OMQ

SOLUTIONS

1 RELATIONS AND FUNCTIONS

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

I. Select the correct option from the given choices. $1 \times 1 = 1$ Mark

1. Let R be the relation in the set {1, 2, 3, 4} given by [2]

R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

1) R is reflexive and symmetric but not transitive.

2) R is reflexive and transitive but not symmetric.

3) R is symmetric and transitive but not reflexive. 4)R is an equivalence relation.

2. Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}. [3]

Choose the correct answer.

1) $(2, 4) \in R$ 2) $(3, 8) \in R$ **3) $(6, 8) \in R$** 4) $(8, 7) \in R$

3. Let f : R →R be defined as $f(x) = x^4$. Choose the correct answer. [4]

1) f is one-one onto 2) f is many-one onto

3) f is one-one but not onto **4) f is neither one-one nor onto.**

4. Let f : R →R be defined as f (x) = 3x. Choose the correct answer. [1]

1) f is one-one onto 2) f is many-one onto

3) f is one-one but not onto 4) f is neither one-one nor onto.

5. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) [1]

which are reflexive and symmetric but not transitive is

1) 1 2) 2 3) 3 4) 4

6. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is [2]

1) 1 **2) 2** 3) 3 4) 4

7. Set A has 3 elements and the set B has 4 elements. Then the number of [3]

injective mappings that can be defined from A to B :

1) 144 2) 12 **3) 24** 4) 64

8. Let A = {3, 5}. Then the number of reflexive relations on set A is [2]

1) 2 **2) 4** 3) 0 4) 8

9. Let f : R→ R be the function defined by $f(x)=\frac{2x-1}{2}$ and g: R→ R be another function defined by g(x)=x+2, then (gof)$\left(\frac{3}{2}\right)$ is [2]

1) 2 **2) 3** 3) $\frac{7}{2}$ 4) $\frac{5}{2}$

10. If $f(x)=\frac{2x+3}{4}$ then $f^{-1}(x)$ is [4]

1) $\frac{4}{2x+3}$ 2) $\frac{2x-3}{4}$ 3) $2x+3$ **4) $\frac{4x-3}{2}$**

[☞ Answers for MCQ are kept in **bold font]**

HINTS & SOLUTIONS FOR MCQ

1. **(2)**

Let A= {1, 2, 3, 4} and $R=\{(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$

(i) $(a,a)\in R$ for every $a\in A$. So, R is reflexive.

(ii) $(1,2)\in R$ but $(2,1)\notin R$. So, R is not symmetric.

(iii) $(a,b),(b,c)\in R \Rightarrow (a,c)\in R\ \forall\ a,b,c\in A$. So, R is transitive.

R is reflexive and transitive but not symmetric.

2. **(3)**

Given set is the set of Natural numbers N

Given that $b > 6 \Rightarrow b=7,8,9,\ldots$ and $a = b - 2$,

Now, $b= 7 \Rightarrow a = 7 - 2= 5$; $(5,7)\in R$

$b= 8 \Rightarrow a = 8 - 2= 6$; $(6,8)\in R$

3. **(4)**

Sol: $f : R \to R$ and $f(x) = x^4$

$\Rightarrow f(-1) =(-1)^4 =1;\ f(1) = 1^4 =1\ \therefore$ f is not - one - one

Also $x^4 \ge 0$ (Domain) $\Rightarrow$ Range $= [0, \infty) \ne$ co-domain $\forall\ x \in R$

$\Rightarrow$ f is not onto $\therefore$ f is neither one-one nor onto

4. **(1)**

Sol: $f : R \to R$ and $f(x) = 3x$. This is a linear function of the form $f(x) = ax + b$

$\Rightarrow$ f is both one-one and onto

5. **(1)**

Sol: The given set is A = {1, 2, 3}

The smallest relation containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is given by R ={(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)}

(i) {(1, 1), (2, 2), (3, 3)} $\in$ R $\Rightarrow$R is reflexive.

(ii) {(1, 2), (2, 1)}$\in$ R and {(1, 3)(3, 1)}$\in$ R$\Rightarrow$ R is symmetric.

(iii) {(3, 1), (1, 2)} $\in$ R but (3,2) $\notin$ R $\Rightarrow$ R is not transitive.

Now, if we add only any of the two pairs (3,2) and (2,3) (or both) to relation R,

then relation R will become transitive.

Hence, the total number of desired relations is one.

6. **(2)**

Sol: The given set is $A=\{1, 2, 3\}$

The smallest equivalence relation containing (1,2) is given by:

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

Now, we are left with only four other pairs (2,3),(3,2), (1,3) and (3,1).

If we add any one pair say(2, 3) to R_1 , then for symmetry we must add (3,2), and for transitivity we are required to add (1,3) and (3,1).

Hence, the only equivalence relation (bigger than R_1) is the universal relation.

This shows that the total number of equivalence relations containing (1.2) is two.

7. **(3)**

Sol: n(A) = 3, n(B) = 4

no.of injective mappings from A to B $= {}^{n(B)}P_{n(A)} = {}^4P_3 = 4\times3\times2 = 24$

8. **(2)**

Sol: $A = \{3, 5\}$

$A\times A=\{3, 5\}\times\{3, 5\}== \{(3, 3), (5, 5), (3, 5), (5, 3)\}$

Here, reflexive relations are

$R_1 = \{(3, 3) (5, 5)\}$; $R_2 = \{(3, 3) (5, 5) (3, 5)\}$;

$R_3 = \{(3, 3) (5, 5) (5, 3)\}$; $R_4 = \{(3, 3) (5, 5) (3, 5) (5, 3)\}$;

$\therefore$ Number of reflexive relations on set A is 4.

9. **(2)**

Sol: $$gof\left(\frac{3}{2}\right)=g\left[f\left(\frac{3}{2}\right)\right]=g\left[\frac{2\times\frac{3}{2}-1}{2}\right]=g\left[\frac{2}{2}\right]=g(1)=1+2=3$$

10. **(4)**

Sol: Put $f^{-1}(x) = y$ ....(1)$\Rightarrow x = f(y)$

$$x=\frac{2y+3}{4} \Rightarrow 4x=2y+3\Rightarrow 2y=4x-3\Rightarrow y=\frac{4x-3}{2}$$

$$\therefore f^{-1}(x)=\frac{4x-3}{2} \qquad (\because \text{from}(1))$$

2 INVERSE TRIG.FUNCTIONS

1 OMQ + 1 SAQ [1 M + 4M = 5 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1 = 1$ Mark

I. Select the correct option from the given choices.

1. If $\sin^{-1}x = y$, then [2]

1) $0 \le y \le \pi$ 2) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ 3) $0 < y < \pi$ 4) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

2. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to [2]

1) π 2) $-\frac{\pi}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{2\pi}{3}$

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to [2]

1) $\frac{7\pi}{6}$ 2) $\frac{5\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

4. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to [4]

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) 1

5. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to [2]

1) π 2) $-\frac{\pi}{2}$ 3) 0 4) $2\sqrt{3}$

6. $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to [4]

1) $\frac{x}{\sqrt{1-x^2}}$ 2) $\frac{1}{\sqrt{1-x^2}}$ 3) $\frac{1}{\sqrt{1+x^2}}$ 4) $\frac{x}{\sqrt{1+x^2}}$

7. If $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to [3]

1) $0, \frac{1}{2}$ 2) $1, \frac{1}{2}$ 3) 0 4) $\frac{1}{2}$

8. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{-1}{2}\right)\right]$ is equal to : [2]

1) 1 **2) $\frac{1}{2}$** 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

9. The principal value of $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is [1]

1) $\frac{\pi}{12}$ 2) π 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

10. The principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ is [1]

1) $\frac{\pi}{8}$ 2) $\frac{3\pi}{8}$ 3) $-\frac{\pi}{8}$ 4) $-\frac{3\pi}{8}$

11. The domain of the function $\cos^{-1}(2x-3)$ is [4]

1) $[-1,1]$ 2) $(1,2)$ 3) $(-1,1)$ **4) $[1,2]$**

12. The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is **[BMP2]** [3]

1) $\frac{\pi}{10}$ 2) $\frac{3\pi}{5}$ **3) $-\frac{\pi}{10}$** 4) $-\frac{3\pi}{5}$

13. The principal value of $\tan^{-1}(-1)$ is [2]

1) $\frac{\pi}{4}$ **2) $-\frac{\pi}{4}$** 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

14. The principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is [4]

1) $\frac{13\pi}{6}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ **4) $\frac{\pi}{6}$**

15. The simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is [3]

1) $\frac{\pi}{4}-\frac{\pi}{2}$ 2) $\frac{\pi}{4}+\frac{\pi}{2}$ **3) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$** 4) $\frac{\pi}{4}+\frac{\pi}{2}\cos^{-1}x$

[☞ Answers for MCQ are kept in **bold font]**

HINTS & SOLUTIONS FOR MCQ

1. **(2) Hint:** We know that range of the principal value of $\sin^{-1}x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Sol: Given that $\sin^{-1}x = y$, $\therefore -\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

2. **(2) Hint Formula :** $\sec^{-1}(-x) = \pi - \sec^{-1}x$;

Sol: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - (\pi - \sec^{-1}2) = 60^\circ - 180^\circ + 60^\circ = -60^\circ = \frac{-\pi}{3}$

3. **(2)Hint:** $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$ and $\cos^{-1}(-x) = \pi - \cos^{-1}x$

Sol: $\therefore \cos^{-1}\cos\frac{7\pi}{6} = \cos^{-1}\left[\cos\left(\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[-\cos\frac{\pi}{6}\right] = \pi - \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

4. **(4)Hint Formula :** $\sin^{-1}(-x) = -\sin^{-1}(x)$

Sol: $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left[60^\circ + \sin\left(\frac{1}{2}\right)\right] = \sin[60^\circ + 30^\circ] = \sin 90^\circ = 1$

5. **(2)Hint Formula :** $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

Sol: $\mathbf{\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})} = \tan^{-1}(\sqrt{3}) - (\pi - \cot^{-1}(\sqrt{3})) = 60^\circ - 180^\circ + 30^\circ = -90^\circ = \frac{-\pi}{2}$

6. **(4)**

Sol: $\tan^{-1}x = \theta \Rightarrow \frac{x}{1} = \tan\theta \Rightarrow \sin\theta = \frac{AB}{BC} = \frac{x}{\sqrt{1+x^2}}$

7. **(3)Hint Formula :** $\sin(90^\circ+\theta) = \cos\theta$; $\cos 2\theta = 1 - 2\text{ins}^2\theta$

Sol: $G.E = \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x \Rightarrow (1-x) = \sin\left[\frac{\pi}{2} + 2\sin^{-1}x\right]$

$\Rightarrow 1 - x = \cos(2\sin^{-1}x) = 1 - 2(\sin(\sin^{-1}x))^2 = 1 - 2x^2$

$\therefore 1 - x = 1 - 2x^2 \Rightarrow 2x^2 - x = 0 \Rightarrow x(2x-1) = 0 \Rightarrow x = 0, \frac{1}{2}$

8. **(2)Hint:** $\sin^{-1}\left(-\frac{1}{2}\right)=-30^\circ$ and $\cos^{-1}(-x)=\pi-\cos^{-1}x$

Sol: $\sin(60^\circ-30^\circ)=\sin 30^\circ=\frac{1}{2}$

9. **(1)**

Sol: $\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=60^\circ-45^\circ=15^\circ=\frac{\pi}{12}$

10. **(1)**

Sol: $\tan^{-1}\left(\tan\left(\pi+\frac{\pi}{8}\right)\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right)=\frac{\pi}{8}$

11. **(4) Hint Formula :** $\cos^{-1}x$ is defined for $x\in(-1,1)$

Sol: $-1\le(2x-3)\le 1\Rightarrow(3-1)\le 2x\le(3+1)\Rightarrow 2\le 2x\le 4\Rightarrow 1\le x\le 2\Rightarrow x\in[1,2]$

12. **(3)Hint Formula :** $\cos\theta=\sin(90^\circ-\theta)$

Sol: $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right]=\sin^{-1}\left[\sin\left(\frac{-\pi}{10}\right)\right]=-\frac{\pi}{10}$

13. **(2) Hint Formula :** $\tan^{-1}(-x)=-\tan^{-1}x$

Sol: $\tan^{-1}(-1)=-\tan^{-1}(1)=-\frac{\pi}{4}$

14. **(4)**

Sol: $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)=\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]=\cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]=\frac{\pi}{6}$

15. **(3)Hint:** Put $x=\cos 2\theta\Rightarrow 2\theta=\cos^{-1}x\Rightarrow\theta=\frac{1}{2}\cos^{-1}x$

Sol: $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}=\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}=\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}$

$$=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\tan\theta}{1+\tan\theta}=\tan\left(\frac{\pi}{4}-\theta\right)$$

$$\therefore\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\theta=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

3 MATRICES

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1=$ 1 Mark

I. Select the correct option from the given choices.

1. $A=[a_{ij}]_{m\times n}$ is a square matrix, if [3]

1) m<n 2) m>n **3) m=n** 4) $m=n^2$

2. Which of the given values of x and y make the following pair of matrices [3]

equal $\begin{bmatrix} 3x+2 & 5 \\ y+1 & 2-3x \end{bmatrix}=\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

1) $x=\frac{-1}{3}, y=7$ 2) $y=7, x=\frac{2}{3}$ **3) $y=7, x=\frac{-2}{3}$** 4) $x=\frac{-1}{3}, y=\frac{-2}{3}$

3. The number of all possible matrices of order 3×3 with each entry 0 or 1 is: [4]

1) 27 2) 18 3) 81 **4) 512**

4. Assume Y, W, P are matrices of order $3\times k$, $n\times3$, $p\times k$ respectively. [1]
The restriction on n, k and p so that PY + WY will be defined are:

1) k = 3, p = n 2) k is arbitrary, p = 2

3) p is arbitrary, k = 3 4) k = 2, p = 3

5. Assume X, Z are matrices of order $2\times n$, $2\times p$ respectively. [2]
If n=p, then the order of the matrix 7X–5Z is

1) p×2 **2) 2×n** 3) n×3 4) p×n

6. If A, B are symmetric matrices of same order, then AB – BA is a [1]

1) Skew symmetric matrix 2) Symmetric matrix

3) Zero matrix 4) Identity matrix

7. If $A=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A+A'=I$, then the value of α is [2]

1) $\frac{\pi}{6}$ **2) $\frac{\pi}{3}$** 3) π 4) $\frac{3\pi}{2}$

8. Matrices A and B will be inverse of each other only if [4]

1) AB = BA 2) AB = BA = O 3) AB = O, BA = I **4) AB = BA = I**

9. If $A=\begin{bmatrix} \alpha & \beta \\ -\gamma & \alpha \end{bmatrix}$ is such that $A^2=I$, then [3]

1) $1+\alpha^2+\beta\gamma=0$ 2) $1-\alpha^2+\beta\gamma=0$ **3) $1-\alpha^2-\beta\gamma=0$** 4) $1+\alpha^2-\beta\gamma=0$

10. If the matrix A is both symmetric and skew symmetric, then [2]

1) A is a diagonal matrix **2) A is a zero matrix**

3) A is a square matrix 4) None of these

11. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to[BMP1] [3]

1) A 2) I – A **3) I** 4) 3A

HINTS & SOLUTIONS FOR MCQ

1. **(3)**

Sol: In a square matrix, the number of rows is equal to the number of columns.

$\therefore$ $A=[a_{ij}]_{m\times n}$ is a square matrix, if m = n.

2. **(3)**

Sol: Equating the corresponding elements, in the given matrices, we get

$$3x+2=0 \Rightarrow 3x=-2 \Rightarrow x=\frac{-2}{3};\ 2-3x=4 \Rightarrow x=\frac{-2}{3}$$

$$y-2=5 \Rightarrow y=7;\ y+1=8 \Rightarrow y=7$$

3. **(4)**

Sol: Matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Hence, by the multiplication principle, the required number of possible matrices is $2^9 = 512$.

4. **(1)**

Sol: $\underset{p\times k}{P}\ \underset{3\times k}{Y} + \underset{n\times 3}{W}\ \underset{3\times k}{Y}$ has to be defined

i) If k = 3 then PY is defined $\left(PY_{p\times k}\right)$

ii) If n = p then WY is defined $\left(WY_{p\times k}\right)$

$\Rightarrow$ PY + WY (addition of matrices of same order) will be defined when k=3, p=n

5. **(2)**

Sol: Given $X_{2\times n} \Rightarrow$ order of 7X is $2\times n$

Given $Z_{2\times p} \Rightarrow$ order of 5Z is $2\times p$. Also $p = n \Rightarrow$ order of 7X – 5Z is $2\times p$ (or) $2\times n$

6. **(1)**

Sol: If A, B are symmetric matrices of same order, then A' =A and B' =B........(1)

Now consider $(AB-BA)' = (AB)' - (BA)'$ $[\because (A-B)' = A' - B']$

$= B'A' - A'B'$ $[\because (AB)' = B'A']$

$= BA - AB$ [from(1)]$= -(AB-BA)$ $\quad \therefore (AB-BA)' = -(AB-BA)$

Thus, AB–BA is a skew symmetric matrix.

7. **(2)**

Sol: Given that $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix} \Rightarrow A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

Now, $A+A'=I$

$$\therefore \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\Rightarrow\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we get

$$2\cos\alpha=1\Rightarrow\cos\alpha=\frac{1}{2}\Rightarrow\alpha=\cos^{-1}\left(\frac{1}{2}\right)\Rightarrow\alpha=\frac{\pi}{3}$$

8. **(4)**

Sol: From the definition of Inverse of a matrix, two matrices A and B are inverses of each other only when AB = BA = I.

9. **(3)**

Sol: Given that $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

$$A^2=A.A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}=\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta\\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}$$

Now A^2=I. Hence, $\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Equating the corresponding elements, we get

$$\alpha^2+\beta\gamma=1\Rightarrow\alpha^2+\beta\gamma-1=0\Rightarrow1-\alpha^2-\beta\gamma=0$$

10. **(2)**

Sol: Given that the matrix A is both symmetric and skew symmetric.

So A' = A and A' = – A

$\therefore A + A = O \Rightarrow 2A = O \Rightarrow A = O$ $\quad\therefore$ A is a zero matrix.

11. **(3)**

Sol: Given that the matrix A is a square matrix such that A^2=A

Now, $(I+A)^3-7A=I^3+A^3+3I^2A+3A^2I-7A$

$=I+A^2.A+3A+3A^2-7A$

$=I+A.A+3A+3A-7A[\because A^2=A]$

$=I+A^2-A \quad =I+A-A=I \qquad [\because A^2=A]$

Hence, $(I+A)^3-7A=I$

4 DETERMINANTS

1 OMQ + 1 VSAQ + 1 SAQ + 1 LAQ [1 M + 2M +4M + 8M = 15M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1=1$ Mark

1. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to [BMP2] [2]

   1) 6 **2) ± 6** 3) – 6 4) 0

2. If A is 3×3 matrix and det (3A) = k (detA), then k = [4]

   1) 9 2) 6 3) 1 **4) 27**

3. Value of k, for which $\begin{bmatrix} k & 2 \\ 8 & k \end{bmatrix}$ is singular [3]

   1) 4 2) –4 **3) ± 4** 4) 0

4. The area of a triangle with vertices (–3, 0), (0, 3) and (0, *k*) is 9 sq. units the value of *k* will be [1]

   1) 9 2) 3 3) –9 4) 6

5. If A is square matrix of order 3 and |A| = –4, then |adj A| is equal to [4]

   1) –4 2) 4 3) –16 **4) 16**

6. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (*k*, 4). Then *k* is [4]

   1) 12 2) –2 3) –12, –2 **4) 12, –2**

7. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij} then value of Δ is given by [4]

   1) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ 2) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

   3) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ **4) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$**

8. Let A be a nonsingular square matrix of order 3 × 3. Then | adj A| is equal to [2]

   1) | A| **2) $|A|^2$** 3) $|A|^3$ 4) 3|A|

9. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to [2]

1) det (A) **2) $\frac{1}{\det(A)}$** 3) 1 4) 0

10. If x, y, z are nonzero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is [1]

1) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ 2) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ 3) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ 4) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta \le 2\pi$. Then [4]

1) $\text{Det}(A)=0$ 2) $\text{Det}(A)\in(2,\infty)$

3) $\text{Det}(A)\in(2,4)$ **4) $\text{Det}(A)\in[2,4]$**

HINTS & SOLUTIONS FOR MCQ

1. **(2)**

Sol: G.E $= x^2 - 36 = 36 - 36 \Rightarrow x^2 - 36 = 0 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

2. **(4) Hint Formula:** $|kA| = k^n |A|$, where n = order of A

Sol: Given $A_{3\times3}$ $\therefore |3A| = 3^3 |A| = 27$ (det A)= k (det A) $\Rightarrow k = 27$

3. **(3)**

Sol: If A is singular $|A| = 0 \Rightarrow \begin{vmatrix} k & 2 \\ 8 & k \end{vmatrix} = 0 \Rightarrow k^2 - 16 = 0 \Rightarrow k = \pm 4$

4. **(1)**

Sol: Area of the triangle formed by (x_1, y_1) (x_2, y_2) (x_3, y_3) is

$$= \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = 9 \Rightarrow \frac{1}{2}\begin{vmatrix} 1 & -3 & 0 \\ 1 & 0 & 3 \\ 1 & 0 & k \end{vmatrix} = 9 \Rightarrow \begin{vmatrix} 1 & -3 & 0 \\ 1 & 0 & 3 \\ 1 & 0 & k \end{vmatrix} = 18 \Rightarrow 3|k-3| = 18 \Rightarrow |k-3| = 6$$

$k-3=6 \Rightarrow k=9$; $k-3=-6 \Rightarrow k=-3$

5. **(4) Hint Formula:** If $A_{n\times n}$, then $|AdjA| = |A|^{n-1}$

Sol: We have $A_{3\times3}$ $\therefore |AdjA| = (|A|)^2 = (-4)^2 = 16$

6. **(4)**

Sol: Area of triangle = 35

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 2 & -6 \\ 1 & 5 & 4 \\ 1 & k & 4 \end{vmatrix} = 35 \quad \Rightarrow |1(20-4k) - 2(4-4) - 6(k-5)| = 70$$

$\Rightarrow 20 - 4k + 0 - 6k + 30 = \pm 70 \Rightarrow 50 - 10k = \pm 70 \Rightarrow 5 - k = \pm 7$

$5-k=7 \Rightarrow k=-2$; $5-k=-7 \Rightarrow k=12$

7. **(4)**

Sol: Δ = Determinant of a matrix

= Sum of the products of the elements of a row (or) column with the corresponding co-factors $= (a_{11})A_{11} + (a_{12})A_{12} + (a_{31})A_{31}$ [using 1^{st} row]

8. **(2) Hint Formula:** If $A_{n\times n}$, then $|AdjA| = |A|^{n-1}$

Sol: We have $A_{3\times 3}$ $\therefore$ $|AdjA| = |A|^2$

9. **(2)**

Sol: $A_{2\times 2}$, and A^{-1}, exists $\because$ $AA^{-1} = A^{-1}A = I$

Consider $AA^{-1} = I$

$|AA^{-1}| = |I|$ $\Rightarrow |A|\,|A^{-1}| = I \Rightarrow |A^{-1}| = \frac{1}{|A|} \Rightarrow \det(A^{-1}) = \frac{1}{\det A}$

10. **(1)**

Sol: If $A = \text{diag}\begin{bmatrix} a & b & c \end{bmatrix}$ then $A^{-1} = \frac{1}{|A|}\text{Adj}A = \text{diag}\begin{bmatrix} a^{-1} & b^{-1} & c^{-1} \end{bmatrix} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$

$\therefore A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \text{diag}\begin{bmatrix} x & y & z \end{bmatrix}$

$\Rightarrow A^{-1} = \text{diag}\begin{bmatrix} x^{-1} & y^{-1} & z^{-1} \end{bmatrix} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

11. **(4)**

Sol: $|A| = 1(1+\sin^2\theta) - \sin\theta[-\sin\theta + \sin\theta] + 1[\sin^2\theta + 1]$

$= 1 + \sin^2\theta - 0 + \sin^2\theta + 1 = 2 + 2\sin^2\theta$

$\Rightarrow \det A = 2 + 2\sin^2\theta$

$\because 0 \le \sin^2\theta \le 1 \Rightarrow 0 \le 2\sin^2\theta \le 2 \Rightarrow 0 + 2 \le (2 + 2\sin^2\theta) \le 2 + 2 \Rightarrow 2 \le |A| \le 4$

$\Rightarrow |A| \in [2, 4]$

5 CONTINUITY & DIFFERENTIABILITY

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

1×1= 1 Mark

I. Select the correct option from the given choices.

1. Which of the following statement is true? [4]

1) Every polynomial function is continuous 2) The function $f(x) = 5x + 3$ is continuous at $x = 0$
3) The function $f(x) = |x|$ is continuous at $x = 0$ **4) All of the options are correct**

2. If $f(x) = \begin{cases} 3ax - 2b, & x > 1 \\ ax + b + 1, & x < 1 \end{cases}$ and $\underset{x\to 1}{\text{Lt}}\, f(x)$ exists. **[BMP1]** [2]

Then the relation between *a* and *b* is

1) $3a - 2b = 1$ **2) $2a - 3b = 1$** 3) $2a + 3b = 1$ 4) $2a + 3b = -1$

3. The function $f(x) = \begin{cases} \dfrac{2}{5-x}, & x < 3 \\ 5 - x, & x \geq 3 \end{cases}$, is [1]

1) Left discontinuous at $x = 3$ 2) Left continuous at $x = 3$
3) Right discontinuous at $x = 5$ 4) Discontinuous at $x = 5$

4. If the function $f(x) = \dfrac{\sqrt{1+x} - 1}{x}$ is continuous at $x = 0$.Then $f(0) =$ [3]

1) $-\dfrac{1}{2}$ 2) $\dfrac{1}{3}$ **3) $\dfrac{1}{2}$** 4) $-\dfrac{1}{3}$

5. If a function f(x) defined on [a,b] is discontinuous at $x = \alpha \in [a,b]$.Then [2]

1) $\underset{x\to\alpha^-}{\text{Lt}}\, f(x) = \underset{x\to\alpha^+}{\text{Lt}}\, f(x) = f(\alpha)$ **2) $\underset{x\to\alpha}{\text{Lt}}\, f(x) \neq f(\alpha)$**

3) $\underset{x\to a^-}{\text{Lt}}\, f(x) = f(a)$ 4) $\underset{x\to b^+}{\text{Lt}}\, f(x) = f(b)$

6. If the function f defined by $f(x) = \begin{cases} \cos x, & \text{if } x \leq 0 \\ 3x + \alpha, & \text{if } 0 < x < 2 \\ \beta x + 3, & \text{if } 2 \leq x \leq 4 \\ 11, & \text{if } x > 4 \end{cases}$ [3]

where α, β are real constants is continuous on R. Then $\alpha^2 + \beta^2 =$

1) 3 2) 9 **3) 5** 4) 1

7. In the interval [0, 3]. The function $f(x) = |x-1| + |x-2|$ is [4]

1) discontinuous 2) differentiable

3) continuous but not differentiable at $x = 2$ only

4) continuous but not differentiable at $x = 1$ and $x = 2$.

8. If $y = \sqrt{x+\sqrt{x+\sqrt{x+.....\infty}}}$. Then $\frac{dy}{dx}$ is equal to [4]

1) $\frac{1}{y}$ 2) $\frac{1}{x}$ 3) $\frac{1}{2x-1}$ **4) $\frac{1}{2y-1}$**

9. The set of all points where the function $f(x) = 2x|x|$ is differentiable is [1]

1) $(-\infty,\infty)$ 2) $(-\infty,0)\cup(0,\infty)$ 3) $(0,\infty)$ 4) $(-\infty,0)$

10. Differentiation of $(x^2 - 5x + 8)(x^3 + 7x + 9)$ can be done [4]

1) only by using product rule 2) only by obtaining a single polynomial expanding it

3) only by using logarithmic differentiation **4) All of the options are correct**

11. If $y = \cos^{-1}(\cos x)$ then find $\frac{dy}{dx}$ at $x = \frac{5\pi}{4}$ [2]

1) 1 **2) -1** 3) 0 4) $-\frac{1}{\sqrt{2}}$

12. If $f(x) = x^4 - x^3 + 7x^2 + 14$, then what is the value of $f^1(5)$? [4]

1) 594 2) 549 3) 954 **4) 495**

13. If $y = x + \frac{1}{x}$ then which anong the following holds? [3]

1) $x^2y^1 + xy = 0$ 2) $x^2y^1 + xy + 2 = 0$ **3) $x^2y^1 - xy + 2 = 0$** 4) $x^2y^1 + xy - 2 = 0$

14. $\frac{d}{dx}\left(e^{\log_e\sqrt{1+\tan^2x}}\right)$ when $x \in Q_1$ [3]

1) $\sec^2(x)\tan x$ 2) $\sec x\tan^2(x)$ **3) $\sec x\tan x$** 4) $\tan^2(x)$

15. If $y = \log(\cosh x)$ then $\frac{d^2y}{dx^2} =$ [1]

1) $\text{sech}^2 x$ 2) $-\text{sech}^2 x$ 3) $\sinh x$ 4) $-\sinh x$

16. If $f(x) = \begin{cases} \frac{\sin^2(ax)}{x^2}; & x \neq 0 \\ 1; & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of 'a' is [4]

1) -1 2) 1 3) 0 **4) ± 1**

17. If $y=\sinh^{-1}\left[\frac{1-x}{1+x}\right]$. Then $\frac{dy}{dx}$ is equal to [1]

1) $\frac{-\sqrt{2}}{(1+x)\sqrt{1+x^2}}$ 2) $\frac{-1}{(1+x)\sqrt{x}}$ 3) $\frac{1}{(1+x^2)\sqrt{1+x}}$ 4) $\frac{-\sqrt{2}}{1+x\sqrt{1-x^2}}$

18. [x] represents the greatest integer function of x. At $x=-1$ $\frac{d}{dx}(\sin\pi[x])=$ [1]

1) 0 2) 2 3) –2 4) 1/2

19. If $3\cdot\sin(xy)+4\cdot\cos(xy)=5$, then $\frac{dy}{dx}$ is equal to [3]

1) $\frac{3\sin xy+4\cos xy}{3\cos xy-4\sin xy}$ 2) $\frac{3\cos xy+4\sin xy}{4\cos xy-3\sin xy}$ 3) $\frac{-y}{x}$ 4) $\frac{x}{y}$

20. If $y=\log_y^x$ then $\frac{dy}{dx}$ is equal to [3]

1) $\frac{1}{x\log y}$ 2) $\frac{\log y}{x(1+\log y)}$ 3) $\frac{1}{x(1+\log y)}$ 4) $\frac{1}{1+\log y}$

HINTS & SOLUTIONS FOR MCQ

1. **(4)**

Sol: By definition, all are correct

2. **(2)**

Sol: $\lim\limits_{x\to 1} f(x)$ exists $\Rightarrow$ LHL = RHL $\Rightarrow \lim\limits_{\substack{x\to 1-\\(x<1)}} f(x)=\lim\limits_{\substack{x\to 1+\\(x>1)}} f(x)$

$\Rightarrow a+b+1=3a-2b \Rightarrow 2a-3b=1$

3. **(1)**

Sol: At x = 3, f(3) = 5 – 3 = 2

LHL $=\lim\limits_{\substack{x\to 3-\\(x<3)}} f(x)=\lim\limits_{x\to 3-}\dfrac{2}{5-3}=\dfrac{2}{2}=1\neq f(3)$

LHL $\neq$ f(3) $\Rightarrow$ f(x) is Left discontinuous at x = 3

4. **(3)**

Sol: f(x) is continuous at x = 0

$\Rightarrow f(0)=\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}=\lim\limits_{x\to 0}\dfrac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)}$

$=\lim\limits_{x\to 0} f(x)\dfrac{1+x-1}{x(\sqrt{1+x}+1)}=\lim\limits_{x\to 0}\left(\dfrac{1}{\sqrt{1+x}+1}\right)=\dfrac{1}{\sqrt{1+0}+1}=\dfrac{1}{2}$

5. **(2)**

Sol: f(x) is discontinuous at x = α $\Rightarrow \lim\limits_{x\to\alpha} f(x)\neq f(x)$ [by definition]

6. **(3)**

Sol: Given f is continuous on R f is continuous at every real number.
Consider continuity of f(x) at x = 0

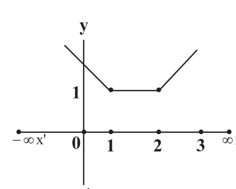
$\Rightarrow \lim\limits_{x\to 0-} f(x)=\lim\limits_{x\to 0+} f(x) \Rightarrow \cos 0^\circ=3(0)+\alpha \Rightarrow 1=0+\alpha \Rightarrow \alpha=1$

Now, consider continuity at x = 4 $\Rightarrow \lim\limits_{x\to 4-} f(x)=\lim\limits_{x\to 4+} f(x)$

$\Rightarrow \beta(4)+3=11 \Rightarrow 4\beta=8 \Rightarrow \beta=2$ Now, $\alpha^2+\beta^2=1^2+2^2=5$

7. **(4)**

Sol: Graph f(x) = |x – 1| + |x – 2| is



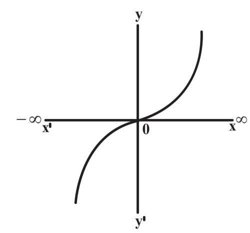
$\therefore$ f(x) is continuous on [0, 3]
but not differentiable at x = 1 and x = 2 (turning points)

8. **(4) Hint Formula:** If $y=\sqrt{f(x)+\sqrt{f(x)+\sqrt{f(x)+......\infty}}}$, then $\frac{dy}{dx}=\frac{f'(x)}{2y-1}$

Sol: Given $f(x)=x \Rightarrow f'(x)=1 \quad \therefore \frac{dy}{dx}=\frac{1}{2y-1}$

9. **(1)**

Sol: $f(x)=2x|x|=\begin{cases}-2x^2 & \forall x\le 0\\ 2x^2 & \forall x>0\end{cases}$



$\Rightarrow f'(x)=\begin{cases}-4x & \forall x\le 0\\ 4x & \forall x>0\end{cases}$ exists $\forall x\in R \Rightarrow$ f(x) is differentiable $\forall x\in R \Rightarrow x\in(-\infty,\infty)$

10. **(4)**

Sol: All are correct.

11. **(2) Hint Formula:** $\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$

Sol: $y=\cos^{-1}(\cos x) \Rightarrow \frac{dy}{dx}=\frac{-1}{\sqrt{1-(\cos x)^2}}.\frac{d}{dx}(\cos x)=\frac{(-1)(-\sin x)}{\sqrt{-1(\cos x)^2}}=\frac{\sin x}{\sqrt{1-(\cos x)^2}}$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=\frac{5\pi}{4}}=\frac{-\sin\left(\frac{\pi}{4}\right)}{\sqrt{1-\left(-\cos\left(\frac{\pi}{4}\right)\right)^2}}=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\left(\frac{1}{2}\right)}}=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}}=-1$$

12. **(4)**

Sol: $f(x)=x^4-x^3+7x^2+14 \Rightarrow f'(x)=4x^3-3x^2+14x$

at $x=5$; $f'(5)=4(5)^3-3(5)^2+14(5)=4(125)-3\times 25+70=500-75+70=495$

13. **(3)**

Sol: Given $y=x+\frac{1}{x}$ ....(1); $\quad y'=1-\frac{1}{x^2}$

$\Rightarrow x^2y^1=x^2-1$ ....(2) $\quad \Rightarrow x^2y^1-x^2+1=0 \Rightarrow x^2y^1-[xy-1]+1=0$

$\Rightarrow x^2y^1-xy+1+1=0 \Rightarrow x^2y^1-xy+2=0$

14. **(3)**

Sol: Given $y=e^{\log_e\sqrt{1+\tan^2 x}}$ $\quad \left[\because e^{\log_e^N}=N\right]$

$y=\sqrt{1+\tan^2 x}=\sec x \quad \therefore \frac{dy}{dx}=\frac{d}{dx}(\sec x)=\sec x\tan x$

15. **(1)**

Sol: $y=\log(\cosh x)$

$\Rightarrow \frac{dy}{dx}=\frac{1}{\cosh x}(\sinh x)=\tanh x \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}(\tanh x) \Rightarrow \frac{d^2y}{dx^2}=\operatorname{sech}^2 x$

16. **(4)**

Sol: f(x) is continuous at x = 0 $\Rightarrow \lim\limits_{x\to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0}\frac{\sin^2(ax)}{x^2} = 1 \quad (\text{given}) \Rightarrow \left(\lim_{x\to 0}\frac{\sin(ax)}{x}\right)^2 = 1 \Rightarrow (a)^2 = 1 \Rightarrow a = \pm 1$$

17. **(1) Hint Formula:** $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{x^2+1}}$

Sol: Given $y = \sinh^{-1}\left[\frac{1-x}{1+x}\right] \quad \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{1-x}{1+x}\right)^2 + \frac{1}{1}}}.\frac{d}{dx}\left(\frac{1-x}{1+x}\right)$

$$= \frac{1+x}{\sqrt{(1-x)^2+(1+x)^2}}\left[\frac{(1+x)[-1]-[(1-x)(1)]}{(1+x)^2}\right]$$

$$= \frac{-1-x-1+x}{\sqrt{2(1^2+x^2)}(1+x)} = \frac{-2}{\sqrt{2}\sqrt{1+x^2}(1+x)} = \frac{-2}{\sqrt{1+x^2}(1+x)}$$

18. **(1)**

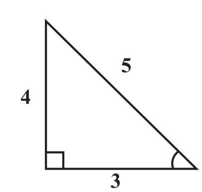
Sol: Let y = sin π[x] $\quad \frac{d}{dx} = \frac{d}{dx}[\sin \pi[x]] = \frac{d}{dx}[\sin(n\pi)] = \frac{d}{dx}(0) = 0$

where n = [x] = An integer $\in Z \ \ \forall x \in R$.

G.S of $\theta = n\pi \ \forall n \in Z \Rightarrow \sin(n\pi) = 0 \ \forall n \in Z$

19. **(3)**

Sol: Given 3 sin(xy) + 4 cos(xy) = 5



$$\Rightarrow \frac{3}{5}\sin(xy) + \frac{4}{5}\cos(xy) = \frac{5}{5} \Rightarrow \sin(xy)\cos\alpha + \cos(xy)\sin\alpha = 1$$

$$\Rightarrow \sin(xy+\alpha) = \sin 90^\circ \quad \Rightarrow xy = \frac{\pi}{2} - \alpha = \text{A constant}$$

Diff. w.r.t x

$$\Rightarrow x\frac{dy}{dx} + y(1) = 0 \Rightarrow x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

20. **(3) Hint Formula:** $\log_b^a = \frac{\log a}{\log b}, \ \frac{d}{dx}(U.V) = U.\frac{dU}{dx} + V\frac{dV}{dx}$

Sol: Given $y = \log_y^x \quad \Rightarrow y = \frac{\log x}{\log y} \Rightarrow y.(\log y) = \log x$

$$\Rightarrow y\left(\frac{1}{y};\frac{dy}{dx}\right) + (\log y)\frac{dy}{dx} = \frac{1}{x} \quad \Rightarrow (1+\log y)\frac{dy}{dx} = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{1}{x(1+\log y)}$$

6 APPLICATION OF DERIVATIVES

1 OMQ + 1 VSAQ + 1 LAQ [1 M + 2M +8M = 11 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1=$ 1 Mark

I. Select the correct option from the given choices.

1. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is [2]

   1) 10π **2) 12π** 3) 8π 4) 11π

2. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when x = 15 is [4]

   1) 116 2) 96 3) 90 **4) 126**

3. Which of the following functions are decreasing on $\left(0, \frac{\pi}{2}\right)$? [1]

   1) cos x 2) cos 2x 3) cos 3x 4) tan x

4. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ is decreasing ? [4]

   1) $(0,1)$ 2) $\left(\frac{\pi}{2}, \pi\right)$ 3) $\left(0, \frac{\pi}{2}\right)$ 4) $\left(-\pi, \frac{-\pi}{2}\right)$

5. In which interval $y = x^2 e^{-x}$ is increases [4]

   1) $(-\infty, \infty)$ 2) $(-2, 0)$ 3) $(2, \infty)$ **4) $(0, 2)$**

6. On the curve $x^2 = 2y$ which is nearest to the point (0, 5) is [1]

   1) $(2\sqrt{2}, 4)$ 2) $(2\sqrt{2}, 0)$ 3) $(0, 0)$ 4) $(2, 2)$

7. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is [4]

   1) 0 2) 1 3) 3 **4) 1/3**

8. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \le x \le 1$ is [3]

   1) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$ 2) $\frac{1}{2}$ **3) 1** 4) 0

9. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of [1]

1) 1 m/h 2) 0.1 m/h 3) 1.1 m/h 4) 0.5 m/h

10. The function $f(x) = x^3 + 3x$ is increasing in interval [3]

1) $(-\infty,0)$ 2) $(0,\infty)$ **3) R** 4) $(0,1)$

11. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x-1$ is decreasing___ [2]

1) $(-1,\infty)$ **2) $(-2,-1)$** 3) $(-\infty,-2)$ 4) $(-1,1)$

12. At which point the function $f(x) = |x-3|$ attains minimum value [3]

1) $x = 1$ 2) $x < 3$ **3) $x = 3$** 4) $x > 3$

13. Minimum value of the function $f(x) = |x-2| + |x-5|$ is [3]

1) 1 2) 2 **3) 3** 4) 4

14. The maximum value of $\frac{\log x}{x}$ is $0<x<\infty$ is [BMP2] [4]

1) ∞ 2) e 3) 1 **4) e^{-1}**

15. The minimum value of $(x-\alpha)(x-\beta)$ is [4]

1) 0 2) $\alpha\beta$ 3) $\frac{1}{4}(\alpha-\beta)^2$ **4) $\frac{-1}{4}(\alpha-\beta)^2$**

☺ HINTS & SOLUTIONS FOR MCQ ☺

1. (2)

Sol: Area of a circle $A = \pi r^2$; Diff w.r.t 'r'

Rate of change of Area $= \frac{dA}{dr} = \pi(2r)$; Now, $= \frac{dA}{(dr)} = \pi(2)(6) = 12\pi$ at r = 6

2. (4)

Sol: Revenue = $R(x) = 3x^2 + 36x + 5$; Marginal Revenue $= \frac{dR}{dx} = 3(2x)+36$

$$\left.\frac{dR}{dx}\right|_{at\ x=15} = 6(15)+36 = 90+36 = 126$$

3. (1)

Sol: Let f(x) = cosx. For decreasing interval $f'(x) < 0 \Rightarrow -\sin x < 0 \Rightarrow \sin x > 0 \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

4. **(4)**

Sol: Give $f(x) = x^{100} + \sin x - 1 \Rightarrow f'(x) = 100x^{99} + \cos x$. For decreasing interval $f'(x) < 0$ check option

1) In (0, 1) = (0, radian) ≡ (0, 57°) $f'(x) = 100x^{99} + \cos x > 0$ (+ve)

2) In $\left(\frac{\pi}{2}, \pi\right)$, $f'(x) = 100x^{99} + \cos x$ = a large +ve value + ve ($\because -1 \ge \cos$ + ve)

3) In $\left(0, \frac{\pi}{2}\right)$, $f'(x) = +ve + +ve\ (+ve)$; 4) In $\left(-\pi, \frac{-\pi}{2}\right)$, $f'(x) = -ve - = -ve < 0$

5. **(4)**

Sol: $f(x) = x^2.e^{-x} \Rightarrow f'(x) = x^2(-e^{-x}) + e^{-x}(2x)$

For increasing interval $f'(x) > 0 \Rightarrow e^{-x}(2x - x^2) > 0$

$\Rightarrow 2x - x^2 > 0 \quad \because e^{-x} > 0\ \forall x \in R \Rightarrow x^2 - 2x < 0 \Rightarrow x(x-2) < 0 \Rightarrow x \in (0, 2)$

6. **(1)**

Sol: Let $P\left(t, \frac{t^2}{2}\right)$ is a point on $x^2 = 2y$ and A = (0, 5)

consider $PA^2 = (t-0)^2 + \left(\frac{t^2}{2} - 5\right)^2$ ....(1) $\Rightarrow PA^2 = f(x) = t^2 + \left(\frac{t^2}{2} - 5\right)^2$

For maxima (or) minimum $f'(x) = 0 \Rightarrow 2t + 2\left(\frac{t^2}{2} - 5\right)\left[\frac{2t}{2}\right] = 0 \quad \Rightarrow 2t + (t^2 - 10)t = 0$

$\Rightarrow 2t + t^3 - 10t = 0 \Rightarrow t^3 - 8f = 0 \Rightarrow f(t^2 - 8) = 0 \Rightarrow t = 0$ (or) $t = \sqrt{8} = 2\sqrt{2}$

From (1)at t = 0 $\Rightarrow PA^2 = 0 + (-5)^2 = 25$; at $t = \sqrt{8} \Rightarrow PA^2 = 8 + (-1)^2 = 9$ minimum

$\therefore$ at $t = \sqrt{8} \Rightarrow P = (\sqrt{8}, 4) = (2\sqrt{2}, 4)$ is nearest

7. **(4)**

Sol: $f(x) = \frac{1 - x + x^2}{1 + x + x^2} \Rightarrow f'(x) = \frac{(1 + x + x^2)(-1 + 2x) - (1 - x + x^2)(1 + 2x)}{(1 + x + x^2)^2} = \frac{2(x^2 - 1)}{(1 + x + x^2)^2}$

$\therefore f'(x) = 0 \Rightarrow 2(x^2 - 1) = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

By second derivative test, f is the minimum at x=1 and $f(1) = \frac{1 - 1 + 1}{1 + 1 + 1} = \frac{1}{3}$

8. **(3)**

Sol: $y = f(x) = [x(x-1) + 1]^{\frac{1}{3}} = (x^2 - x + 1)^{\frac{1}{3}} = \left(\left(x - \frac{1}{2}\right) + \frac{3}{4}\right)^{\frac{1}{3}}$

Since, extreme values (maximum (or) minimum) occurs at critical points (or) at the end of the interval. Solving, $f'(x) = 0$ we get $x = \frac{1}{2}$ (critical value)

$\therefore f_{max} = \text{Max of } \left\{(f(0), f(1), f\left(\frac{1}{2}\right)\right\} = \text{Max of } \left\{1, 1, \left(\frac{3}{4}\right)^{\frac{1}{3}}\right\} \Rightarrow f_{max} = 1$

9. **(1)**

Sol: Given $r = 10 = \text{radius}, \frac{dv}{dt} = 314, h = \text{depth}, \frac{dh}{dt} = ?$

$\text{Volume} = V = \pi r^2 h \Rightarrow V = \pi(100)h \Rightarrow V = (3.14)100h \Rightarrow V = (314)h$

Diff w.r.t 'f' $\frac{dv}{dt} = (314)\frac{dh}{dt} \Rightarrow (314) = (314)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 1$ $\therefore \frac{dh}{df} = 1\ m/h$

10. **(3)**

Sol: $f(x) = x^3 + 3x$

For increasing interval $f'(x) > 0 \Rightarrow 3x^2 + 3 > 0 \quad \forall x \in R$

11. **(2)**

Sol: $f(x) = 2x^3 + 9x^2 + 12x - 1$

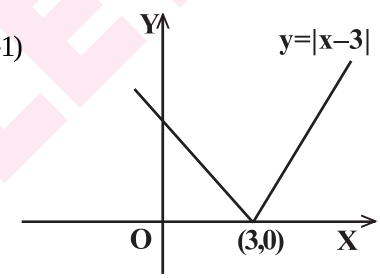
For decreasing interval $f'(x) < 0 \Rightarrow 2(3x^2) + 9(2x) + 12 < 0$

$\Rightarrow x^2 + 3x + 2 < 0 \Rightarrow (x+1)(x+2) < 0$ $x \in (-2, -1)$

12. **(3)**

Sol: $y = f(x) = |x - 3|$ graph

clearly f(x) is maximum at x = 3



13. **(3)**

Sol: $f(x) = |x-2| + |x-5| = |x-a| + |x-b|$

Range of f(x) is $[|a-b|, \infty) \Rightarrow f_{Minimum} = |a-b|$

$f_{min} = |2-5| = |3| = 3$

14. **(4)**

Sol: $f(x) = \frac{\log x}{x} \Rightarrow f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x(1)}{x^2} = \frac{1 - \log x}{x^2}$

For maxima (or) Minima $f'(x) = 0 \Rightarrow 1 - \log x = 0 \Rightarrow \log_e x = 1 \Rightarrow x = e$

f_{max} at $x = e = \frac{\log_e}{e} = \frac{1}{e} = e^{-1}$

15. **(4)**

Sol: $f(x) = (x-\alpha)(x-\beta) = x^2(\alpha+\beta)x + \alpha\beta = ax^2 + bx + c$

$\Rightarrow A = 1$ (+ve)

$\Rightarrow f_{min} = \frac{4ac - b^2}{4a} = \frac{4(1)(\alpha\beta) - (\alpha+\beta)^2}{4} = \frac{-[(\alpha+\beta)^2 + 4\alpha\beta]}{4} = -\left(\frac{(\alpha-\beta)^2}{4}\right)$

7 INTEGRALS

1 OMQ + 1 VSAQ + 1 SAQ + 1 LAQ [1 M + 2M + 4M + 8M= 15 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1=$ 1 Mark

1) INDEFINITE INTEGRALS

1. The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=$ [3]

1) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+C$ 2) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+C$ **3)** $\mathbf{\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C}$ 4) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$

2. The anti derivative of $e^{2\log\cot x}$ [3]

1) $\cot x-1$ 2) $\tan x-\cot x$ **3)** $\mathbf{-\cot x-x}$ 4) $-1-\cot x$

3. If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ such that $f(2)=0$. Then $f(x)$ is [1]

1) $\mathbf{x^4+\frac{1}{x^3}-\frac{129}{8}}$ 2) $x^3+\frac{1}{x^4}+\frac{129}{8}$ 3) $x^4+\frac{1}{x^3}+\frac{129}{8}$ 4) $x^3+\frac{1}{x^4}-\frac{129}{8}$

4. $\int\frac{10x^9+10^x\log_e 10}{x^{10}+10^x}dx=$ [4]

1) $10^x-x^{10}+C$ 2) $10^x+x^{10}+C$ 3) $(10^x-x^{10})^{-1}+C$ **4)** $\mathbf{\log(10^x+x^{10})+C}$

5. $\int\frac{x^2}{1+x^3}dx=$ [1]

1) $\mathbf{\frac{1}{3}\log\left|1+x^3\right|+c}$ 2) $\frac{2}{3}\log\left|1+x^3\right|+c$ 3) $\log\left|1+x^3\right|+c$ 4) $\tan^{-1}(x^{3/2})+c$

6. $\int\frac{dx}{\sin^2 x\cos^2 x}=$ [2]

1) $\tan x+\cot x+C$ **2)** $\mathbf{\tan x-\cot x+C}$ 3) $\tan x\cot x+C$ 4) $\tan x-\cot 2x+C$

7. $\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=$ [1]

1) $\mathbf{\tan x+\cot x+C}$ 2) $\tan x+\operatorname{cosec} x+C$ 3) $-\tan x+\cot x+C$ 4) $\tan x+\sec x+C$

8. $\int\frac{\cos x+x\sin x}{x(x+\cos x)}dx=\log|f(x)|+c$ then $f(x)=$ [3]

1) $x(x+\cos x)$ 2) $\frac{x+\cos x}{x}$ **3)** $\mathbf{\frac{x}{x+\cos x}}$ 4) $\frac{1}{x(x+\cos x)}$

9. $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx =$ [2]

1) $-\cot(e^x x)+C$ 2) $\tan(xe^x)+C$ 3) $\tan(e^x)+C$ 4) $\cot(e^x)+C$

10. $\int \frac{dx}{x^2+2x+2} =$ [2]

1) $x\tan^{-1}(x+1)+C$ 2) $\tan^{-1}(x+1)+C$ 3) $(x+1)\tan^{-1}x+C$ 4) $\tan^{-1}x+C$

11. $\int \frac{dx}{\sqrt{9x-4x^2}} =$ [2]

1) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ 2) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

3) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ 4) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

12. $\int \frac{xdx}{(x-1)(x-2)} =$ [2]

1) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$ 2) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$ 3) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$ 4) $\log|(x-1)(x-2)|+C$

13. $\int \frac{dx}{x(x^2+1)} =$ **[BMP1]** [1]

1) $\log|x|-\frac{1}{2}\log|x^2+1|+C$ 2) $\log|x|+\frac{1}{2}\log|x^2+1|+C$

3) $-\log|x|+\frac{1}{2}\log|x^2+1|+C$ 4) $\frac{1}{2}\log|x|+\log|x^2+1|+C$

14. $\int \frac{x^2}{1-x^4}dx =$ [1]

1) $\frac{1}{4}\log\left|\frac{1+x}{1-x}\right|-\frac{1}{2}\tan^{-1}x+C$ 2) $\frac{1}{4}\log\left|\frac{1+x}{1-x}\right|+\frac{1}{2}\tan^{-1}x+C$

3) $\frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right|-\frac{1}{2}\tan^{-1}x^2+C$ 4) $\frac{1}{4}\log\left|\frac{1-x^2}{1+x^2}\right|-\frac{1}{2}\tan^{-1}x^2+C$

15. $\int x^2 e^{x^3} dx =$ [1]

1) $\frac{1}{3}e^{x^3}+C$ 2) $\frac{1}{3}e^{x^2}+C$ 3) $\frac{1}{2}e^{x^3}+C$ 4) $\frac{1}{2}e^{x^2}+C$

16. $\int x\sec^2 x dx =$ [1]

1) $x\tan x-\log|\sec x|+C$ 2) $x\tan x-\log|\cos x|+C$

3) $x\tan x-\log|\operatorname{cosec} x|+C$ 4) $x\tan x-\log|\sin x|+C$

17. $\int e^x \sec x(1+\tan x)dx =$ [2]

1) $e^x \cos x + C$ **2) $e^x \sec x + C$** 3) $e^x \sin x + C$ 4) $e^x \tan x + C$

18. $\int e^x \left(\frac{1+x\log x}{x}\right)dx =$ [2]

1) $xe^{\log x} + C$ **2) $e^x \log x + C$** 3) $e^x \log x^2 + C$ 4) None

19. $\int \sqrt{1+x^2}dx =$ [1]

1) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right| + C$

2) $\frac{2}{3}(1+x^2)^{\frac{3}{2}} + C$

3) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}} + C$

4) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right| + C$

20. $\int \sqrt{x^2-8x+7}dx =$ [4]

1) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} + 9\log|x-4+\sqrt{x^2-8x+7}| + C$

2) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7} + 9\log|x+4+\sqrt{x^2-8x+7}| + C$

3) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - 3\sqrt{2}\log|x-4+\sqrt{x^2-8x+7}| + C$

4) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}| + C$

21. $\int \frac{dx}{e^x+e^{-x}} =$ [1]

1) $\tan^{-1}(e^x) + C$ 2) $\tan^{-1}(e^{-x}) + C$

3) $\log(e^x - e^{-x}) + C$ 4) $\log(e^x + e^{-x}) + C$

22. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx =$ [2]

1) $\frac{-1}{\sin x+\cos x} + C$ **2) $\log|\sin x+\cos x| + C$**

3) $\log|\sin x - \cos x| + C$ 4) $\frac{1}{(\sin x+\cos x)^2}$

2) DEFINITE INTEGRALS

1. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} =$ [4]

1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{6}$ **4) $\frac{\pi}{12}$**

2. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} =$ [3]

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{12}$ **3) $\frac{\pi}{24}$** 4) $\frac{\pi}{4}$

3. The value of the integral $\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^2} dx$ is [1]

1) 6 2) 0 3) 3 4) 4

4. If $f(x) = \int_0^x t \sin t dt$, then $f'(x)$ is [2]

1) $\cos x + x \sin x$ **2) $x \sin x$** 3) $x \cos x$ 4) $\sin x + x \cos x$

5. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)dx$ is [3]

1) 0 2) 2 **3) π** 4) 1

6. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is [3]

1) 2 2) 3/4 **3) 0** 4) –2

7. If $f(a+b-x) = f(x)$, then $\int_a^b f(x)dx =$ [4]

1) $\frac{(a+b)}{2}\int_a^b f(b-x)dx$ 2) $\frac{(a+b)}{2}\int_a^b f(b+x)dx$

3) $\frac{b-a}{2}\int_a^b f(x)dx$ **4) $\frac{a+b}{2}\int_a^b f(x)dx$**

8. $\int_0^{\pi/2} \frac{3\sin x + 5\cos x}{\sin x + \cos x} dx =$ [1]

1) 2π 2) π 3) 4π 4) 8π

9. $\int_0^4 |2-x| dx =$ [2]

1) 12 **2) 4** 3) 8 4) 2

10. $\int_{-2}^{2} (4-x^2)^{\frac{3}{2}} dx =$ [3]

1) 2π 2) 4π **3) 6π** 4) 8π

HINTS & SOLUTIONS FOR MCQ

1. **(3)**

Sol: Anti derivative of $\sqrt{x}+\frac{1}{\sqrt{x}}=\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

$$\Rightarrow I=\int x^{\frac{1}{2}}\,dx+\int x^{-\frac{1}{2}}\,dx=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c\;=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+c$$

2. **(3)**

Sol: $I=\int e^{2\log\cot x}dx=\int e^{\log_e \cot^2 x}dx=\int \cot^2 x\,dx=\int(\operatorname{cosec}^2 x-1)dx=-\cot x-x+c$

3. **(1)**

Sol: $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}\quad\Rightarrow f(x)=\int\left(4x^3-\frac{3}{x^4}\right)dx=\cancel{4}.\frac{x^4}{\cancel{4}}-\cancel{3}\left(\frac{-1}{\cancel{3}x^3}\right)=x^4+\frac{1}{x^3}+c$ ....(1)

Given, $f(x)=0\;\Rightarrow 0=16+\frac{1}{8}+c\;\Rightarrow c=-\left(\frac{129}{8}\right)\;\therefore (1)\Rightarrow f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}$

4. **(4)Hint:** $\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+c$

Sol: $I=\int\frac{10x^9+10^x\log_e^{10}}{x^{10}+10^x}dx=\log(x^{10}+10^x)+c$

5. **(1) Hint:** Put $1+x^3=t\;\Rightarrow 0+3x^2dx=dt\;\Rightarrow x^2dx=\frac{1}{3}dt$

Sol: $I=\int\frac{x^2}{1+x^3}dx=\frac{1}{3}\int\frac{1}{t}dt=\frac{1}{3}\log|t|+c=\frac{1}{3}\log|1+x^3|+c$

6. **(2)**

Sol: $I=\int\frac{1}{\sin^2 x\cos^2 x}dx=\int\frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}dx$

$$=\int\frac{\cancel{\sin^2}x}{\cancel{\sin^2}x\cos^2 x}dx+\int\frac{\cos^2 x}{\sin^2 x\cos^2 x}dx=\int\sec^2 dx+\int\operatorname{cosec}^2 dx=\tan x-\cot x+c$$

7. **(1)**

Sol: $I=\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=\int\frac{1}{\cos^2 x}dx-\int\frac{1}{\sin^2 x}dx$

$$=\int\sec^2 x\,dx-\int\operatorname{cosec}^2 x\,dx=\tan x+\cot x+c$$

8. **(3)**

Sol: $I=\int\frac{\cos x+x\sin x}{x(x+\cos x)}dx=\int\frac{(x+\cos x)-(x-x\sin x)}{x(x+\cos x)}dx=\int\frac{1\cancel{(x+\cos x)}}{x\cancel{(x+\cos x)}}dx-\int\frac{\cancel{x}(1-\sin x)}{\cancel{x}(x+\cos x)}dx$

$=\int\frac{1}{x}dx-\int\frac{f'(x)}{f(x)}dx=\log|x|-\log|x+\cos x|=\log\left|\frac{x}{x+\cos x}\right|+c=\log|f(x)|+c\Rightarrow f(x)=\frac{x}{x+\cos x}$

9. **(2)** Put, $x.e^x=t\Rightarrow\ (xe^x+e^x)dx=dt\Rightarrow e^x(x+1)dx=dt$

Sol: $I=\int\frac{e^x(1+x)}{\cos^2(e^x x)}dx\ \Rightarrow I=\int\frac{dx}{\cos^2 t}=\int\sec^2 dt=\tan t+c=\tan(xe^x)+c$

10. **(2)**

Sol: $I=\int\frac{dx}{x^2+2x+2}dx=\int\frac{1}{x^2+2x+1+1}dx=\int\frac{1}{(x+1)^2+1^2}dx=\tan^{-1}(x+1)+c$

11. **(2)** $9x-4x^2=-4\left[x^2-\frac{9}{4}x\right]=-4\left[x^2-2.x.\frac{9}{8}+\left(\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]=4\left[\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2\right]$

Sol: $\therefore I=\int\frac{dx}{9x-4x^2}=\int\frac{1}{\sqrt{4\left[\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2\right]}}dx=\frac{1}{2}\int\frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$

$=\frac{1}{2}\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)+c=\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+c\qquad\left[\because\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c\right]$

12. **(2)** Using partial fractions $\frac{x}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}\Rightarrow x=A(x-2)+B(x-1)$

at $x=1$ we get $A=-1$; at $x=2$ we get $B=2$

Sol: $I=\int\frac{x}{(x-1)(x-2)}dx\ =\int\frac{-1}{x-1}dx+\int\frac{2}{x-2}dx\ =-\log|x-1|+2\log|x-2|$

$=-\log|x-1|+\log|(x-2)^2|=\log\left|\frac{(x-2)^2}{(x-1)}\right|+c$

13. **(1) Hint:** $\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$ we get $A=1;\ B=-1;\ C=0$

Sol: $I=\int\frac{1}{x(x^2+1)}dx=\int\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx\ =\int\frac{1}{x}dx-\frac{1}{2}\int\frac{2x}{x^2+1}dx=\log|x|-\frac{1}{2}\log|x^2+1|+c$

14. **(1)**

Sol: $I=\int\frac{x^2}{1-x^4}dx=\int\frac{x^2}{(1+x^2)(1-x^2)}dx=\frac{1}{2}\int\frac{2x^2}{(1+x^2)(1-x^2)}dx=\frac{1}{2}\left[\int\frac{(1+x^2)-(1-x^2)}{(1+x^2)(1-x^2)}dx\right]$

$$=\frac{1}{2}\left[\int\frac{\cancel{1+x^2}}{(\cancel{1+x^2})(1-x^2)}dx-\int\frac{\cancel{1-x^2}}{(1+x^2)(\cancel{1-x^2})}dx\right]=\frac{1}{2}\left[\int\frac{1}{1^2-x^2}dx-\int\frac{1}{1+x^2}dx\right]$$

$$=\frac{1}{2}\left[\int\frac{1}{1^2-x^2}dx\right]-\frac{1}{2}\left[\int\frac{1}{1+x^2}dx\right]=\frac{1}{2}\left[\frac{1}{2}\log\frac{1+x}{1-x}\right]-\frac{1}{2}\tan^{-1}x+c=\frac{1}{4}\log\left|\frac{1+x}{1-x}\right|-\frac{1}{2}\tan^{-1}x+c$$

15. **(1)** Put, $x^3=t\Rightarrow 3x^2dx=dt\Rightarrow x^2dx=\frac{1}{3}dt$

Sol: $I=\int x^2e^{x^3}dx=\frac{1}{3}\int e^tdt=\frac{1}{3}.e^t+c=\frac{1}{3}e^{x^3}+c$

16. **(1)Hint:** Integration by parts we have

Sol: $I=x(\tan x)-\int\tan x\,dx=\int x\sec^2x\,dx=x(\tan x)-\log|\sec|+c$

17. **(2)Hint:** $\int e^x(f(x)+f'(x))\,dx=e^xf(x)+c$

Sol: $I=\int e^x\sec x(1+\tan x)dx=\int e^x[\sec x+\sec x\tan x]dx=e^x\sec x+c$

18. **(2)Hint:** I= $\int e^x[f(x)+f'(x)]dx=e^xf(x)+c$

Sol: $I=\int e^x\left(\frac{1+x\log x}{x}\right)dx=\int e^x\left(\frac{1}{x}+\log x\right)dx=\int e^x\left(\log x+\frac{1}{x}\right)dx=e^x(\log x)+c$

19. **(1)Hint:** $\int\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}+1}\right|+c$

Sol: $I=\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{x^2+1}\right|+c$

20. **(4)Hint:** $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}=\frac{a^2}{2}\log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}-1}\right|+c$

Sol: $I=\int\sqrt{x^2-8x+7}\,dx=\int\sqrt{(x^2-2.x.4+16)-9}\,dx=\int\sqrt{(x-4)^2-3^2}\,dx$

$$=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+c$$

21. **(1)**

Sol: $I=\int\frac{dx}{e^x+e^{-x}}=\int\frac{dx}{e^x+\frac{1}{e^x}}=\int\frac{e^x}{(e^x)^2+1^2}dx$. Put $e^x=t \Rightarrow e^x dx=dt$

$I=\int\frac{dt}{t^2+1}=\tan^{-1}(t)+c=\tan^{-1}(e^x)+c$

22. **(2)**

Sol: $I=\int\frac{\cos 2x}{(\sin x+\cos x)^2}=dx=\int\frac{\cos^2 x-\sin^2 x}{(\cos x+\sin x)^2}dx$

$=\int\frac{\cancel{(\cos x+\sin x)}(\cos x-\sin x)}{\cancel{(\cos x+\sin x)}(\cos x+\sin x)}dx=\log|\cos x+\sin x|+c$

DEFINITE INTEGRALS

1. **(4). Textual given key is 1 .**

Sol: $I=\int\frac{1}{(1+x^2)}dx=\left(\tan^{-1}(x)\right)_1^{\sqrt{3}}=\tan^{-1}(\sqrt{3})-\tan^{-1}(1)=60^\circ-45^\circ=15^\circ=\frac{\pi}{12}$

2. **(3) Textual given key is 4 .**

Sol: $I=\int_0^{2/3}\frac{1}{4+9x^2}dx=\int_0^{2/3}\frac{1}{9\left(x^2+\frac{4}{9}\right)}dx=\frac{1}{9}\left[\int_0^{2/3}\frac{1}{x^2+\left(\frac{2}{3}\right)^2}dx\right]=\frac{1}{9}\left[\frac{1}{\left(\frac{2}{3}\right)}\tan^{-1}\left(\frac{x}{\frac{2}{3}}\right)\right]_0^{2/3}$

$=\frac{1}{9}\times\frac{3}{2}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{2/3}=\frac{1}{6}\left[\tan^{-1}\left(\frac{3}{2}\times\frac{2}{3}\right)-\tan^{-1}(0)\right]=\frac{1}{6}\tan^{-1}(1)=\frac{1}{6}\times\frac{\pi}{4}=\frac{\pi}{24}$

3. **(1)**

Sol: $I=\int_{1/3}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^2}dx=\int_{1/3}^{1}\frac{\left(x^3\left[\frac{x}{x^3}-1\right]\right)^{1/3}}{x^2x^4}dx=\int_{1/3}^{1}\frac{x\left(\frac{1}{x^2}-1\right)^{1/3}}{x^2}dx=\int_{1/3}^{1}\frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x}dx$

Put, $\frac{1}{x^2}-1=t \Rightarrow \frac{-2}{x^3}dx=dt; \frac{1}{x^3}dx=-\frac{1}{2}dt$

at $x=\frac{1}{3}$ we have $t=8$; at $x=1$ we have $t=0$

$=-\frac{1}{2}\int_8^0 t^{1/3}\,dt=\frac{1}{2}\int_0^8 t^{1/3}\,dt=\frac{1}{2}\left[\frac{t^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_0^8=\frac{1}{2}\times\frac{3}{4}\left[t^{\frac{4}{3}}\right]_0^8=\frac{3}{8}\times2^4=3\times2=6$

4. **(2)**

Sol: $f(x)=\int_0^x t\sin t\,dx$ Diff. wr.t.x we get f′(x) =xsinx

5. **(3)**

Sol: $I=\int_{-\pi/2}^{\pi/2}(x^3+x\cos x+\tan^5 x+1)=\int_{-\pi/2}^{\pi/2}1\,dx=(x)_{-\pi/2}^{\pi/2}=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

6. **(3) Hint**: $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

Sol: $I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ ....(1) $I=\int_0^{\pi/2}\log\left[\frac{4+3\cos x}{4+3\sin x}\right]dx$ ....(2)

$I+I=\int_0^{\pi/2}\left[\log\left(\frac{4+3\sin x}{4+3\cos x}\right)+\log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right]dx\Rightarrow 2I=\int_0^{\pi/2}\log(1)\,dx\Rightarrow 2I=0\Rightarrow I=0$

7. **(4)**

Sol: $\int_a^b xf(x)\,dx=\int_a^b(a+b-x)f(a+b-x)\,dx=\int_a^b[(a+b)-x]f(x)\,dx=\int_a^b(a+b)f(x)\,dx-\int_a^b xf(x)\,dx$

$\int_a^b xf(x)\,dx=(a+b)\int_a^b f(x)\,dx-\int_a^b xf(x)\,dx$ $\Rightarrow 2\int_a^b xf(x)\,dx=(a+b)\int_a^b f(x)\,dx$

$\Rightarrow\int_a^b xf(x)\,dx=\left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$

8. **(1) Hint:** $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

Sol: $I=\int_0^{\pi/2}\frac{2\sin x+5\cos x}{\sin x+\cos x}dx$............(1) $I=\int_0^{\pi/2}\frac{3\cos x+5\sin x}{\cos x+\sin x}dx$ ....(2)

$\Rightarrow I+I=\int_0^{\pi/2}\left(\frac{3\sin x+5\cos x}{\sin x+\cos x}\right)+\left(\frac{3\cos x+5\sin x}{\cos x+\sin x}\right)dx$

$\Rightarrow 2I=\int_0^{\pi/2}\frac{3(\sin x+\cos x)+5(\cos x+\sin x)}{\sin x+\cos x}dx\Rightarrow 2I=\int_0^{\pi/2}\frac{(\sin x+\cos x)(3+5)}{(\sin x+\cos x)}dx$

$\Rightarrow \cancel{2}I=\cancel{8}\int_0^{\pi/2}(1)\,dx\Rightarrow I=4(x)_0^{\pi/2}=4\times\frac{\pi}{2}=2\pi$

9. **(2)**

Sol: $I=\int_0^4 |2-x|dx \qquad \because |2-x| = 2x \text{ if } 2-x\ge 0;\ 2\ge x;\ x\le 2$

$$=\int_0^2 |2-x|dx+\int_2^4 |2-x|dx \quad =\int_0^2 (2-x)dx+\int_2^4 -(2-x)dx=\left(2x-\frac{x^2}{2}\right)_0^2-\left(2x-\frac{x^2}{2}\right)_2^4$$

$$=(4-2)-0-[(8-8)-(4-2)] \quad = 2-[0-2] \quad = 2+2=4$$

10. **(3)**

Sol: $I=\int_{-2}^{2}(4-x^2)^{3/2}dx=2\int_0^2 (4-x^2)^{3/2}dx$

$\because f(x)=(4-x^2)^{3/2}$ is even $\qquad \Rightarrow f(-x)=f(x)$

Put $x=2\sin\theta \Rightarrow dx=2\cos\theta\, d\theta$; at x = 0, 2sinθ = θ, θ = 0; at x = 2, 2sinθ = 2, θ = π/2

$$I=2\left[\int_0^{\pi/2}(4-4\sin^2\theta)^{3/2}2\cos\theta\, d\theta\right]=2\left[\int_0^{\pi/2}(4)^{3/2}\left[1-\sin^2\theta\right]^{3/2}.2\cos\theta\, d\theta\right]$$

$$=2(2)^2\int_0^{\pi/2}(\cos^2\theta)^{3/2}.\,2\cos\theta\, d\theta \ =2.2^3.2\int_0^{\pi/2}\cos^3\theta.\cos\theta\, d\theta=2^5\int_0^{\pi/2}\cos^4\, d\theta$$

$$=2^5\left[\frac{3}{4}.\frac{1}{2}\right]\frac{\pi}{2}=2\times3\times\pi=6\pi \qquad \left[\because \int_0^{\pi/2}\cos^n x\, dx=\left(\frac{n-1}{n}\right)I_{n-2} \qquad \because n=4 \text{ even}\right]$$

8 APPLICATION OF INTEGRALS

1 OMQ + 1 SAQ [1 M + 4M =5 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1 \times 1 = 1$ Mark

I. Select the correct option from the given choices.

1. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is [1]

   1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

2. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is [2]

   1) 2 **2) $\frac{9}{4}$** 3) $\frac{9}{3}$ 4) $\frac{9}{2}$

3. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is [4]

   1) -9 2) $\frac{-15}{4}$ 3) $\frac{15}{4}$ **4) $\frac{17}{4}$**

4. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by [3]
   [Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

   1) 0 2) $\frac{1}{3}$ **3) $\frac{2}{3}$** 4) $\frac{4}{3}$

5. Area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$ is [1]

   1) $\frac{\pi a^2}{2}$ 2) $\frac{\pi a^2}{4}$ 3) $\frac{\pi a}{2}$ 4) $\frac{\pi a}{4}$

6. The area bounded by $y = \sin 2x$ the x - axis and the lines $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ is [1]

   1)1 sq units 2)2sq.units 3)4sq.units 4) $\frac{3}{2}$sq.units

7. The area bounded by the curve $y = x^2 - 4$, and the lines $y = 0$ and $y = 5$ is **[BMP2]** [2]

   1) $\frac{38}{3}$ **2) $\frac{76}{3}$** 3) $\frac{16}{3}$ 4) $\frac{8}{3}$

8. The area of the region bounded by parabola $y^2 = 8x$ and latus rectum is [3]

   1) $\frac{4}{3}$ 2) $\frac{16}{3}$ **3) $\frac{32}{3}$** 4) $\frac{8}{3}$

9. The area bounded by the curve $y = 2x - x^2$ and the line $y = -x$ is [3]

1) $\frac{7}{2}$ 2) 7 **3)** $\mathbf{\frac{9}{2}}$ 4) 9

10. The area enclosed between the graph of $y = x^3$ and the lines $x = 0, y = 1, y = 8$ is [3]

1) 7 2) 14 **3)** $\mathbf{\frac{45}{4}}$ 4) $\frac{54}{4}$

11. The area of the region bounded by the curve $y^2 = x$, the Y - axis and between $y = 2$ and $y = 4$ [3]

1) $\frac{52}{3}$ 2) $\frac{54}{3}$ **3)** $\mathbf{\frac{56}{3}}$ 4) $\frac{58}{3}$

12. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ and the X-axis is [2]

1) 1 **2) 2** 3) 3 4) 4

13. Area of the region bounded by the curve $x = 2y + 3$, the Y-axis and between $y = -1$ and $y = 1$ is [1]

1) 6 2) 4 3) 8 4) 3/2

14. The area bounded by the curve $y = x^3$, X - axis and two ordinates $x = 1$ and $x = 2$ is

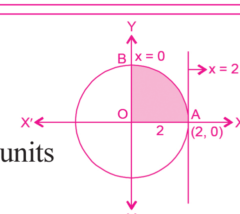
1) $\frac{15}{2}$ **2)** $\mathbf{\frac{15}{4}}$ 3) $\frac{17}{2}$ 4) $\frac{17}{4}$ [2]

15. The area bounded by the curves $y^2 = 4x$ and $y = x$ is equal to [2]

1) $\frac{1}{3}$ **2)** $\mathbf{\frac{8}{3}}$ 3) $\frac{35}{6}$ 4) $\frac{7}{3}$

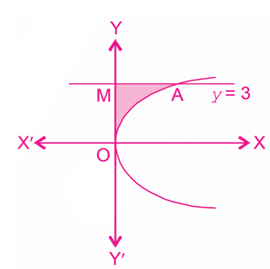
HINTS & SOLUTIONS FOR MCQ

1. **(1)**



Sol: $$\text{Area(OAB)} = \int_0^2 y\,dx = \int_0^2 \sqrt{4-x^2}\,dx = \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 = 2\left(\frac{\pi}{2}\right) = \pi \text{ sq.units}$$

2. **(2)**



Sol: $$\text{Area(OAM)} = \int_0^3 x\,dy = \int_0^3 \frac{y^2}{4}\,dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}(27) = \frac{9}{4} \text{ sq.units}$$

3. (4)

Sol: Required area $=-\int_{-2}^{0} y dx+\int_{0}^{1} y dx$

$$=-\int_{-2}^{0} x^3 dx+\int_{0}^{1} x^3 dx=-\left[\frac{x^4}{4}\right]_{-2}^{0}+\left[\frac{x^4}{4}\right]_{0}^{1}=-\left[0-\frac{(-2)^4}{4}\right]+\left[\frac{1}{4}-0\right]=\left(4+\frac{1}{4}\right)=\frac{17}{4}\text{ sq.units}$$

4. (3)

Sol: Required area $=\int_{-1}^{1} y dx=\int_{-1}^{1} x|x| dx=-\int_{-1}^{0} x^2 dx+\int_{0}^{1} x^2 dx$

$$=-\left[\frac{x^3}{3}\right]_{-1}^{0}+\left[\frac{x^3}{3}\right]_{0}^{1}=-\left(-\frac{1}{3}\right)+\frac{1}{3}=\frac{2}{3}\text{ sq.units}$$

5. (2)

Sol: Area $\int_0^a \sqrt{a^2-x^2}\,dx$ = Area of the circle $x^2+y^2=a^2$ in 1st quadrant $=\frac{1}{4}(\pi a^2)$

6. (1)

Sol: $y=\sin 2x \Rightarrow y>0$ if $0<2x<\pi$; i.e., $0<x<\frac{\pi}{2}$ and $y<0$ if $\pi<2x<2\pi$; i.e., $\frac{\pi}{2}<x<\pi$

$$A=\int_{\pi/4}^{3\pi/4} \sin(2x)\,dx=\int_{\pi/4}^{\pi/2} \sin(2x)\,dx-\int_{\pi/2}^{3\pi/4} \sin(2x)\,dx=-\left[\frac{\cos(2x)}{2}\right]_{\pi/4}^{\pi/2}-\left[-\frac{\cos(2x)}{2}\right]_{\pi/2}^{3\pi/4}$$

$$=-\frac{1}{2}\left[\cos\pi-\cos\frac{\pi}{2}\right]+\frac{1}{2}\left[\cos\left(\frac{3\pi}{2}\right)-\cos\pi\right]=-\frac{1}{2}[-1-0]+\frac{1}{2}[0-(-11)]=\frac{1}{2}+\frac{1}{2}(1)=1\text{ sq.units}$$

7. (2)

Sol: Given $y=x^2-4 \Rightarrow x^2=y+4 \quad \Rightarrow x=\sqrt{y+4}$

Required Area $A=2\left[\int_0^5 x\,dx\right]=2\left[\int_0^5 \sqrt{y+4}\,dy\right]=2\left[\frac{2}{3}(y+4)\sqrt{y+4}\right]_0^5$

$$=\frac{4}{3}\left[9\sqrt{9}-(4\sqrt{4})\right]=\frac{4}{3}[27-8]=\frac{4\times 19}{3}=\frac{76}{3}\text{ sq.units}$$

8. (3)

Sol: $y^2=8x \quad y=\sqrt{8x}=2\sqrt{2x}$

Area $=2\int_0^2 (y)dx=2\left[\int_0^2 2\times 2\sqrt{x}\,dx\right]=2\times 2\sqrt{2}\left(\frac{2}{3}x\sqrt{x}\right)_0^2=\frac{8\sqrt{2}}{3}(2\sqrt{2}-0)=\frac{16\times 2}{3}=\frac{32}{3}$ sq.units

9. **(3)**

Sol: Given y = $2x - x^2$ ....(1) (Upper curve) y = –x ....(2) (Lower curve)

Solving (1) and (2)

$-x = 2x - x^2 \Rightarrow x^2 - x - 2x = 0 \Rightarrow x^2 - 3x = 0 \Rightarrow x(x-3) = 0 \Rightarrow x = 0 \quad x = 3$

$$\text{Area} = \int_0^3 (2x - x^2) - (-x)\,dx = \int_0^3 (3x - x^2)\,dx = \left(3\frac{x^2}{2} - \frac{x^3}{3}\right)_0^3$$

$$= \frac{3}{2}\times 9 - \frac{27}{3} - (0) = \frac{27}{2} - \frac{27}{3} = 27\left(\frac{1}{6}\right) = \frac{9}{2}\text{sq.units}$$

10. **(3)**

Sol: $y = x^3$ ....(1)

x = 0 (y-axis), y = 1, y = 8

$$A = \int_1^8 (x)\,dx = \int_1^8 y^{\frac{1}{3}}\,dx = \left(\frac{y^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right)_1^8 = \frac{3}{4}\left(y^{\frac{4}{3}}\right)_1^8 = \frac{3}{4}\left[2^4 - 1\right] = \frac{3\times 15}{4} = \frac{45}{4}\text{sq.units}$$

11. **(3)**

Sol: $y^2 = x$; y-axis (x = 0), y = 2, y = 4

$$\text{Area} = \int_2^4 (x)\,dx = \int_2^4 y^2\,dx = \left[\frac{y^3}{3}\right]_2^4 = \frac{1}{3}[64-8] = \frac{1}{3}[56] = \frac{56}{3}\text{sq.units}$$

12. **(2)**

Sol: y = cosx, x = 0 (y-axis), x = π, x-axis (y = 0)

$$\text{Required Area} = 2\int_0^{\pi/2} (y)\,dx = 2\int_0^{\pi/2} \cos x\,dx = 2[\sin x]_0^{\pi/2} = 2[\sin 90^\circ - \sin 0^\circ] = 2[1-0] = 2$$

13. **(1)**

Sol: Given x = 2y + 3

$$\text{Area } A = \int_{-1}^{1} (x)\,dy = \int_{-1}^{1} (2y+3)\,dy = \left(\frac{2y^2}{2} + 3y\right)_{-1}^{1} = 1+3-[1-3] = 4 - (-2) = 6 \text{ sq.units}$$

14. **(2)**

Sol: $y = x^3$ x-axis (y = 0) x = 1, x = 2

$$\text{Area} = \int_1^2 (y)\,dx = \int_1^2 x^3\,dx = \left(\frac{x^4}{4}\right)_1^2 = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}\text{sq.units}$$

15. **(2)**

Sol: $y^2 = 4x \Rightarrow y = 2\sqrt{x}$ ..(1) (Upper curve) y = x ....(2) (Lower curve)

Solving (1) and (2) $y^2 = 4y$ y(y – 4) = 0 y = 0 ; y = 4

$$\text{Area} = \int_1^4 (2\sqrt{x} - x)\,dx = 2\int_1^4 \sqrt{x}\,dx = \int_1^4 x\,dx = 2\frac{2}{3}\left(x\sqrt{x}\right)_0^4 - \left(\frac{x^4}{2}\right)_0^4 = \frac{4}{3}[4\sqrt{4}] - \frac{1}{2}(16)$$

$$= \frac{32}{3} - \frac{16}{2} = \frac{32}{3} - 8 = \frac{32-24}{3} = \frac{8}{3}\text{sq.units}$$

9 DIFFERENTIAL EQUATIONS

1 OMQ + 1 VSAQ + 1 LAQ [1 M + 2M +8M = 11 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1=$ 1 Mark

I. Select the correct option from the given choices.

1. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$ is **[BMP2]** [4]

   1) 3 2) 2 3) 1 **4) not defined**

2. The order of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ is [1]

   1) 2 2) 1 3) 0 4) not defined

3. The number of arbitrary constants in the general solution of a differential equation of fourth order is [4]

   1)0 2)2 3)3 **4)4**

4. The number of arbitrary constants in the particular solution of a differential equation of third order is [4]

   1)3 2)2 3)1 **4) 0**

5. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ is [1]

   1) $e^x+e^{-y}=C$ 2) $e^x+e^y=C$ 3) $e^{-x}+e^y=C$ 4) $e^{-x}+e^{-x}=C$

6. A homogeneous differential equation of the from $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$ can be solved by making the substitution. [3]

   1) $y=vx$ 2) $v=yx$ **3) $x=vy$** 4) $x=v$

7. Which of the following is a homogeneous differential equation? [4]

   1) $(4x+6y+5)\,dy-(3y+2x+4)\,dx=0$ 2) $(xy)\,dx-(x^3+y^3)\,dy=0$

   3) $(x^3+2y^2)\,dx+2xy\,dy=0$ **4) $y^2\,dx+(x^2-xy-y^2)\,dy=0$**

8. The Integrating Factor of the differential equation $x\frac{dy}{dx}-y=2x^2$ is [3]

   1) e^{-x} 2) e^{-y} **3) $\frac{1}{x}$** 4) x

9. The Integrating Factor of the D.E $(1-y^2)\frac{dx}{dy}+yx=ay,\ (-1<y<1)$ is [4]

   1) $\frac{1}{y^2-1}$ 2) $\frac{1}{\sqrt{y^2-1}}$ 3) $\frac{1}{1-y^2}$ **4) $\frac{1}{\sqrt{1-y^2}}$**

10. The general solution of the differential equation $\frac{ydx - xdy}{y} = 0$ is [3]

1) $xy = C$ 2) $x = Cy^2$ **3) $y = Cx$** 4) $y = Cx^2$

11. The general solution of a D.E of the type $\frac{dx}{dy} + P_1x = Q_1$ (P_1, Q_1 are functions of y) is [3]

1) $ye^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)dy + C$ 2) $y.e^{\int P_1 dx} = \int\left(Q_1 e^{\int P_1 dx}\right)dx + C$

3) $xe^{\int P_1 dy} = \int\left(Q_1 e^{\int P_1 dy}\right)dy + C$ 4) $xe^{\int P_1 dx} = \int\left(Q_1 e^{\int P_1 dx}\right)dx + C$

12. The general solution of the differential equation $e^x\,dy + (y\,e^x + 2x)\,dx = 0$ is [3]

1) $x\,e^y + x^2 = C$ 2) $x\,e^y + y^2 = C$
3) $y\,e^x + x^2 = C$ 4) $y\,e^y + x^2 = C$

13. General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ is [3]

1) $e^{-y} = \frac{1}{2}e^{2x} + C$ 2) $\frac{1}{e^y} + \frac{1}{2}e^{2x} = C$ **3) $-e^{-y} = \frac{1}{2}e^{2x} + C$** 4) $e^y = \frac{1}{2}e^{2x} + C$

14. General solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ is [2]

1) $\log y = Cx$ **2) $y = Cx$** 3) $xy = C$ 4) $y = C\log x$

15. The degree of the differential equation $\left(1 + \frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ is [3]

1) 1 2) 2 **3) 3** 4) 4

16. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is [4]

1) 1 2) 2 3) 4 **4) Not defined**

17. Order of differential equation corresponding to family of curves [1]

$y = Ae^{2x} + Be^{-2x}$ is

1) 2 2) 1 3) 3 4) 4

18. The general solution of differential equation $\frac{dy}{dx} = e^{x-y}$ is [1]

1) $e^y = e^x + C$ 2) $e^x + e^y = C$ 3) $e^{x+y} = C$ 4) $e^{x-y} = C$

19. The order and degree of the differential equation $\frac{dy}{dx} = \left(\frac{d^2y}{dx^2} + 2\right)^{1/2} + \frac{d^2y}{dx^2} + 5$ [3]

are respectively

1) 2,1 2) 2,4 **3) 2,2** 4) 2,3

20. The differential equation for which $ax + by = 1$ is general solution [3]
(a, b are arbitrary constants) is

1) $\frac{dy}{dx} = x + C$ 2) $y\frac{d^2y}{dx^2} + x = 1$ **3) $\frac{d^2y}{dx^2} = 0$** 4) $\frac{d^3y}{dx^3} = 0$

HINTS & SOLUTIONS FOR MCQ

1. **(4)** Given D.E is not a polynomial equation in its derivatives. Its degree is not defined.

2. **(1)** Highest order derivative present in the given D.E is $\frac{d^2y}{dx^2}$. Its order is two.

3. **(4)** Number of constants in the G.S= Order

Number of constants in the general solution of D.E of order n is equal to its order.

The number of constants in fourth order differential equation is 4.

4. **(4)**In a particular solution of a differential equation, there are no arbitrary constants.

5. **(1)** Given D.E is $\frac{dy}{dx}=e^{x+y}=e^x.e^y \Rightarrow \frac{dy}{e^y}=e^x dx \Rightarrow e^{-y}dy=e^x dx$ Integrating both sides,

$x\int e^{-y}dy=\int e^x dx \Rightarrow -e^{-y}=e^x+k \Rightarrow e^x+e^{-y}=-k \Rightarrow e^x+e^{-y}=C$

6. **(3)** For solving homogeneous equation of form $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$, we need to make substitution as $x=vy$

Thus, the correct option is C.

7. **(4) Hint:** F(x, y) is homogeneous function of degree n, if $F(\lambda x,\lambda y)=\lambda F \backslash (x,y)$

Consider D.E in (D) $y^2dx+(x^2-xy^2-y^2)dy=0 \Rightarrow \frac{dy}{dx}=\frac{y^2}{y^2+xy^2-x^2}$ $F(x,y)=\frac{y^2}{y^2+xy^2-x^2}$

$$F(\lambda x,\lambda y)=\frac{(\lambda y)^2}{(\lambda y)^2+(\lambda x)(\lambda y)^2-(\lambda x)^2}=\frac{\lambda^2y^2}{\lambda^2(y^2+xy^2-x^2)}=\lambda^2\left(\frac{y^2}{y^2+xy^2-x^2}\right)=\lambda^0F(x,y)$$

Differential equation given in D is a homogeneous equation

8. **(3)** Given D.E is $x\frac{dy}{dx}-y=2x^2 \Rightarrow \frac{dy}{dx}-\frac{y}{x}=2x$ This is in the $\frac{dy}{dx}+Py=Q$ form

where, $P=-\frac{1}{x}$ and $Q=2x$ $\therefore IF=e^{-\int\frac{1}{x}dx}=e^{-\log x}=e^{\log(x^{-1})}=x^{-1}=\frac{1}{x}$

9. **(4)** Given D.E is $(1-y^2)\frac{dx}{dy}+yx=ay \Rightarrow \frac{dx}{dy}+\frac{yx}{1-y^2}=\frac{ay}{1-y^2}$. This is in the $\frac{dx}{dy}+Px=Q$ form

where, $P=\frac{y}{1-y^2}$ and $Q=\frac{ay}{1-y^2}$

$$\therefore IF=e^{\int P_1dy}=e^{\int\frac{y}{1-y^2}dy}=e^{-\frac{1}{2}\log(1-y^2)}=e^{\log\left[\frac{1}{\sqrt{1-y^2}}\right]}=\frac{1}{\sqrt{1-y^2}}$$

10. **(3)** Given D.E is $\frac{ydx-xdy}{y}=0 \Rightarrow \frac{ydx-xdy}{xy}=0 \Rightarrow \frac{1}{x}dx-\frac{1}{y}dy=0$

$\Rightarrow \log|x|-\log|y|=\log k \Rightarrow \log\left|\frac{x}{y}\right|=\log k \Rightarrow \frac{x}{y}=k \Rightarrow y=\frac{1}{k}x \Rightarrow y=Cx\left(\text{where}, C=\frac{1}{k}\right)$

11. **(3)** IF for $\frac{dx}{dy}+P_1x=Q_1$ is $e^{\int P_1 dy} \Rightarrow x(\text{I.F.})=\left(\int Q_1\times IF\right)dy+C \Rightarrow x.e^{\int P_1 dy}=\int(Q_1e^{\int P_1 dy})dy+C$

12. **(3)** Given D.E is $e^xdy+(ye^x+2x)dx=0 \Rightarrow e^x\frac{dy}{dx}+ye^x+2x=0 \Rightarrow \frac{dy}{dx}+y+\frac{2x}{e^x}=0$

$\Rightarrow \frac{dy}{dx}+y+2xe^{-x}=0 \Rightarrow \frac{dy}{dx}+y=-2xe^{-x}$

This is a Linear D.E form $\frac{dy}{dx}+Py=Q$ where, $P=1$ and $Q=-2xe^{-x}$

Now, $IF=e^{\int Pdx}=e^{\int dx}=e^x \Rightarrow y(IF)=\int(Q\times IF)dx+C$

$\therefore ye^x=\int(-2xe^{-x}.e^x)dx+C \Rightarrow ye^x=-\int 2xdx+C \Rightarrow ye^x=-x^2+C \Rightarrow ye^x+x^2=C$

13. **(3)** $\log_e\left[\frac{dy}{dx}\right]=2x+y \Rightarrow \frac{dy}{dx}=e^{2x+y} \Rightarrow \frac{dy}{dx}=e^{2x}.e^y \quad \frac{1}{e^y}dy=e^{2x}dx$

Integrating $\int e^{-y}dy=\int e^{2x}dx \Rightarrow -e^{-y}=\frac{e^{2x}}{2}+c$

14. **(2)** $\frac{dy}{dx}=\frac{y}{x} \Rightarrow \frac{dy}{y}=\frac{dx}{x} \Rightarrow \int\frac{1}{y}dy=\int\frac{1}{x}dx \Rightarrow \log|y|=\log|x|+\log|c| \Rightarrow \log|y|=\log|cx| \Rightarrow y=cx$

15. **(3)** order = 1; degree = 3

16. **(4)** The given equation is not a polynomial equation in $\left(\frac{dy}{dx}\right)$.

Here, its degree is not defined. Hence, degree not defined

17. **(1)**

Sol: $y=Ae^{2x}+Be^{-2x}$ arbitrary constants = 2

$\therefore$ Order of D.E is '2'

18. **(1)** $\frac{dy}{dx}=e^x.e^{-y} \Rightarrow \frac{1}{e^{-y}}dy=e^xdx \Rightarrow e^ydy=e^xdx \Rightarrow \int e^ydy=\int e^xdx \Rightarrow e^y=e^x+c$

19. **(3)** Transposing the terms properly and squaring on both sides we get $\left[\left(\frac{dy}{dx}\right)-\left(\frac{d^2y}{dx^2}\right)-5\right]^2=\frac{d^2y}{dx^2}+2$

$\therefore$ order = 2 ; degree = 2

20. **(3)** Given ax + by = 1 $\Rightarrow a(1)+b\left(\frac{dy}{dx}\right)=0$ Again diff w.r.t 'x', $0+b\left(\frac{d^2y}{dx^2}\right)=0 \Rightarrow \frac{d^2y}{dx^2}=0$

10 VECTOR ALGEBRA

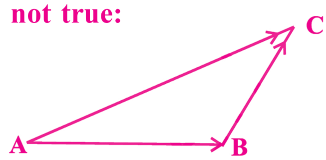
1 OMQ + 1 VSAQ + 1 SAQ + 1 LAQ [1 M + 2M +4M + 8M = 15 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1\times1=$ 1 Mark

1. In triangle ABC (Fig), which of the following is not true: [3]



1) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ 2) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$

3) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{AC}=\vec{0}$ 4) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

2. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following is correct [1]

1) $\vec{b}=\lambda\vec{a}$, for some scalar $\lambda\neq0$. 2) $\vec{a}=\pm\vec{b}$

3) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

4) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

3. If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if [4]

1) $\lambda=1$ 2) $\lambda=-1$ 3) $a=|\lambda|$ **4) $a=1/|\lambda|$**

4. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3, |\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is [2]

1) $\pi/6$ **2) $\pi/4$** 3) $\pi/3$ 4) $\pi/2$

5. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is [3]

1) 1/2 2) 1 **3) 2** 4) 4

6. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b}\geq0$ only when [2]

1) $0<\theta<\frac{\pi}{2}$ **2) $0\leq\theta\leq\frac{\pi}{2}$** 3) $0<\theta<\pi$ 4) $0\leq\theta\leq\pi$

7. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if [4]

1) $\theta=\frac{\pi}{4}$ 2) $\theta=\frac{\pi}{3}$ 3) $\theta=\frac{\pi}{2}$ **4) $\theta=\frac{2\pi}{3}$**

8. The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ is [3]

1) 0 2) -1 **3) 1** 4) 3

9. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to [2]

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

10. The value of the dot product of $\vec{a}-\vec{b}$ and $\vec{a}+\vec{b}$ is [1]

1) $a^2 - b^2$ 2) $(\vec{a} \times \vec{b})$ 3) $\vec{a} \times \vec{b}$ 4) $\vec{b} \times \vec{a}$

11. The position vector of the point $(1,2,0)$ is [3]

1) $\vec{i}+\vec{j}+\vec{k}$ 2) $\vec{i}+2\vec{j}+\vec{k}$ 3) $\vec{i}+2\vec{j}$ 4) $2\vec{j}+\vec{k}$

12. If $|(\vec{a}\times\vec{b})| = 4$ and $|\vec{a}\cdot\vec{b}| = 2$ then $|\vec{a}|^2|\vec{b}|^2$ is equal to [BMP1] [3]

1) 4 2) 2 3) 20 4) 2

13. The points with position vectors $10\vec{i}+3\vec{j}$, $12\vec{i}-5\vec{j}$ and $a\vec{i}+11\vec{j}$ are collinear, if a is

1) 2 2) –8 3) 4 4) 8 [4]

14. The vector $\cos\alpha\cos\beta\,\vec{i}+\cos\alpha\sin\beta\,\vec{j}+\sin\alpha\,\vec{k}$ is [2]

1) null vector 2) unit vector 3) constant vector 4) vector with magnitude >1

15. If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular unit vectors, then the value of $|\vec{a}+\vec{b}+\vec{c}|$ is [3]

1) 1 2) $\sqrt{2}$ 3) $\sqrt{3}$ 4) 2

16. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$ then the angle between $\vec{a}$ and $\vec{b}$ is [4]

1) $\frac{\pi}{6}$ 2) $\frac{2\pi}{3}$ 3) $\frac{5\pi}{3}$ 4) $\frac{\pi}{3}$

17. If $\vec{a}$, and $\vec{b}$ are two unit vectors inclined at an angle θ then the value of $|\bar{a}-\bar{b}|$ is [1]

1) $2\sin\frac{\theta}{2}$ 2) $2\sin\theta$ 3) $2\cos\frac{\theta}{2}$ 4) $2\cos\theta$

18. If $|\vec{a}|=3$ and $-1\le k\le 2$ then $|k\vec{a}|$ lies in the internal [1]

1) $[0,6]$ 2) $[-3,6]$ 3) $[3,6]$ 4) $[1,2]$

HINTS & SOLUTIONS FOR MCQ

1. **(3)**

Sol: By Triangle Law of Addition of Vectors we have

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ (or) $\overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CA} \Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$.

2. **(1)**

Sol: If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then $\vec{b}=\lambda\vec{a}$.
The other options (2) & (4) are only true for particular values of λ

3. **(4)**

Sol: $|\lambda\vec{a}|=1 \Rightarrow |\lambda||\vec{a}|=1 \Rightarrow |\vec{a}|=\frac{1}{|\lambda|} \Rightarrow a=\frac{1}{|\lambda|}$

4. **(2)**

Sol: Given that $|\vec{a}|=3, |\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector. $\Rightarrow |\vec{a}\times\vec{b}|=1 \Rightarrow |\vec{a}||\vec{b}|\sin\theta=1$

$\Rightarrow 3\left(\frac{\sqrt{2}}{3}\right)\sin\theta=1 \quad \Rightarrow \sqrt{2}\sin\theta=1 \Rightarrow \sin\theta=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4} \quad \Rightarrow \theta=\frac{\pi}{4}$

5. **(3)**

Sol: Given ABCD is a rectangle.

Now $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\hat{\mathbf{i}}+\frac{1}{2}\hat{\mathbf{j}}+4\hat{\mathbf{k}}-\left(-\hat{\mathbf{i}}+\frac{1}{2}\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)=\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}+\hat{i}-\frac{1}{2}\hat{j}-4\hat{k}=2\hat{i}+0\hat{j}+0\hat{k}$

$|\overrightarrow{AB}|=\sqrt{4+0+0}=\sqrt{4}=2$

$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}-\left(-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right)=-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}+\hat{i}-\frac{1}{2}\hat{j}-4\hat{k}=-\hat{j}=0\hat{i}-\hat{j}+0\hat{k}$

$\therefore |\overrightarrow{AD}|=\sqrt{0+1+0}=\sqrt{1}=1$

Area of rectangle ABCD= = Length $\times$ Breadth = (AB) $\times$ (AD) = 2(1)=2 sq.units.

6. **(2)**

Sol: We have $\vec{a}.\vec{b}\geq 0 \quad \Rightarrow |\vec{a}||\vec{b}|\cos\theta\geq 0 \Rightarrow \cos\theta\geq 0 \left[\because |\vec{a}|\geq 0 \text{ and } |\vec{b}|\geq 0\right]$

$\Rightarrow 0\leq\theta\leq\frac{\pi}{2}$ Hence $\vec{a}.\vec{b}\geq 0$ if $0\leq\theta\leq\frac{\pi}{2}$

7. **(4)**

Sol: We have $\vec{a}$ and $\vec{b}$ two unit vectors and θ is the angle between them. Then, $|\vec{a}|=|\vec{b}|=1$

Now $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}|=1 \Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b})=1 \Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=1$

$\Rightarrow |\vec{a}|^2+2\vec{a}\vec{b}+|\vec{b}|^2=1 \Rightarrow 1^2+2|\vec{a}||\vec{b}|\cos\theta+1^2=1$

$$\Rightarrow 1+2(1)(1)\cos\theta+1=1 \Rightarrow \cos\theta=-\frac{1}{2} \Rightarrow \theta=\frac{2\pi}{3}$$

8. **(3)**

Sol: $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})=\hat{i}.\hat{i}+\hat{j}.(-\hat{j})+\hat{k}.\hat{k}=1-1+1=1$

9. **(2)**

Sol: $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}| \Rightarrow |\vec{a}||\vec{b}|\cos\theta=|\vec{a}||\vec{b}|\sin\theta \Rightarrow \cos\theta=\sin\theta \Rightarrow \tan\theta=1 \Rightarrow \theta=\frac{\pi}{4}$

10. **(1)**

Sol: $(\bar{a}-\bar{b}).(\bar{a}+\bar{b})=\bar{a}.\bar{a}+\bar{a}-\bar{b}-\bar{b}.\bar{a}-\bar{b}.\bar{b}=|\bar{a}|^2-|\bar{b}|^2=a^2-b^2$ where $|\bar{a}|=a$; $|\bar{b}|=b$

11. **(3)**

Sol: PV of P = (1, 2, 0) is $\overline{OP}=\bar{i}+2\bar{j}+0\bar{k}$

12. **(3)**

Sol: Relation between $\bar{a}\times\bar{b}$ and $\bar{a}.\bar{b}$ is $|\bar{a}\times\bar{b}|^2+(\bar{a}.\bar{b})^2=(\bar{a})^2(\bar{b})^2$

$\Rightarrow (4)^2+(2)^2=(\bar{a})^2(\bar{b})^2 \Rightarrow (\bar{a})^2.(\bar{b})^2=16+4=20$

13. **(4)**

Sol: Let $\overline{OA}=10\bar{i}+3\bar{j}$, $\overline{OB}=12\bar{i}-5\bar{j}$, $\overline{OC}=a\bar{i}+11\bar{j}$

$\overline{AB}=\overline{OB}-\overline{OA}=2\bar{i}-8\bar{j}$; $\overline{BC}=\overline{OC}-\overline{OB}=(a-12)\bar{i}+16\bar{j}$

$$\overline{AB}\,||\,\overline{BC} \Rightarrow \frac{a-12}{2}=\frac{16}{8} \Rightarrow \frac{a-12}{2}=-2 \Rightarrow a-12=-4 \Rightarrow a=8$$

14. **(2)**

Sol: Consider $|(\cos\alpha.\cos\beta)\bar{i}+(\cos\alpha.\sin\beta)\bar{j}+(\sin\alpha)\bar{k}|$

$$=\sqrt{\cos^2\alpha\cos^2\beta+\cos^2\alpha\sin^2\beta+\sin^2\alpha}=\sqrt{\cos^2\alpha(\cos^2\beta+\sin^2\beta)+\sin^2\alpha}$$

$=\sqrt{\cos^2\alpha+\sin^2\alpha}=\sqrt{1}=1$. Hence a Unit vector

15. (3)

Sol: Given $|\bar{a}|=|\bar{b}|=|\bar{c}|=1$ and $\bar{a}.\bar{b}=\bar{b}-\bar{c}=\bar{c}.\bar{a}=0$

$\Rightarrow |\bar{a}+\bar{b}+\bar{c}|^2=(\bar{a})^2+(\bar{b})^2+(\bar{c})^2+2(\bar{a}.\bar{b}+\bar{b}-\bar{c}+\bar{c}-\bar{a})=1+1+1+0=3$

$\Rightarrow |\bar{a}+\bar{b}+\bar{c}|=\sqrt{3}$

16. (4)

Sol: $\bar{a}+\bar{b}+\bar{c}=0 \Rightarrow \bar{a}+\bar{b}=-\bar{c} \quad \Rightarrow |\bar{a}+\bar{b}|=|\bar{c}|$. Squaring on both sides, we get

$\Rightarrow (\bar{a})^2+(\bar{b})^2+2\bar{a}.\bar{b}=(\bar{c})^2 \Rightarrow 9+25+2\bar{a}.\bar{b}=49 \Rightarrow 2\bar{a}.\bar{b}=15 \Rightarrow 2(\bar{a})(\bar{b})\cos(\overline{ab})=15$

$\Rightarrow 2(3)(5)\cos\theta=15 \Rightarrow 2\cos\theta=1 \Rightarrow \cos\theta=\frac{1}{2}=\cos 60^\circ$

$\therefore \theta=60^\circ=\pi/3$

17. (1)

Sol: Given $|\bar{a}|=|\bar{b}|=1 < \bar{a},\bar{b} >=\theta$

consider $|\bar{a}-\bar{b}|^2=(\bar{a})^2+(\bar{b})^2-2\bar{a}-\bar{b}\ =1+1-2(\bar{a})(\bar{b})\cos\theta=2-2\cos\theta$

$$=2(1-\cos\theta)=2\ 2\sin^2\theta/2 \Rightarrow |\bar{a}-\bar{b}|=\sqrt{4\sin^2(\theta/2)}=2\sin(\theta/2)$$

18. (1)

Sol: $|k\bar{a}| \Rightarrow |k||\bar{a}| \Rightarrow 3|k|$

$-1\le k\le 2$

$0\le |k|\le 2$

$0\times 3\le 3|k|\le 2\times 3$

$0\le 3|k|\le 6 \quad \Rightarrow |k\bar{a}|\in[0,6]$

12 LINEAR PROGRAMMING

1 OMQ + 1 SAQ [1M + 4M = 5 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1 \times 1 = 1$ Mark

1. **Region represented by $x \geq 0, y \geq 0$ is** [1]
   1) First quadrant 2) Second quadrant 3) Third quadrant 4) Fourth quadrant
2. **If the objective function $Z = ax + by$ has both a maximum and a minimum value on the region R then R is** [1]
   1) Bounded 2) Unbounded 3) Concave polygon 4) Infeasible
3. **The linear inequalities or equations or restrictions on the variables of a linear programming problem are called** [1]
   1) constriants 2) Decision variables 3) objective function 4) Linear relations
4. **The optimal value of the objective function is attained at the points** [3]
   1) On X-axis 2) On Y-axis
   3) Which are at the corner points of the feasible region
   4) Which are at the points of intersection of the inequation with Y-axis
5. **In a linear programming problem the objective function and constraints must be** [2]
   1) Non-linear **2) Linear** 3) Exponential 4) Logarithmic
6. **The maximum value of $Z = 3x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0, y \geq 0$ is** [3]
   1) 12 2) 14 **3) 16** 4) 10
7. **Maximize $Z = 3x + 5y$, subject to constriants $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \leq 9$, $x \geq 0$, $y \geq 0$.** [3]
   1) 20 at (1, 0) 2) 30 at (0, 6) **3) 37 at (4, 5)** 4) 33 at (6, 3)
8. **The point which does not lie in the half plane $2x + 3y - 12 < 0$ is** [4]
   1) (2, 1) 2) (1, 2) 3) (–2, 3) **4) (2, 3)**
9. **Which of the following is not a component of linear programming problem?** [4]
   1) Objective function 2) Constriant
   3) Decision variable **4) Differential equation**
10. **The position of points O(0, 0) and P(2, –3) in the region of graph of inequation $2x - 3y < 5$ will be** [1]
    1) O inside and P outside 2) O and P both inside
    3) O and P both outside 4) O outside and P inside

HINTS & SOLUTIONS FOR MCQ

1. **(3)**

Sol: **Region $x \geq 0$, $y \geq 0$ (non-negative)** $\Rightarrow$ Both x and y positive $\rightarrow$ first quadrant.

2. **(1)**

Sol: Both maximum and minimum exist only if region is closed and bounded.

3. **(1)**

Sol: **Linear restrictions in LPP are called constraints.**

4. **(3)**

Sol: In LPP, optimum value occurs at vertices of feasible region.

5. **(2)**

Sol: **Objective function & constraints must be linear in LPP**

6. **(3)**

Sol: **Corner check points:** (0,0), (4,0), (0,4) then $Z = 3x + 4y$ values: 0, 12, 16.

Maximum value is 16.

7. **(3)**

Sol: **Corner check points** of feasible region. $Z = 3x + 5y$

Best point: (4,5) $\Rightarrow Z = 12 + 25 = 37$

8. **(4)**

Sol: **Point NOT in $2x + 3y - 12 < 0$**

Substitute each point$\rightarrow$LHS < 0

Check (2,3): $4 + 9 - 12 = 1$ (NOT < 0)

9. **(4)**

Sol: **Not a component of LPP.** LPP uses linear equations, not calculus.

10. **(1)**

Sol: **Check points in $2x - 3y < 5$**

O(0,0): $0 < 5 \rightarrow$ inside

P(2,–3): $4 + 9 = 13 > 5 \rightarrow$ outside

13 PROBABILITY

1 OMQ + 1 VSAQ + 1 SAQ + 1 LAQ [1 M + 2M +4M +8M = 15 M]

MULTIPLE CHOICE Q'S(MCQ)

OMQ'S

$1 \times 1 =$ 1 Mark

1. If $P(A) = \frac{1}{2}, P(B) = 0$, then $P(A|B)$ is [3]

1) 0 2) $\frac{1}{2}$ **3) not exist** 4) 1

2. If A and B are events such that P(A|B) = P(B|A), then [4]

1) $A \subset B$ but $A \neq B$ 2) $A = B$ 3) $A \cap B = \phi$ **4) P(A) = P(B)**

3. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is **[BMP1]** [4]

1) 0 2) $\frac{1}{3}$ 3) $\frac{1}{12}$ **4) $\frac{1}{36}$**

4. Two events A and B will be independent, if [2]

1) A and B are mutually exclusive **2) $P(A'B') = [1 - P(A)] [1 - P(B)]$**

3) $P(A) = P(B)$ 4) $P(A) + P(B) = 1$

5. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct? [3]

1) $P(A|B)=\frac{P(B)}{P(A)}$ 2) $P(A|B)<P(A)$ **3) $P(A|B) \geq P(A)$** 4) $P(A)=P(B)$

6. If A and B are two events such that $P(A) \neq 0$ and $P(B | A) = 1$, then [1]

1) $A \subset B$ 2) $B \subset A$ 3) $B = \phi$ 4) $A = \phi$

7. If P(A|B) > P(A), then which of the following is correct : [3]

1) $P(B|A) < P(B)$ 2) $P(A \cap B) < P(A) . P(B)$ **3) $P(B|A) > P(B)$** 4) $P(B|A) = P(B)$

8. If A and B are any two events such that P(A) + P(B) – P(A and B) = P(A), then [2]

1) $P(B|A) = 1$ **2) $P(A|B) = 1$** 3) $P(B|A) = 0$ 4) $P(A|B) = 0$

9. A bag contains 7 red and 3 white balls. Three balls are drawn one after other without replacement. Then the probability that the first two are red and third one is white is **[BMP2]** [1]

1) $\frac{7}{40}$ 2) $\frac{33}{40}$ 3) $\frac{23}{40}$ 4) $\frac{17}{40}$

10. A book consists of 20 pages. If two pages are drawn (opened) at random, then the probability that both numbers are prime numbers is [4]

1) $\frac{17}{95}$ 2) $\frac{16}{95}$ 3) $\frac{2}{15}$ **4) $\frac{14}{95}$**

11. A fair coin is tossed 3 times, then the probability of getting one head and two tails is [3]

1) $\frac{1}{8}$ 2) $\frac{1}{4}$ **3) $\frac{3}{8}$** 4) $\frac{1}{2}$

12. A person appears for an interview for two posts A and B. The selection of the posts are independent. If $P(A)=\frac{1}{5}, P(B)=\frac{1}{8}$ then $P(A\cup B)$ is [2]

1) $\frac{7}{10}$ **2) $\frac{3}{10}$** 3) $\frac{9}{10}$ 4) $\frac{1}{10}$

13. Three events A, B, C are mutually exclusive and exhaustive and P(A) = 0.4, then P(B) + P(C) = [3]

1) 0.4 2) 0.5 **3) 0.6** 4) 0

14. If $P(A \cup B) = 0.65$ and $P(A \cap B) = 0.15$, then $P(\overline{A})+P(\overline{B})$ = [3]

1) 0.8 2) 0.6 **3) 1.2** 4) 1.4

15. When two dice are rolled, the probability of getting unequal numbers on the faces is [3]

1) $\frac{1}{6}$ 2) $\frac{35}{36}$ **3) $\frac{5}{6}$** 4) $\frac{1}{3}$

16. A fair coin whose faces are marked with 1 and 2 is thrown for four times. Then the probability of throwing a total of atleast 5 is [3]

1) $\frac{1}{16}$ 2) $\frac{5}{16}$ **3) $\frac{15}{16}$** 4) $\frac{3}{16}$

17. If A and B are independent events of an experiment then which of the following statements is true [4]

1) $P(A \cup B) = P(A) + P(B) - P(A) . P(B)$ 2) $P(A|B) = P(A)$ and $P(B|A) = P(B)$

3) $P(A \cap B) = P(A) . P(B)$ **4) All the above**

18. If A and B are two events of a random experiment of throwing a die given by "A": throwing an odd face and B: throwing a composite face. Then which of the following statements is correct. [2]

1) A and B are equally likely **2) A and B are mutually exclusive**

3) A and B are mutually exhaustive 4) A and B are linearly independent

19. If A, B and C are independent events of a random experiment such that $P(A) = p$, $P(B) = q$, $P(C) = r$, where $p, q, r \in (0, 1)$. Then the probability of the event A only occurs is [2]

1) $p \cdot q \cdot r$ **2) $p(1-q)(1-r)$** 3) $p.\ q(1-r)$ 4) $(1-p)(1-q)(1-r)$

20. A fair die is rolled. Consider the events $A = \{1, 3, 5\}$ and $B = \{2, 3\}$, then $P(A|B)$ is [1]

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{5}{6}$

HINTS & SOLUTIONS FOR MCQ

1. (3)

Sol: $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{\mathbf{0}}$ which is not defined

2. (4)

Sol: Given P(A/B) = P(B/A)

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)} \Rightarrow \frac{\cancel{P(A \cap B)}}{P(B)} = \frac{\cancel{P(A \cap B)}}{P(A)} \qquad [\because A \cap B = B \cap A]$$

$$\Rightarrow \frac{1}{P(B)} = \frac{1}{P(A)} \Rightarrow P(A) = P(B)$$

3. (4)

Sol: When two dice are rolled, the number of outcomes $n(S)=6^2= 36$.

The only even prime number is 2.

Let E be the event of getting an even prime number on each die. $\therefore E=(2,2) \Rightarrow P(E) = \frac{1}{36}$

4. (2)

Sol: A and B are independent $\Rightarrow A'$ and B' are independent

$\Rightarrow P(A' \cap B') = [1 - P(A)]\ [1 - P(B)]$ are independent

5. (3)

Sol: If $A \subset B$, then $A \cap B = A \Rightarrow P(A \cap B) = P(A)$. Also, $P(A) < P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \quad .....(1)$$

Since $P(B) \le 1 \Rightarrow \frac{1}{P(B)} \ge 1 \Rightarrow \frac{P(A)}{P(B)} \ge P(A)$

From (1), we have $P(A|B) \ge P(A)$

6. (1)

Sol: Given $P(A) \neq 0$ and $P(B \mid A) = 1$,

$$\therefore P(B|A) = \frac{P(B \cap A)}{P(A)} \quad \Rightarrow 1 = \frac{P(B \cap A)}{P(A)} \Rightarrow P(A) = P(B \cap A) \Rightarrow A = A \cap B \Rightarrow A \subset B$$

7. (3)

Sol: Given that Given, $P(A|B) > P(A) \Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$

$$\Rightarrow P(A \cap B) > P(A) \times P(B) \Rightarrow \frac{P(A \cap B)}{P(A)} > P(B) \Rightarrow P(B|A) > P(B)$$

8. (2)

Sol: Given that P(A) + P(B) – P(A and B) = P(A),

$\Rightarrow P(A)+P(B)-P(A\cap B)=P(A) \Rightarrow P(B)-P(A\cap B)=0 \Rightarrow P(A\cap B)=P(B)$

$\therefore P(A|B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{P(B)}{P(B)}=1$

9. (1)

Sol: 7R + 3W = Total 10 balls

$P(E) = P(\text{Red and Red and White})=\left(\dfrac{7}{10}\right)\times\dfrac{6}{9}\times\dfrac{3}{8}=\dfrac{7}{40}$ ($\because$ Drawn ball is not replaced)

10. (4)

Sol: Total no.of pages = 20

Primes up to 20 are 2, 3, 5, 7, 11, 13, 17, 19 & the no.of these primes =8

$\therefore P(E)=\dfrac{{}^{8}C_2}{{}^{20}C_2}=\dfrac{8\times7}{20\times19}=\dfrac{2\times7}{5\times19}=\dfrac{14}{95}$

11. (3)

Sol: $S=\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E=\{HTT,\ THT,\ TTH\} \Rightarrow n(E)=3;\ n(S)=8$ $\qquad P(E)=\dfrac{n(E)}{n(S)}=\dfrac{3}{8}$

12. (2)

Sol: Given A, B are independent event $\Rightarrow P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\dfrac{1}{5}+\dfrac{1}{8}-[P(A).P(B)]=\dfrac{13}{40}-\left(\dfrac{1}{5}\times\dfrac{1}{8}\right)=\dfrac{13}{40}-\dfrac{1}{40}=\dfrac{12}{40}=\dfrac{3}{10}$

13. (3)

Sol: A, B, C are mutually exclusive and exhaustive $\Rightarrow A\cup B\cup C=S$ ....(1)

Given P(A) = 0.4 ....(2)

$(1)\Rightarrow P(A\cup B\cup C)=P(S)\Rightarrow P(A)+P(B)+P(C)=1 \Rightarrow 0.4 + P(B) + P(C) = 1$

$\Rightarrow P(B) + P(C) = 1 - 0.4 = 0.6$

14. (3)

Sol: $P(\bar{A})+P(\bar{B})=1-P(A)+1-P(B)$

$=2-[P(A)+P(B)]=2-[P(A\cup B)+P(A\cap B)]$

$= 2 - [0.65 + 0.15] = 2 - [0.80] = 1.20 = 1.2$

15. **(3)**

Sol: Two dice are rolled n(S) = 36

Equal number faces $=\{(1,1)\ (2,2)\ (3,3)\ (4,4)(5,5)(6,6)\} \Rightarrow n(S)=6$

$\Rightarrow$ no.of unequal faces = 36 – 6 = 30 = n(E)

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{30}{36}=\frac{5}{6}$$

16. **(3)**

Sol: Coin with faces 1. (say H); 2. (say T)
thrown 4 - times
Getting total at least 5 $\Rightarrow$ total $\geq 5 \Rightarrow x \geq 5$
Now, $P(x \geq 5)=1-P(x<5)$ = 1 – P (getting a total 4 faces 4 - times)

= 1 – P (every time a face 1) $=1-\left[\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right]=1-\frac{1}{16}=\frac{16-1}{16}=\frac{15}{16}$

17. **(4)**

Sol: By definition $P(A\cap B)=P(A).P(B)$

18. **(2)**

Sol: When a die is thrown

A : odd face (1, 3, 5); B : composite face (4, 6). Then $P(A)=\frac{3}{6}=\frac{1}{2}$ and $P(B)=\frac{2}{6}=\frac{1}{3}$

1) A, B are likely ($\times$)

2) A, B are mutually exclusive($\checkmark$)

$A\cap B=\phi$ (or) $P(A\cap B)=0$

3) A, B mutually exhaustive ($\times$) $\because A\cup B\neq S$

4) $P(A\cap B)\neq P(A).P(B)\Rightarrow$ NOT independent ($\times$)

19. **(2)**

Sol: A, B, C are independent

P(A only occurs) $=P(A\cap\bar{B}\cap\bar{C})=P(A).P(\bar{B}).P(\bar{C})$ ($\because$ A, B, C are independent)

$=P(A).[1-P(B)][1-P(C)]=P[1-q][1-r]$

20. **(1)**

Sol: $S=\{1, 2, 3, 4, 8, 6\}$; $A=\{1, 3, 5\}, B=\{2, 3\}\Rightarrow P(B)=\frac{2}{6}=\frac{1}{3}$

$$\therefore (A\cap B)=\{3\}\Rightarrow P(A\cap B)=\frac{1}{6}$$

$$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{1/6}{1/3}=\frac{1}{6}\times\frac{3}{1}=\frac{1}{2}$$