11 3D-GEOMETRY

1 VSAQ + 1 LAQ [2M + 8M = 10 M]

11a) D.C's & D.R' s of a Line

CONCEPTS & FORMULAS

1. **Direction cosines (d.c's) of a line:** If α,β,γ are are the angles made by a line with the 3 coordinate axes then the direction cosines of the line are $l=\cos\alpha$, $m=\cos\beta$, $n=\cos\gamma$

2. **Direction ratios (d.r's) of a line** are the numbers (a,b,c) which are proportional to the direction cosines of the line. d.r's are multiples of d.c's of a line.

3. If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.

4. If a, b, c are the direction ratios of a line then the direction cosines l, m, n are

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}};m=\frac{b}{\sqrt{a^2+b^2+c^2}};n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$ (Here, $l^2 + m^2 + n^2 = 1$.)

5. Direction cosines of a line joining two points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ are

$$\frac{x_2-x_1}{PQ},\frac{y_2-y_1}{PQ},\frac{z_2-z_1}{PQ}$$. Here $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

6. If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of two lines; and θ is the acute angle between the two lines then $\cos\theta=|l_1l_2+m_1m_2+n_1n_2|$

7. If (a_1, b_1, c_1) and (a_2, b_2, c_2) are the direction ratios of two lines and θ is the acute angle between the two lines then $\cos\theta=\left|\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

11b) SHORTEST DISTANCE BETWEEN LINES

CONCEPTS & FORMULAS - I

1. **Skew lines** are lines in space which are neither parallel nor intersecting. They lie in different planes.

2. **Angle between skew lines** is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.

3. Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

4. Equation of a line through a point (x_1, y_1, z_1) and having direction cosines l, m, n is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

5. The acute angle between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ is given by $\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

6. The acute angle between the two lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ is given by $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$.

7. The distance between parallel lines $\vec{r} = \vec{a}_1 + \mu\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is $\left|\frac{\vec{b}\times(\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$

8. The distance between skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $\left|\frac{(\vec{b}_1\times\vec{b}_2)(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

9. The shortest distance between the lines: $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is

$$\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2 + (c_1a_2-c_2a_1)^2 + (a_1b_2-a_2b_1)^2}}$$