9 STRAIGHT LINES

1 OMQ + 1 SAQ + 1 LAQ [1 M + 4M +8 M = 13 M]

✎ CONCEPTS & FORMULAS ✎

1. If P(x,y) is a point in the cartesian plane (R^2) then
(i) |x| denotes the perpendicular distance from P to the Y-axis.
(ii) |y| denotes the perpendicular distance from P to the X-axis.

2. The distance between the points $A(x_1,y_1)$, $B(x_2,y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The distance between the origin O(0,0) and P(x,y) is $OP = \sqrt{x^2 + y^2}$

3. **Inclination of a line:** The angle θ ($0° \le θ < 180°$) made by a line with the positive x − axis measured in anti - clockwise direction is called the inclination of the line.

4. **Slope of a line:**

- If θ is the inclination of a line then its slope is m = tanθ.

- Slope of the line passing through the points (x_1, y_1), (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

- The slope of the line ax + by + c = 0 is $\dfrac{-a}{b} = \dfrac{-\text{coefficient of } x}{\text{coefficient of } y}$

☞   1) The slope of a horizontal line is zero.
    2) The slope of a vertical line is 'un defined'.(or ∞?)
    3) The slopes of any two parallel lines (non − vertical) are equal.
    4) If m_1, m_2 are the slopes of two perpendicular lines then $m_1 m_2 = -1$.
    5) If m is the slope of a line then the slope of its perpendicular line is $-1/m$

5. The area of the **Triangle** formed by the vertices $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ is

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = \frac{1}{2}\left|(x_1 - x_2)(y_1 - y_3) - (y_1 - y_2)(x_1 - x_3)\right|$$

- The area of the triangle with vertices O(0,0), $A(x_1,y_1)$, $B(x_2,y_2)$ is $\Delta = \dfrac{1}{2}|x_1 y_2 - x_2 y_1|$

**6.** The area of the **Quadrilateral** formed by the vertices $A(x_1,y_1),B(x_2,y_2),C(x_3,y_3),D(x_4,y_4)$ in order, is $\Delta = \dfrac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix} = \dfrac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (y_1 - y_3)(x_2 - x_4)\right|$ Sq.units

7. The point collinear with A,B dividing the line segment joining $A(x_1,y_1)$,$B(x_2,y_2)$ in the ratio *l*:m
(i) internally is $\left(\dfrac{lx_2 + mx_1}{l + m}, \dfrac{ly_2 + my_1}{l + m}\right)$ (ii) externally is $\left(\dfrac{lx_2 - mx_1}{l - m}, \dfrac{ly_2 - my_1}{l - m}\right)$

8. The midpoint of the line segment joining $A(x_1,y_1)$, $B(x_2,y_2)$ is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

9. If θ is the acute angle between the lines with slopes m_1 and m_2 then $\tan θ = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$
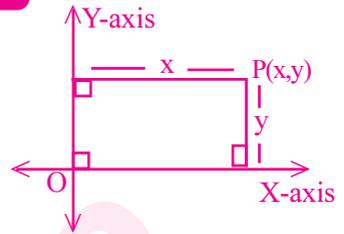
10. <u>**Collinearity of 3 points:**</u>

- Three points $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$ are collinear $\Leftrightarrow$ Area of ΔABC is zero

- Three points A,B,C in order are collinear $\Leftrightarrow AB + BC = AC$

- Slope of AB =Slope of BC =Slope of AC

- Equation of the line passing through any two points, say A, B satisfies the other point C.

11. **Equation of straight line in various forms**:

- **Slope form:** $y = mx$;   [slope is m, line passes through the origin O(0,0)]
- **Slope- intercept form 1:** $y = mx+c$;   [slope is m and Y-intercept is c]
- **Slope- intercept form 2:** $y = m(x-d)$ [slope is m and X-intercept is d]
- **Point - slope form:** $y - y_0 = m(x - x_0)$;   [Point is (x_0, y_0) and slope is m]
- **Two point form:** $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1} \right)(x - x_1)$;   [Two Points are (x_1, y_1) and (x_2, y_2)]
- **Intercept form:** $\dfrac{x}{a} + \dfrac{y}{b} = 1$ [x-intercept is a; y-intercept is b]
- Equation of the x−axis is $y = 0$ [∵ All y-coordinates on the x-axis are 0]
- Equation of the **horizontal line** passing through the point $A(x_1,y_1)$ is $y = y_1$.
- Equation of the y−axis is $x = 0$ [∵ All x-coordinates on the y-axis are 0]
- Equation of the **vertical line** passing through the point $A(x_1, y_1)$ is $x = x_1$.

12. The distance between the point $P(x_1, y_1)$ and the line $Ax+By+C=0$ is $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

13. The distance between the parallel lines $Ax + By + C_1 = 0$ & $Ax + By + C_2 = 0$ is $\dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

14. The point of intersection of the lines $a_1x+b_1y+c_1=0$; $a_2x+b_2y+c_2=0$ is $\left(\dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right)$

┤ **TIT BITS** ├

If the slope of a line is **m** then slope of its perpendicular line is $\dfrac{-1}{m}$ (i.e., Negative reciprocal)

Practice Test

Slopes of a few lines are given below. Write slopes of their perpendicular lines.

(i) 2   **(ii)** 1   **(iii)** $\dfrac{3}{2}$ **(iv)** $\dfrac{5}{12}$ **(v)** $-\dfrac{1}{5}$ **(vi)** $-\dfrac{2}{3}$ **(vii)** 7   **(viii)** $-\dfrac{5}{14}$ **(ix)** $-\dfrac{3}{2}$ **(x)** 0

Key: (i) $\dfrac{-1}{2}$ (ii) -1 (iii) $\dfrac{-2}{3}$ (iv) $\dfrac{-12}{5}$ (v) 5   (vi) $\dfrac{3}{2}$ (vii) $\dfrac{-1}{7}$ (viii) $\dfrac{14}{5}$ (ix) $\dfrac{2}{3}$ (x) undefined (or) ∞?

DEAR MATHS FACULTY!

- **Straight lines** chapter is the first chapter in Geometry. It is a concept based chapter. Students get confused if it is not introduced in a proper way.

- So, inorder to make smooth running of this chapter, for both lecturers and students aswell, all the jumbled and zigzag, misplaced problems are set properly in this material.

- All the Textual Problems are covered and are arranged in a smooth teaching method.

- Problems are arranged concept wise and order of difficulty level is maintained.

- Unimportant and Tough models are sent to the last exercise

   Here, is the order of concepts and contents for the four exercises 9(a), 9(b), 9(c), 9(d),

- **Exercise 9(a): i)** Point $P(x,y)$ **ii)** Distance between two points

   iii) Section Formula   **iv)** Area of triangle   **v)** Slope of a line joining two points

   vi) Condition for Parallelism and Perpendicularity

- **Exercise 9(b): Various forms of equation of straight line:**

   i) Equations of horizontal and vertical lines **ii)** Two point form **iii)** Point-Slope form

   iv(a) Slope-y- intercept form ($y=mx+c$)   **iv(b)** Slope-x- intercept form ($y=m(x–d)$)

   v) Intercept form **vi)** Parametric Equations   * Collinearity of three points

- **Exercise 9(c):**

   i) Distance of a point from a line   **ii)** Distance between parallel lines

   iii) Point of intersection of lines   **iv)** Angle between intersecting lines

   v) Ratio of a line dividing two points   **vi)** Foot of the perpendicular, Image

- **Exercise 9(d):** Miscellaneous models and Higher level problems.