

Now $P'=\begin{bmatrix}2&\frac{-3}{2}&\frac{-3}{2}\\\frac{-3}{2}&3&1\\\frac{-3}{2}&1&-3\end{bmatrix}=P$ Thus P is a symmetric matrix.

Also let, $Q=\frac{1}{2}(B-B')=\frac{1}{2}\begin{bmatrix}0&-1&-5\\1&0&6\\5&-6&0\end{bmatrix}=\begin{bmatrix}0&\frac{-1}{2}&\frac{-5}{2}\\\frac{1}{2}&0&3\\\frac{5}{2}&-3&0\end{bmatrix}$

$Q'=\begin{bmatrix}0&\frac{1}{2}&\frac{5}{2}\\\frac{-1}{2}&0&-3\\\frac{-5}{2}&3&0\end{bmatrix}=-Q$ Thus Q is a skew symmetric matrix.

Now $P+Q=\begin{bmatrix}2&\frac{-3}{2}&\frac{-3}{2}\\\frac{-3}{2}&3&1\\\frac{-3}{2}&1&-3\end{bmatrix}+\begin{bmatrix}0&\frac{-1}{2}&\frac{-5}{2}\\\frac{1}{2}&0&3\\\frac{5}{2}&-3&0\end{bmatrix}=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}=B$

Thus, B is expressed as the sum of a symmetric and a skew symmetric matrix.

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5. Find area of the triangle with vertices (2, 7), (1, 1), (10, 8)

Sol: Area of the triangle with vertices $A(x_1,y_1)=(2,7)$, $B(x_2,y_2)=(1,1)$, $C(x_3,y_3)=(10,8)$ is

$$\Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\frac{1}{2}\begin{vmatrix}2 & 7 & 1\\ 1 & 1 & 1\\ 10 & 8 & 1\end{vmatrix}=\frac{1}{2}|[2(1-8)-7(1-10)+1(8-10)]|$$

$$=\frac{1}{2}|[2(-7)-7(-9)+1(-2)]|=\frac{1}{2}|-14+63-2|=\frac{1}{2}[47]=\frac{47}{2}\text{ Sq.units}$$

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6. Find equation of line joining (3, 1) and (9, 3) using determinants.

Sol: Let P(x,y) be a point on the line joining points and A $(x_1,y_1)=(3,1)$ and B$(x_2,y_2)=(9,3)$.

Then, the points A,B and P are collinear.

Hence, the area of triangle ABP will be zero.

$$\therefore\ \Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\frac{1}{2}\begin{vmatrix}3 & 1 & 1\\ 9 & 3 & 1\\ x & y & 1\end{vmatrix}=0\Rightarrow\ \frac{1}{2}[3(3-y)-1(9-x)+1(9y-3x)]=0$$

$$\Rightarrow\ 9-3y-9+x+9y-3x=0\Rightarrow\ 6y-2x=0\Rightarrow x-3y=0$$

$\therefore$ The equation of the line joining the given points is $x-3y=0$

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7. Find the inverse of the matrix $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$

Sol: Let $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$

Then $|A|=1(8-6)-0+3(3-4)=2-3=-1\neq 0$. So, A is non singular , hence A^{-1} exists.

$A_{11}=2;\quad A_{12}=-9;\quad A_{13}=-6$

$A_{21}=0;\quad A_{22}=-2;\quad A_{23}=-1$

$A_{31}=-1;\quad A_{32}=3;\quad A_{33}=2$

Hence, $\text{adj }A=\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & 3\\ -6 & -1 & 2\end{bmatrix}\quad \therefore A^{-1}=\frac{1}{|A|}\text{adj}A=-1\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & 3\\ -6 & -1 & 2\end{bmatrix}=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$

8. **Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.**

Sol: Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

Then, $|A| = 15 - 14 = 1$

Now, $A_{11} = 5;\ A_{12} = -2;\ A_{21} = -7\ \ A_{22} = 3;$

Hence, $\text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj}A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Now, Let $B = \begin{pmatrix} 6 & 8 \\ 7 & 9 \end{pmatrix}$. Then, $|B| = 54 - 56 = -2$

Now $A_{11} = 9;\ A_{12} = -7;\ A_{21} = -8;\ \ A_{22} = 6$. Hence, $\text{adj } B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$$\therefore B^{-1} = \frac{1}{|B|}\text{adj}B = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}$$

$$\text{Now } B^{-1}A^{-1} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & \frac{-67}{2} \end{bmatrix} \text{........(1)}$$

$$\text{Also, } AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18 + 49 & 24 + 63 \\ 12 + 35 & 16 + 45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

Then, we have $|AB| = 67(61) - 87(47) = 4087 - 4089 = -2$

$$\text{adj }(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$\text{Hence } (AB)^{-1} = \frac{1}{|AB|}\text{adj}(AB) = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \text{......(2)}$$

From (1) and (2) we have $(AB)^{-1} = B^{-1}A^{-1}$

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VECTOR ALGEBRA

9. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Sol: We have $\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

A vector which is perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is given by

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=-2\hat{i}+4\hat{j}-2\hat{k}\ \ (=\vec{c},\ \text{say})$$

Now, $|\vec{c}|=\sqrt{4+16+4}=\sqrt{24}=2\sqrt{6}$

$\therefore$ the required unit vector is $\dfrac{\vec{c}}{|\vec{c}|}=\dfrac{-1}{\sqrt{6}}\hat{i}+\dfrac{2}{\sqrt{6}}\hat{j}-\dfrac{1}{\sqrt{6}}\hat{k}$

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10. Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k})$, $B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear.

Sol: We have $\overline{AB}=(1+2)\hat{i}+(2-3)\hat{j}+(3-5)\hat{k}=3\hat{i}-\hat{j}-2\hat{k}\Rightarrow|\overline{AB}|=\sqrt{3^2+(-1)^2+(-2)^2}=\sqrt{14}$

$\overline{BC}=(7-1)\hat{i}+(0-2)\hat{j}+(-1-3)\hat{k}=6\hat{i}-2\hat{j}-4\hat{k}\Rightarrow|\overline{BC}|=\sqrt{6^2+(-2)^2+(-4)^2}=2\sqrt{14}$

$\overline{AC}=(7+2)\hat{i}+(0-3)\hat{j}+(-1-5)\hat{k}=9\hat{i}-3\hat{j}-6\hat{k}\Rightarrow|\overline{AC}|=\sqrt{9^2+(-3)^2+(-6)^2}=3\sqrt{14}$

Here, $|\overline{AC}|=|\overline{AB}|+|\overline{BC}|$.

Hence, the points A, B and C are collinear.

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PROBABILITY

11. If A and B are two events such that $P(A)=\frac{1}{4}, P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$, find P(not A and not B).

Sol: Given that $P(A)=\frac{1}{4},\ P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$

P(not A and not B) $=P(A'\cap B')=P[(A\cup B)']$ $\quad [\because \mathbf{A'\cap B=(A\cup B)'}]$

$$=1-P(A\cup B)=1-[P(A)+P(B)-P(A\cap B)]$$

$$=1-\left[\frac{1}{4}+\frac{1}{2}-\frac{1}{8}\right]=1-\frac{5}{8}=\frac{3}{8}$$

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12. Determine P(E|F) when a coin is tossed three times, where

E : head on third toss , F : heads on first two tosses

Sol: When a coin is tossed 3 times S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$\Rightarrow n(S)=8$

E: Head on third toss $\Rightarrow$ E = {HHH, HTH, THH, TTH} $\Rightarrow n(E)=4$

F: Heads on first two tosses $\Rightarrow$ F = {HHH, HHT} $\Rightarrow n(F)=2$

Hence $E\cap F$ = {HHH} $\Rightarrow n(E\cap F)=1$

Now $P(E)=\frac{n(E)}{n(S)}=\frac{4}{8}=\frac{1}{2},\quad P(F)=\frac{n(F)}{n(S)}=\frac{2}{8}=\frac{1}{4},\ P(E\cap F)=\frac{1}{8}$

$$\therefore P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1/8}{1/4}=\frac{1}{8}\times\frac{4}{1}=\frac{1}{2}$$

BABY BULLET-Q AP

LINEAR PROGRAMMING

13. Solve the following linear programming problem graphically:

Maximise Z = 4x + y subject to the constraints: $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$.

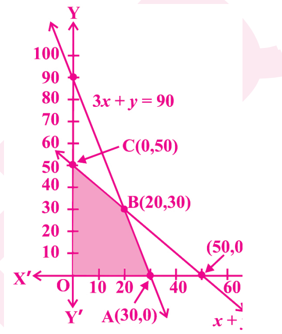
Sol: The feasible region determined by the system of constraints, $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$

is given by the shaded region.

The coordinates of the corner points O, A, B, C are (0, 0), (30, 0), (20, 30), (0, 50).

The value of Z at these corner points are as follows:

Corner Point	Corresponding value of Z	
(0,0)	0	
(30,0)	120	← Maximum
(20,30)	110	
(0,50)	50	



Hence, maximum value of Z is 120 at the point (30, 0).

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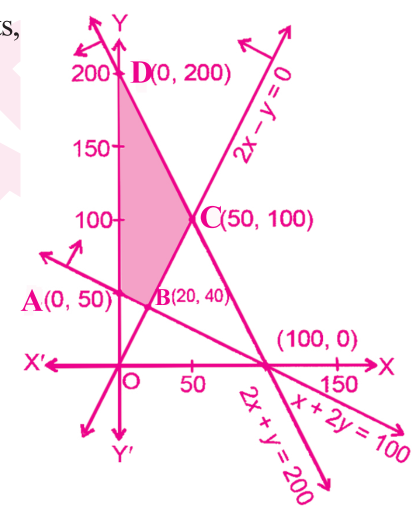
14. Show that the minimum of Z occurs at more than two points.

Minimise and Maximise Z = x + 2y subject to $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$; $x, y \ge 0$.

Sol: The feasible region determined by the system of constraints, $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$ and $x, y \ge 0$ is given by the shaded region.

The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200)

The values of Z at these corner points are as follows:



Corner Point	Z = x + 2y	
A(0,50)	100	→ Minimum
B(20,40)	100	→ Minimum
C(50,100)	250	
D(0,200)	400	→ Maximum

The minimum value of Z is 100 at A (0,50), B(20,40) and the maximum value of Z is 400 at D(0,200)

CONTINUITY & DIFFERENTIABILITY

15. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases} \quad \text{at } x = 5$$

Sol: The given function is $f(x) = \begin{cases} kx+1, \text{if } x \le 5 \\ 3x-5, \text{if } x > 5 \end{cases}$

It is clear that f is defined at x=5 and f(5)=kx+1=5k+1

For the given function f(x) to be continuous at x=5, we have

$$\underset{x\to 5-}{\text{Lt}} f(x) = \underset{x\to 5+}{\text{Lt}} f(x) = f(5) \Rightarrow \underset{x\to 5-}{\text{Lt}} (kx+1) = \underset{x\to 5+}{\text{Lt}} (3x-5) = 5k+1$$

$$\Rightarrow 5k+1 = 3(5)-5 = 5k+1 \Rightarrow 5k+1 = 15-5 = 5k+1$$

$$\Rightarrow 5k+1 = 10 = 5k+1 \quad \Rightarrow 5k+1 = 10 \Rightarrow 5k = 9 \Rightarrow k = \frac{9}{5}$$ $\therefore$ the value of k=9/5.

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16. Find $\frac{dy}{dx}$ in $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Sol: Given that $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$ Put x=tanθ.

$$\therefore \quad y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta)$$

$$y = 2\theta = 2\tan^{-1}x (: \quad x = \tan\theta \Rightarrow \theta = \tan^{-1}x) \Rightarrow \frac{dy}{dx} = 2\frac{1}{1+x^2} = \frac{2}{1+x^2}$$

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17. Find $\frac{dy}{dx}$ in $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Sol: Given that $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

To simplify the given inverse t-function, we put x=tanθ.

$$\therefore \quad y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta \Rightarrow y = 2\tan^{-1}x \ (\because x = \tan\theta \Rightarrow \theta = \tan^{-1}x)$$

$$\therefore \frac{dy}{dx} = 2\frac{1}{1+x^2} = \frac{2}{1+x^2}$$

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18. Differentiate the function $x\ (\log x)^{\log x}$, $x > 1$ w.r.t x.

Sol: Let $y=(\log x)^{\log x}, x>1$ ....(i)

Taking log of both sides of (i), we have

$$\log y=\log(\log x)^{\log x}=\log x\log(\log x)$$

Differentiating both sides w.r.t x, we have $\frac{d}{dx}(\log y)=\frac{d}{dx}(\log x\log(\log x))$

$$\therefore \frac{1}{y}\frac{dy}{dx}=\log x\frac{d}{dx}\log(\log x)+\log(\log x)\frac{d}{dx}\log x \quad \text{(By Product Rule)}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\cancel{\log x}\,\frac{1}{\cancel{\log x}}\frac{d}{dx}\log x+\log(\log x)\frac{1}{x} \quad \left[\because \frac{d}{dx}\log f(x)=\frac{1}{f(x)}\frac{d}{dx}f(x)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{\log(\log x)}{x}=\frac{1+\log(\log x)}{x}$$

$$\therefore \frac{dy}{dx}=y\left(\frac{1+\log(\log x)}{x}\right)$$

BABY BULLET-Q AP

INTEGRATION

19. Find $\int \frac{3x-2}{(x+1)^2(x+3)}dx$

Sol: Let $\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3} = \frac{A(x+1)(x+3)+B(x+3)+C(x+1)^2}{(x+1)^2(x+3)}$

$\Rightarrow A(x+1)(x+3) + B(x+3) + C(x+1)^2 = 3x - 2$ ....(1)

Put $x = -1$ in (1)

$\Rightarrow A(0) + B(-1+3) + C(0)^2 = 3(-1) - 2 \Rightarrow 2B = -5 \Rightarrow B = -5/2$

Put $x = -3$ in (1) $\Rightarrow A(0) + B(0) + C(9-6+1) = -9 - 2 \Rightarrow 4C = -11 \Rightarrow C = -11/4$

Comparing coefficient of x^2 from (1) we have

$A + C = 0 \Rightarrow A = -C \Rightarrow A = \frac{11}{4}$

$$\therefore \frac{3x-2}{(x+1)^2(x+3)} = \frac{11}{4(x+1)} - \frac{5}{2(x+1)^2} - \frac{11}{4(x+3)}$$

$$\int \frac{3x-2}{(x+1)^2(x+3)} = \frac{11}{4}\int \frac{dx}{x+1} - \frac{5}{2}\int \frac{dx}{(x+1)^2} - \frac{11}{4}\int \frac{dx}{x+3}$$

$$= \frac{11}{4}\log|x+1| + \frac{5}{2(x+1)} - \frac{11}{4}\log|x+3| + C = \frac{11}{4}\log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + C$$

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20. Find integral of $\frac{1}{1-\tan x}$

Sol: Let $I = \int \frac{1}{1-\tan x}dx = \int \frac{1}{1-\frac{\sin x}{\cos x}}dx = \int \frac{\cos x}{\cos x - \sin x}dx$

$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx = \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)}dx$$

$$= \frac{1}{2}\int 1dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx = \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t} = \frac{x}{2} - \frac{1}{2}\log|t| + C = \frac{x}{2} - \frac{1}{2}\log|(\cos x - \sin x)| + C$$

BABY BULLET-Q AP

21. Find the integral of cos 2x cos 4x cos 6x

Sol: We know that $\cos A \cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$

$$\therefore \int \cos 2x(\cos 4x \cos 6x)dx = \int \cos 2x\left[\frac{1}{2}[\cos(4x+6x)+\cos(4x-6x)]\right]dx$$

$$=\frac{1}{2}\int[\cos 2x \cos 10x + \cos 2x \cos(-2x)]\,dx = \frac{1}{2}\int[\cos 2x \cos 10x + \cos^2 2x]\,dx$$

$$=\frac{1}{2}\int\left[\left(\frac{1}{2}\cos(2x+10x)+\frac{1}{2}\cos(2x-10x)\right)+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$=\frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)dx = \frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

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22. Find the integral of $\frac{1}{\cos(x-a)\cos(x-b)}$

Sol: $$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}$$

$$=\frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]$$

$$\therefore \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx$$

$$=\frac{1}{\sin(a-b)}[-\log|\cos(x-b)|+\log|\cos(x-a)|] = \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

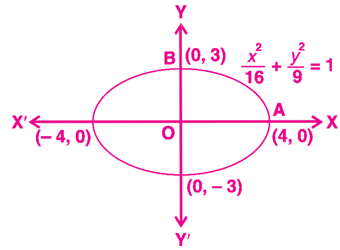
BABY BULLET-Q AP

APPLICATION OF INTEGRALS

23. Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Sol: We have $\frac{x^2}{16}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9}=1-\frac{x^2}{16} \Rightarrow y^2=9\left(\frac{16-x^2}{16}\right) \Rightarrow y=\frac{3}{4}\sqrt{16-x^2}$

Area of the ellipse $= 4\times \text{Area}(OAB)$



$$A = 4\int_0^4 y\,dx = 4\left(\frac{3}{4}\right)\int_0^4 \sqrt{16-x^2}\,dx$$

$$=3\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4 = 3\left[2\sqrt{16-16}+8\sin^{-1}(1)-0-8\sin^{-1}(0)\right]$$

$$=3\left[\frac{8\pi}{2}\right]=3[4\pi]=12\pi$$