13. NUCLEI

Single Page Simple Synopsis

Nuclei chapter covers the composition and properties of atomic nuclei, nuclear forces, mass defect, binding energy, and the mechanics of radioactivity, nuclear fission, and fusion. It explains the core physical principles governing nuclear stability and energy release.

Key Concepts & Formulas

- **Composition of Nucleus :** Nucleus consists of protons and neutrons (collectively called nucleons).
- **Atomic Number (Z):** Number of protons.
- **Mass Number (A):** Total number of protons and neutrons.
- Number of neutrons N= A–Z
- **Nuclear Size:** The radius of nucleus is $R = R_0 A^{1/3}$. Here $R_0 = 1.2\times10^{-15}$ m
- **Mass Defect (Δ_m):** The difference between the actual mass of a nucleus and the sum of the masses of its individual nucleons. **Formula:** $\Delta_m = [Z m_p + (A - Z)m_n] - M$
- **Nuclear Binding Energy (E_b):** The energy required to break a nucleus into its constituent nucleons. **Formula:** $E_b = \Delta_m c^2$ (If Δ_m is in amu, use 1 amu $\approx$ 931.5 MeV)
- **Binding Energy per Nucleon (E_{bn}):** E_{bn} indicates nuclear stability.
  E_{bn} peaks around A = 56(Iron Fe), meaning 'medium-mass nuclei' are the most stable.
- **Radioactivity and Nuclear Reactions:**
- **Radioactive Decay Law:** The number of undecayed nuclei decreases exponentially over time.
  $N(t) = N_0 e^{-\lambda t}$ (Where λ is the decay constant)
- **Half-Life ($T_{1/2}$):** The time required for half of the radioactive nuclei to decay: $T_{1/2} = \frac{\ln(2)}{\lambda} \approx \frac{0.693}{\lambda}$
- **Mean Life (τ):** The average life expectancy of a radioactive nucleus: $\tau = \frac{1}{\lambda}$
- **Nuclear Fission (in nuclear reactors):** The splitting of a heavy nucleus (like Uranium-(235) into lighter fragments, releasing a massive amount of energy and neutrons.
- **Nuclear Fusion (in Sun, Stars) :** The combination of lighter nuclei (like Hydrogen isotopes) to form a heavier nucleus, which powers the Sun and other stars. Extremely high temperatures and pressures are required to overcome electrostatic repulsion.

Real Life Examples

- **Nuclear Power Plants**, Cancer Treatment (Radiotherapy), PET Scan & Medical Diagnosis, Carbon Dating (Archaeology), Smoke Detectors, Nuclear Submarines & Space Missions

IMP. IPE POINTS

1. **Radius** of nucleus $R = R_0 A^{1/3}$

2. **Ratio** of Radii of nuclei $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$

3. **Density** of nucleus $\rho = \frac{m}{\frac{4}{3}\pi R_0^3}$

4. **Mass defect**
$\Delta M = [Zm_P + (A-Z)m_n] - M$

5. Einstein's mass-energy equivalence
$E = mc^2$

6. Binding energy $B.E = \Delta M\, c^2$
Binding energy $\overline{B.E} = \Delta M \times 931.5$ MeV

7. Energy released when 1 amu mass is anhiliated is 931.5 MeV

8. Binding energy per nucleon $\overline{B.E} = B.E / A$

9. Alpha decay ${}_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4$

10. Beta decay
β^- decay : ${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$
β^+ decay : ${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu$
where $\bar{\nu}$ is anti neutrino and ν is neutrino

11. **Gamma decay: When** a nucleus emits γ-rays, there will be reduction in the energy state of the nucleus.

12. Radioactive decay law: $N = N_0\, e^{-\lambda t}$

13. **Activity:** $-\left(\frac{dN}{dt}\right) = \lambda N$

14. **Half life:** $T_{1/2} = \frac{\log_e 2}{\lambda}$ or $T_{1/2} = \frac{0.693}{\lambda}$

15. **Average life:** $\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44T_{1/2}$

16. Relation between half life and average life is $T_{1/2} = 0.693\,\tau$

17. **Discovery of neutron**:
$${}_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1$$

18. **Disintegration of neutron**:
$${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu}$$
where $\bar{\nu}$ is anti neutrino

19. **Nuclear fission**:
$${}_0n^1 + {}_{92}U^{235} \rightarrow {}_{92}U^{236} \rightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3\,{}_0n^1 + 200MeV$$

20. **Nuclear fusion**:
$$4\,{}_1H^1 + 4\,{}_{-1}e^0 \rightarrow {}_2He^4 + 2\,{}_{-1}e^0 + 2\nu + 6\gamma + 26.7MeV$$

21. **Pair annihilation**:
$${}_{+1}e^0 + {}_{-1}e^0 \rightarrow 2\gamma + 1.02\text{ MeV}$$

22. 1 KWH = 36×10^5 J

23. 1 MeV = 1.6×10^{-13} J

24. **Hints for Problems:**
(a) $\log_e e = 1$
(b) $\log_e x = (2.303)\log_{10} x$
(c) $\log_e \frac{N_0}{N} = \frac{\log_e 2}{T_{1/2}}\, t$
(d) $\frac{N_0}{N} = 2^n$; n = no. of half lifes $= \frac{t}{T}$
(e) $\frac{A_0}{A} = 2^n$