13 PROBABILITY

1 OMQ + 1 VSAQ + 1 SAQ + 1 LAQ [1 M + 2M +4M +8M = 15 M]

13(a) CONDITIONAL PROBABILITY

CONCEPTS & FORMULAS

1.1) Compound Event:

The simultaneous occurrence of two or more events is called a Compound Event.

1.2) Compound Probability: If E,F are any two events of a sample space S then the probability of joint occurrence of E,F is called Compound Probability of E,F

The probability of joint occurrence of E,F is denoted by $P(E\cap F)$ (or) $P(EF)$

1.3) Conditional Event: Let E,F be two events of a sample space S. If E occurs only after (from) the occurrence of F, then the event of occurrence of E w.r.to F is called conditional event of E given F and it is denoted by E/ F. Here E is said to be conditioned from F.

In conditional event, the sample space gets reduced.

Ex 1: When a die is thrown 2 times, the event of getting sum 5, given that the first throw was a 3 is a conditional event={(3,2)}

Ex 2: A bag A contains 3 white and 2 black balls and bag B contains 2 white and 4 black balls. Event of choosing a white ball from one of the bags chosen at random is a conditional event.

1.4) Conditional Probability: The conditional probability of an event E, given the occurrence of the event F is given by $P(E/F)=\dfrac{P(E\cap F)}{P(F)}, P(F)\neq 0$

☞ $$P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{n(E\cap F)}{n(S)}}{\frac{n(F)}{n(S)}}=\frac{n(E\cap F)}{n(F)}, \text{ where } P(F)\neq 0$$

Similarly, $$P\left(\frac{F}{E}\right)=\frac{P(E\cap F)}{P(E)}=\frac{n(E\cap F)}{n(E)}, \text{ where } P(E)\neq 0$$

Note: If A and B are any two events of a sample space S and F is an event of S such that $P(F)\neq 0$, then $P((A\cup B)|F) = P(A|F) + P(B|F) - P((A\cap B)|F)$

Ex : A die is rolled once and A be the event of getting an Odd number, B be the event of getting a prime number then find (i) P(A/B) (ii)P(B/A)

Sol: Here $A=\{1,3,5\} \Rightarrow n(A)=3$; $B=\{2,3,5\} \Rightarrow n(B)=3$; $A\cap B=\{3,5\} \Rightarrow n(A\cap B)=2$

(i) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{n(A\cap B)}{n(B)}=\frac{2}{3}$; Note: $P(A)=\frac{3}{6}=\frac{1}{2}$

(ii) $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{n(A\cap B)}{n(A)}=\frac{2}{3}$ Note: $P(B)=\frac{3}{6}=\frac{1}{2}$

13b) MULTIPLICATION THEOREM, INDEPENDENT EVENTS

CONCEPTS & FORMULAS

1) **Independent events:** Two events are said to be independent if the happening of any event is not influenced by the happening of the other event.

2) **Dependent events:** The events which are not independent are called dependent events.

Ex 1: In tossing 2 coins at a time, getting H on one coin and getting T/H on the other are independent events. Also when a coin is tossed twice, getting H in the first toss and getting T/H in the second toss are also independent events.

Ex 2: Let A be the event of drawing a card from pack of cards in the first draw. Let B be the event of drawing another card in the second draw **with replacement** of the first drawn card. Then A,B are **independent events**. Here if the first drawn card is **not replaced** then A,B are **dependent events**.

Ex 3: A coin is tossed. If Head (H) comes toss again and if Tails (T) comes stop tossing. Here both the events are dependent events.

Note 1: Two events E,F in a sample space S are independent iff $P(E\cap F)=P(E)P(F)$

Note 2: Two events E,F in a sample space S are dependent iff $P(E\cap F)\neq P(E)P(F)$

3) **Multiplication Theorem on Compound Probability:**

Statement: If E,F are 2 events of a random experiment such that $P(E)\neq 0$, $P(F)\neq 0$ then $P(E\cap F)=P(E).P(F/E)=P(F).P(E/F)$

Proof: We know $P(E\mid F)=\dfrac{P(E\cap F)}{P(F)}$ (from the definition of conditional probability)

$\Rightarrow P(E\cap F)=P(E).P(F/E)$

Note : Two events E,F in a sample space S are independent iff $P(F|E)=P(F)$

4) **Basic examples to understand the concept of Multiplication theorem, independent, dependent events:**

Ex1: Probability of drawing a King card (K) and Queen card (Q) simultaneously from a pack of 52 playing cards is $P(K\cap Q) = \dfrac{{}^4C_1\times{}^4C_1}{{}^{52}C_2}$ [Compound probability]

Ex2: Probability of drawing a King card and Queen card in successoin with replacement from a pack of 52 playing cards is $P(K\cap Q) = \dfrac{4}{52}\times\dfrac{4}{52}$

[Here, K, Q are **independent events**, as the first drawn card is **replaced**]

Ex3: Probability of drawing a King card and Queen card in succession without replacement from a pack of 52 playing cards is $P(K\cap Q) = P(K)P(Q/K) = \dfrac{4}{52}\times\dfrac{4}{51}$

[Here, K, Q are **dependent events,** as the first drawn card is **not replaced**]

13(c) BAYES THEOREM

CONCEPTS & FORMULAS

Bayes Theorem:

Statement: If E_1, E_2 ,...., E_n are n non empty events which constitute a partition of sample space S, i.e. E_1, E_2 ,...., E_n are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and A is any event of nonzero probability, then $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)}$ for any $i = 1, 2, 3, ., n$

Proof : By formula of conditional probability, we know that $P(E_i \mid A) = \dfrac{P(A \cap E_i)}{P(A)}$

$$= \frac{P(E_i)P(A|E_i)}{P(A)} \text{ (by multiplication rule of probability)}$$

$$= \frac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)} \text{ (by the result of theorem of total probability)}$$

Ex: A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball drawn is from bag B.

Sol: Let A, B be the events of choosing bag A, bag B and E be the event of drawing red ball

$\Rightarrow P(A)=1/2,\ P(B)=1/2,\ P(E/A)=3/5,\ P(E/B)=5/9$

$$\therefore P(B/E) = \frac{P(B)P(E/B)}{P(A)P(E/A) + P(B)P(E/B)} = \frac{(1/2)(5/9)}{(1/2)(3/5) + (1/2)(5/9)} = \frac{25}{52}$$