2 INVERSE TRIG.FUNCTIONS

1 OMQ + 1 SAQ [1 M + 4M = 5 M]

CONCEPTS & FORMULAS

We know that T' functions like sinθ, cosθ tanθ are used to find T'values of the given angles.

Conversely Inverse trigonometric functions are used to **find the angle** for a given real value.

Ex: If $\sin\theta = \frac{1}{2}$ then $\theta = \sin^{-1}\frac{1}{2} = 30^{\circ} = \frac{\pi}{6}$

All the 6 Trigonometric Functions are not bijections in their general domains and codomains. But by restricting the domains and codomains properly they can be converted into bijections and hence inverse trigonometric functions.

1) Domains and ranges (principal value branches) of inverse trigonometric functions.

Function	Domain	Principal value Range
$\sin^{-1}x$	$[-1,1]$	$[-\pi/2,\pi/2]$
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1}x$	R	$(-\pi/2,\pi/2)$
$\cot^{-1}x$	R	$(0,\pi)$
$\sec^{-1}x$	$(-\infty,-1]\cup[1,\infty)$	$[0,\pi]-\{\pi/2\}$
$\csc^{-1}x$	$(-\infty,-1]\cup[1,\infty)$	$[-\pi/2,\pi/2]-\{0\}$

2)
- $\sin^{-1}(\sin\theta) = \theta$ for $\theta \in [-\pi/2, \pi/2]$
- $\tan^{-1}(\tan\theta) = \theta$ for $\theta \in (-\pi/2, \pi/2)$
- $\cos^{-1}(\cos\theta) = \theta$ for $\theta \in [0, \pi]$
- $\cot^{-1}(\cot\theta) = \theta$ for $\theta \in (0, \pi)$

3)
- $\sin^{-1}(-x) = -\sin^{-1}x$;
- $\cos^{-1}(-x) = \pi - \cos^{-1}x$;
- $\tan^{-1}(-x) = -\tan^{-1}x$;
- $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$;
- $\sec^{-1}(-x) = \pi - \sec^{-1}x$;
- $\cot^{-1}(-x) = \pi - \cot^{-1}x$

4) Important remarks on Inverse Trigonometric Functions:

- All the six inverse trigonometric functions are merely angles
- In $\sin^{-1}x$, x is a real number belonging to [-1,1] and functional value of $\sin^{-1}x$ is an angle in radian measure.
- $\sin^{-1}x=\theta \to$ x is a real number and θ is also a real value in radian measure.
- $\sin^{-1}(\sin\theta)=\theta$ and this θ is the measure of angle in radians.
- $\sin(\sin^{-1}x)=x$ and this x is a real number.
- $\sin(\sin^{-1}x) \neq \sin^{-1}(\sin x)$.

  Infact, $\sin^{-1}(\sin\theta)=\theta$ for $-\pi/2 \le \theta \le \pi/2$ and $\sin(\sin^{-1}x)=x$ for $-1\le x\le 1$
- $\sin^{-1}x$ is also written as arc sinx.
- $\sin^{-1}x \neq \frac{1}{\sin x}$ i.e., $\sin^{-1}x$ and $(\sin x)^{-1}$ are different. Infact $(\sin x)^{-1} = \frac{1}{\sin x} = \csc x$

Master Hint Table

MODEL PROBLEMS -I

- $\sin^{-1}\frac{1}{\sqrt{2}}=\sin^{-1}\left(\sin\frac{\pi}{4}\right)=\frac{\pi}{4}$
- $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)=-\sin^{-1}\frac{1}{\sqrt{2}}=-\frac{\pi}{4}$
- $\sin^{-1}\frac{1}{2}=\sin^{-1}\left(\sin\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\sin^{-1}\left(-\frac{1}{2}\right)=-\sin^{-1}\frac{1}{2}=-\frac{\pi}{6}$
- $\cos^{-1}\frac{1}{2}=\cos^{-1}\left(\cos\frac{\pi}{3}\right)=\frac{\pi}{3}$
- $\cos^{-1}\left(-\frac{1}{2}\right)=\pi-\cos^{-1}\frac{1}{2}=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$
- $\cos^{-1}\frac{\sqrt{3}}{2}=\cos^{-1}\left(\cos\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\pi-\cos^{-1}\frac{\sqrt{3}}{2}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$
- $\tan^{-1}\sqrt{3}=\tan^{-1}\left(\tan\frac{\pi}{3}\right)=\frac{\pi}{3}$
- $\tan^{-1}\left(-\sqrt{3}\right)=-\tan^{-1}\sqrt{3}=-\frac{\pi}{3}$
- $\cot^{-1}\sqrt{3}=\cot^{-1}\left(\cot\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\cot^{-1}\left(-\sqrt{3}\right)=\pi-\cot^{-1}\sqrt{3}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$
- $\sec^{-1}\frac{2}{\sqrt{3}}=\sec^{-1}\left(\sec\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)=\pi-\sec^{-1}\frac{2}{\sqrt{3}}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$
- $\operatorname{cosec}^{-1}2=\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\operatorname{cosec}^{-1}(-2)=-\operatorname{cosec}^{-1}2=-\frac{\pi}{6}$

MODEL PROBLEMS -II

- $\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$
- $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$
- $\sin^{-1}\left(\sin\frac{2\pi}{5}\right)=\frac{2\pi}{5}$
- $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=\sin^{-1}\left(\sin\left(\pi-\frac{2\pi}{5}\right)\right)=\sin^{-1}\left(\sin\frac{2\pi}{5}\right)=\frac{2\pi}{5}$
- $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)=-\frac{\pi}{3}$
- $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)=\sin^{-1}\left(\sin\left(\pi+\frac{\pi}{3}\right)\right)=\sin^{-1}\left(-\sin\frac{\pi}{3}\right)=-\sin^{-1}\left(\sin\frac{\pi}{3}\right)=-\frac{\pi}{3}$
- $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)=\frac{2\pi}{3}$
- $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)=\cos^{-1}\left(\cos\left(\pi+\frac{\pi}{3}\right)\right)=\cos^{-1}\left(-\cos\frac{\pi}{3}\right)=\pi-\cos^{-1}\left(\cos\frac{\pi}{3}\right)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$
- $\tan^{-1}\left(\tan\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\tan^{-1}\left(\tan\left(\pi+\frac{\pi}{6}\right)\right)=\tan^{-1}\left(\tan\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)=-\frac{\pi}{4}$
- $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)=\tan^{-1}\left(\tan\left(\pi-\frac{\pi}{4}\right)\right)=\tan^{-1}\left(-\tan\frac{\pi}{4}\right)=-\tan^{-1}\left(\tan\frac{\pi}{4}\right)=-\frac{\pi}{4}$