1 RELATIONS AND FUNCTIONS

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

I) RELATIONS

[Reflexive , Symmetric , Transitive & Equivalence Relations.]

CONCEPTS & FORMULAS

1.1 Relation ($R: A \to A$): In class XI, we learnt about relations defined on a non-empty set A to itself. Thus, any subset of $A \times A$ is a relation on A.

The Relations are generally expressed in 3 ways.

i) Real Life Models (Human Relations) ii) Set of Ordered Pairs (a,b) iii)Expressions like y=f(x)

Illustrations on Relations:

i) Real Life Examples (Human Relations):

Suppose there are three boys A,B,C. Their friendship among themselves is a relation. Also, their height, age, belonging to a class etc., establish various relations among A,B,C.

ii) Set of Ordered Pairs (a,b):

Let A= {1,2,3} then R= {(a,b) such that a<b} is a relation. Here R= { (1,2), (2,3),(1,3)}

iii) Expressions like y=f(x):

$f:N \to N$ such that y= f(x)= 2x is a relation. Here, f= {(1,2), (2,4), (3,6),......}

1.2 Types of Relations : Basing on how the two elements in the order pair (a,b) are connected, the relations are classified into the following types:

i) Reflexive Relation: A relation R on a set A is reflexive, if $(a,a) \in R$ **for all** $a \in A$.

Ex 1: The relation equality (=) on reals. $[\because 1=1,\ 2=2,\ 3=3,...]$

Ex 2: The relation subset ($\subset$) on sets [$\because$ Every set is subset to itself]

Non-Ex: The relation greater than (>) on reals. [2 > 2 is false]

ii) Symmetric Relation: A relation R on a set A is symmetric, if $(a,b) \in R \Rightarrow (b,a) \in R$

Ex: The relation 'parallel of' lines (||) in a plane. (If $L_1 \parallel L_2$ then $L_2 \parallel L_1$)

Non-Ex: The relation greater than (>) on reals. [3 > 2 is true but 2>3 is false]

iii) Transitive Relation: A relation R on a set A is transitive, if $(a,b) \in R$ **and** $(b,c) \in R \Rightarrow (a,c) \in R$

Ex: The relation greater than (>) on reals. $[\because 3>2 \text{ and } 2>1 \Rightarrow 3>1]$

Non-Ex: A is neighbour of B, and B is neighbour of C, then A is not neighbour of C.

☞ **Equivalence Relation:** A relation R on a set A is called an equivalence relation if R is **reflexive, symmetric and transitive.**

Ex: The relation equality (=) on reals. Students in a school having **same** height or **same** blood group or **same** date of birth etc.,

Non-Ex: Perpendicularity of lines; greater than, taller than, father of,...

Note : A given relation may have one, two, three or none of the above three properties.

iv) Empty Relation($\varnothing$): A relation R in a set A is called empty relation, if no element of A is related to any element of A. [$\because$ $\varnothing$ is subset to any set]

v) Universal Relation ($A \times A$): A relation R in a set A is called universal relation, if each element of A is related to every element of A. [$\because$ Every set is subset to itself]

☞ The empty relation and the universal relation are called **trivial relations.**

The RST Relation Table

Property	Condition	How to Check?
i) Reflexive **(R)**	$(a,a) \in R\ ,\ \forall a$	All self-pairs
ii)Symmetric **(S)**	$(a,b) \in R \Rightarrow (b,a) \in R$	All pairs & Reverse pairs
iii)Transitive **(T)**	$(a,b),\ (b,c) \in R \Rightarrow (a,c) \in R$	Chain link completion

A Model Illustration :

Let $A=\{1,2\}$. Then $A\times A = \{1,2\}\times\{1,2\} == \{(1,1),\ (1,2),\ (2,1)\ (2,2)\} \Rightarrow n(A\times A) = 4$

Here the number of relations from A to A = $2^4 = 16$

The 16 relations from A to A = The 16 subsets of $A \times A$. They are given below

$R_1=\{(1, 1)\}$ [This R_1 is Reflexive, symmetric and transitive.]

$R_2=\{(1, 2)\}$ [This R_2 is **not** Reflexive, not symmetric and not transitive.]

$R_3=\{(2, 1)\}$, $R_4=\{ (2, 2)\}$; $R_5=\{(1, 1), (1, 2)\}$, $R_6= \{(1, 1), (2, 1)\}$,

$R_7 =\{(1, 1), (2, 2)\}$ [This R_7 is Reflexive, symmetric and not transitive.]

$R_8 =\{(1, 2), (2, 1)\}$, $R_9 =\{(1, 2), (2, 2)\}$,

$R_{10} =\{ (2, 1), (2, 2)\}$; $R_{11} =\{(1, 1), (1, 2), (2, 1)\}$, $R_{12} =\{ (1, 1), (1, 2), (2, 2) \}$,

$R_{13}=\{ (1, 1), (2, 1), (2, 2)\}$, $R_{14} =\{ (1, 2), (2, 1), (2, 2)\}$;

$R_{15} =\{(1, 1), (1, 2), (2, 1), (2, 2) \}$[This R_{15} is a Universal relation and is Equivalence relation]

$R_{16} =\varnothing$. [This R_{16} is an Empty relation]

THE SLAPPING GAME

- Three boys A,B,C play the 'slapping game' that they slap **themselves** and slap **one another.**
  - i) This slapping relation is Reflexive. [Everyone slaps himself.]
  - ii) This slapping relation is Symmetric. [A slaps B and B slaps A]
  - iii) This slapping relation is Transitive. [A slaps B, B slaps C and also A slaps C]