8 MECHANICAL PROPERTIES OF SOLIDS

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M + 4M = 7 M]

CONCEPTS & DEFINITIONS

1.0 Consider two Balls, one is made fully with a 'Rubber material' and another with 'wet Clay'. When both the Balls are pressed and released for a moment, what change do we observe in their Shape and Size ? The 'Rubber ball' **regains** its original shape and size, where as the ball made with 'wet clay', **do not regain** its original shape and size. Here ,the Rubber ball is said to possess Elastic property and the other ball is said to possess Plastic property.

1.1 In this chapter we study (i) Elasticity (ii) Stress (iii) Strain (iv) Modulus of Elasticity - Types

2.1.1 Perfectly Rigid body : A body which does not under go any change in its Shape or Size under the action of any amount of external force is called a pefectly Rigid body.
In practice, it is not possible to have a perfectly rigid body.

2.1.2 Deforming force : A force which changes or tries to change ,the Shape or Size of a body (without moving it as a whole) is called deforming force.

2.2.1 Elasticity : The property of a body, by virtue of which, it regains its original Size and Shape when the deforming force is removed, is called Elasticity.
Note: Elasticity is a Molecular property of Matter.
In physics, Elasticity stands for 'opposition to change'. Thus, more rigidness leads to more Elastic nature. For this reason, Steel is more Elastic than Rubber.

2.2.2 Elastic body: A body which returns to its original size and shape after the removal of deforming force (when deformed with in the elastic limit) is called Elastic body.
Note1 : There is no perfect Elastic body.
Note2 : 'Quartz fibre' is a close approximation to perfectly Elastic body.

2.3.1 Plasticity : The property of a body by virtue of which, it does not regain its original size and shape on the removal of deforming force, is called Plasticity.

2.3.2 Plastic body : A body which does not regain its size and shape when the deforming force is removed is called a Plastic body. Ex : Wet Clay,Wax, chewing Gum

3.1 Stress : Stress is defined as the restoring force developed in the body per unit area

$$\text{Stress} = \frac{\text{restoring force}}{\text{Area}}, \quad \text{Stress} = \frac{F}{A}$$

SI unit: Nm^{-2} (or) Pascal (Pa) **Dimensional Formula** : $[M^1L^{-1}T^{-2}]$

Stress is of 3 Types :

1. **Longitudinal stress or Tensile stress :** The stress produced with in the body, when its length is changed by applying force normal to the surface of a body is **Longitudinal stress.**

2. **Bulk stress or Volume stress :** The stress produced with in the body ,when its volume is changed by applying force normally and uniformly all over its surfaces is called **Bulk stress.**

3. **Shear stress orTangential stress :** The stress produced with in the body ,when its shape is changed by applying tangential force on its surface is called **Shear stress.**

3.2 Elastic limit : The maximum value of stress with in which the body completely regain its original size and shape when the deforming force is removed is called Elastic limit.

Note: 1. For a perfectly plastic body, the elastic limit is very low.
2. For a perfectly elastic body, the elastic limit is very high.

4 Strain : The ratio of change in dimension of a body to the original dimension is called Strain.
Note: Being a Ratio of same quantities, Strain has no Units & no Dimensions

Strain is of 3 Types :

4.1 Longitudinal strain : Longitudinal strain is defined as, increase in length per unit original length, when deformed by an external force.

$$\text{longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l} = \frac{e}{l}$$

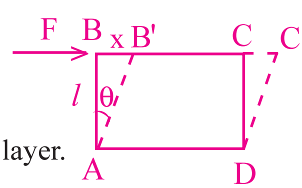
. Here e is extension and 'l' is original length.

4.2 Volume strain (or) Bulk strain : Volume strain is defined as, change in volume per unit original volume, when deformed by external forces.

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta \mathbf{V}}{\mathbf{V}}$$

4.3.1 Shear strain : Shear strain is defined as the angle θ (in radian), through which a line originally perpendicular to the fixed face gets turned on applying tangential deforming force.

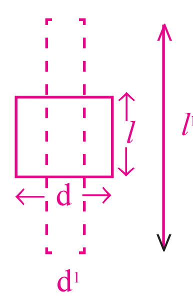
4.3.2 Angle of shear : The angle through which the reference line turns is called Angle of shear.
For solids, angle of shear is very small, so that in triangle ABB'.
So Shear strain may also be defined as the ratio of the lateral displacement of a layer to its perpendicular distance from the fixed layer.



4.4 Poisson's ratio(σ): Poisson's ratio is the ratio of lateral contractional strain to the longitudinal elongational strain.

$$\sigma = \frac{\text{contractional strain}}{\text{elongational strain}} = \frac{(d - d^1)/d}{(l^1 - l)/l}$$

$$\Rightarrow \quad \sigma = -\frac{\Delta d / d}{\Delta l \; / \, l}$$



Note1 : Poisson's ratio (σ) has no units.

Note2 : Theoretical limits of Poisson's ratio are –1 to 0.5

Note3 : Practical limits of Poisson's ratio are 0.2 to 0.4

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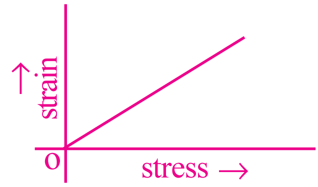
5 **Hooke's law:** With in the elastic limit, stress is directly proportional to Strain applied.
Thus, Stress $\propto$ Strain $\Rightarrow$ Stress = E (Strain).
Where E is proportionality constant called modulus of elasticity.

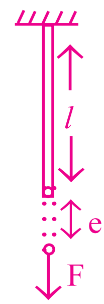


$$E = \cot\theta = \frac{OA}{AB} = \frac{Stress}{Strain}$$

$$\therefore E = \frac{Stress}{Strain}$$

6 **Modulus of Elasticity (E) :** The ratio of Stress and Strain is called Modulus of Elasticity of the material of a body.
Note1 : Modulus of elasticity depends only on the nature of material.
Note2 : SI unit of Modulus of elasticity is Nm^{-2} or Pascal.

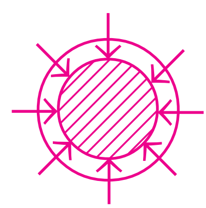
Types of modulus of Elasticity :



6.1 **Young's modulus (Y) :** With in the elastic limit,
Young's modulus is the ratio of longitudinal stress to the longitudinal strain.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{\mathbf{F/A}}{\mathbf{e/l}} = \frac{\mathbf{Fl}}{\mathbf{Ae}}$$

6.2 **Bulk modulus (K) :** With in the elastic limit,
Bulk modulus is the ratio of volume stress to volume strain



$$K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{-\Delta V/V}$$

As Stress increases , the Volume decreases.
That is why , the negative sign is attached.

$$K = \frac{-FV}{A\Delta V} \Rightarrow K = \frac{-PV}{\Delta V} \quad \text{Since, } \frac{F}{A} = P = \text{the pressure over the sphere}$$

Compressibility : The reciprocal of the bulk modulus of a material is called its compressibility.

$$\text{Compressibility} = \frac{1}{K}$$

SI unit of compressibility : $N^{-1}m^2$.; Dimensional Formula: $[M^{-1} L^1 T^2]$

It has negative dimension in mass.

6.3 **Rigidity modulus (η) :** With in the elastic limit, **Rigidity modulus** is the ratio of Tangential stress to the Shear strain

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{(F/A)}{\theta} = \frac{F}{A\theta} = \frac{F}{A\left(\frac{x}{l}\right)} = \frac{Fl}{Ax}$$

7. **Behaviour of a wire under gradually increasing load :**
The elastic behaviour of a material of a wire can be studied by plotting a curve between stress (on y-axis) and strain (on x-axis). The curve is called Stress - Strain curve. By using this curve we can study the behaviour of a wire under gradually increasing load.

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8. **Elastic fatigue :** When a wire is subjected to repeated number of stresses within the elastic limit, it temporarly looses its Elastic nature. This temporary loss of Elastic nature is called "elastic fatigue".

9. **Potential Energy :** The amount of workdone ,in deforming a body is called "Potential Energy" and the amount of workdone per unit volume, in deforming a body is called "strain energy".

Strain Energy $E = \frac{1}{2} \text{x stress x strain x Volume of the wire} = \frac{1}{2} Fe$

Strain Energy per unit volume $= \frac{1}{2} \times \text{Stress} \times \text{Strain}$

Imp. Formulae

1. **Stress** $= \frac{F}{A}$

2. **Longitudinal Strain** $= \frac{\Delta l}{l}$

3. **Young's Modulus** $Y = \frac{F/A}{\Delta l / l} \Rightarrow \frac{Fl}{A\Delta l} = \frac{(mg)l}{(\pi r^2)e} = \frac{gl}{\pi r^2}\left(\frac{M}{e}\right)$

4.1. **Bulk Modulus** $K = \frac{F/A}{-\Delta V/V} \Rightarrow \frac{-FV}{A\Delta V} \text{ (or) } \frac{-pV}{\Delta V}$

4.2. **Compressebility** $C = \frac{1}{K} = \frac{-\Delta V}{pV}$

5. **Rigidity Modulus** $\eta = \frac{F/A}{\theta} = \frac{F}{A\theta}$ where $\theta = \frac{\Delta l}{l}$

6. **Poisson's Ratio** $\sigma = \frac{-\Delta d / d}{\Delta l / l}$

7.1. **Strain Energy** $E = \frac{1}{2} \text{x stress x strain x Volume of the wire} = \frac{1}{2} Fe$

7.2. **Strain Energy per unit volume** $= \frac{1}{2} \times \text{Stress} \times \text{Strain}$

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