7 BINOMIAL THEOREM

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4 M = 7 M]

CONCEPTS & FORMULAS

1. Binomial theorem for Integral index n:

$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + \mathbf{{}^nC_ra^{n-r}b^r} + \ldots\ldots + {}^nC_nb^n,\ n\in N$

2. The **General term** of the binomial expansion is $T_{r+1} = {}^nC_ra^{n-r}b^r$.

3. Middle term(s):

(i) If n is even then the Middle term is $T_{\frac{n}{2}+1}$

(ii) If n is odd then the two Middle terms are $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$.

4. Standard Binomial expansion: $(1+a)^n = 1 + na + {}^nC_2a^2 + \ldots + {}^nC_ra^r + \ldots + a^n$

5. Binomial Coefficients: The coefficients ${}^nC_0, {}^nC_1, \ldots {}^nC_r \ldots {}^nC_n$ in the binomial expansion are called binomial coefficients and these are usually denoted by $C_0, C_1, C_2 \ldots C_r \ldots C_n$
Thus, $(1+a)^n = C_0 + C_1a + C_2a^2 + \ldots + C_na^n$

6. Illustrations:

$(a+b)^0 = 1$

$(a+b)^1 = a + b$

$(a+b)^2 = a^2 + 2ab + b^2$ $\quad [\because (a+b)^2 = {}^2C_0a^2 + {}^2C_1a^{2-1}b + {}^2C_2b^2]$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $\quad [\because (a+b)^3 = {}^3C_0a^3 + {}^3C_1a^{3-1}b + {}^3C_2a^{3-2}b^2 + {}^3C_3b^3]$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

7. Division Algorithm:

For two numbers a and b if we can find numbers q and r such that $a = bq + r,\ 0 \le r < b$ then we say that b divides a with q as quotient and r as remainder.

☞ b divides a $\Leftrightarrow$ a=bq for some integer q. [Here remainder r =0]

TIT BITS

Observations on the Binomial Expansion:

- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

i) The number of terms in the expansion is one more than the index. (Index=3, no.of terms=4)

ii) The powers of a decreases gradually by 1 while the powers of b increases by 1 .

iii) The sum of the powers of a and b in each term is equal to the index (3) of the binomial.

iv) The binomial coefficients of various terms are provided by the Pascal triangle pattern.