6 SYSTEMS OF PARTICLES & ROTATIONAL MOTION

1VSAQ + 2 SAQ [2M + 4M + 4M = 10 M]

CONCEPTS & DEFINITIONS

1. In this chapter we study about centre of mass, centre of gravity, motion of centre of mass, Vector product, Angular velocity, Angular acceleration, Torque, Angular momentum, Moment of inertia, Parallel axis theorem, Perpendicular axis theorem.

2. **Centre of mass:** The point at which the entire mass of the body is supposed to be concentrated is called centre of mass. Here, the body moves, as if the whole external force is acting at that point

   When a body is in complex motion, the point which undergo only translatory motion is known as centre of mass.

   Ex: Centre of mass of a triangular lamina is at its centroid G.

3. **Centre of Gravity:** The centre of gravity of a body is that point where the total gravitational torque on the body is zero.

4. **Motion of centre of mass :**

   Ex 1: The centre of mass of the fragments of an exploded projectile continues along the same parabolic path which it would have followed if there were no explosion.

   Ex 2: In case of a projected Indian club the particle at the centre of mass moves along a parabolic path but all other particles move in various complicated paths.

5. **Vector Product:** If $\overline{a}$ and $\overline{b}$ are two vectors which include an angle is θ then vector product of $\overline{a},\overline{b}$ is given by $\overline{a}\times\overline{b}=|\overline{a}||\overline{b}|\sin\theta\hat{n}$

   Here, $\hat{n}$ is the unit vector normal to the plane containing $\overline{a}$ and $\overline{b}$ and its direction is determined according to the right hand screw rule.

6. **Angular displacement (θ) :** The angle described by the radius vector at the centre, in a given interval of time is called Angular displacement.

   SI unit: radian.

   Dimensional Formula: $[M^0L^0T^0]$

7. **Angular velocity (ω):** The rate of angular displacement is called Angular velocity. $\omega=\dfrac{\theta}{t}$

   SI unit: rad s^{-1}.

   Dimensional Formula: $[M^0L^0T^{-1}]$

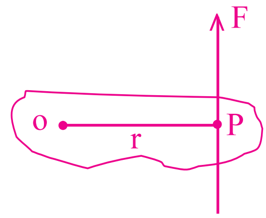
   Ex: Angular velocity of seconds hand of a watch, $\omega=\dfrac{2\pi}{60}\text{rad s}^{-1}\left[\because \omega=\dfrac{\theta}{t}=\dfrac{2\pi}{T}\right]$

8.0 **Angular acceleration (α) :** The rate of angular velocity is called Angular acceleration.

8.1 Torque / Moment of force :

"Torque is the turning effect of the force about a point".

It is the product of magnitude of the force and the perpendicular distance of the point from the line action of force.



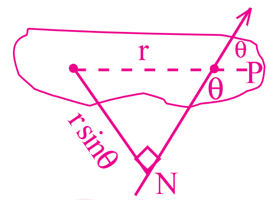
Torque τ = Force × perpendicular distance

$\tau = F\,r$

Note : Moment of force acting through the point N is

τ = Force × perpendicular distance

$= F \times r \sin\theta$

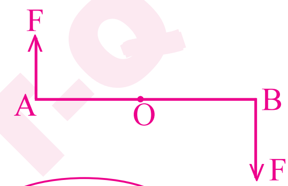


Vectorially, $\vec{\tau} = \vec{r} \times \vec{F}$

SI unit of moment of force : Nm

Dimensional formula: $[M^1L^2T^{-2}]$

8.2 Couple: Two equal and unlike parallel forces acting on a body at two different points in opposite directions constitute a couple. Couple produces rotation.

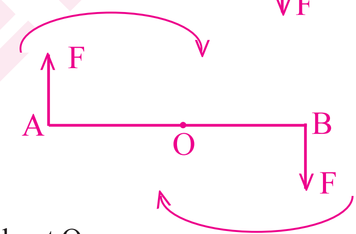


8.3 Moment of couple (Torque): The turning effect of a couple is given by the product of one of the forces and the perpendicular distance between the forces.

Moment of couple (C) $= (F \times OA) + (F \times OB)$

$= F(OA + OB)$

$C = F\,.\,AB$



AOB is called axis of rotation. O is Fulcrum if body is rotating about O

8.4 Moment of Inertia (I): The inherent property of rotation,which opposes change in rotation. The product of mass(M) of the particle and square of the distance between the particle and the axis of rotation is called moment of Inertia.

Formula : $I = Mr^2$; S.I unit : Kg m^2; D.F : $M^1L^2T^0$

Moment of Inertia depends on
- i) the mass of the body
- ii) the position of the axis of rotation
- iii) the distribution of the mass of the body about the axis.

9.0 Parallel axes theorem: The moment of inertia of a rigid body about an axis passing through a point is equal to the sum of moment of inertia about a parallel axis passing through its centre of mass and product of the mass of the body and the square of the perpendicular distance between the axes.

Formula : $I = I_G + Mx^2$

9.1 Perpendicular axes theorem: The moment of inertia of a plane lamina about an axis perpendicular to its plane and passing through a point is equal to the sum of its moment of inertia about any two mutually perpendicular axes in that plane passing through that point.

Formula : $I_z = I_x + I_y$

10.1 Angular momentum (L) : The moment of momentum of a body is angular momentum. The product of moment of inertia of a body and its angular velocity is angular momentum.

Formula : $L = I\,\omega = mvr$

10.2 Law of conservation of angular momentum : The angular momentum of a system always remains constant, when no external torque act on the system.

Thus, $I\,\omega$ = constant Hence, $I_1\omega_1 = I_2\,\omega_2$

13.1 Moment of Inertia of different bodies about different Axes :

	Name of the body	Axis of rotation	Moment of Inertia
1.	**Circular ring of radius R** [Mass: M; Radius:R]	(i) Axis passing through its centre and perpendicular to its plane	MR^2
		(ii) About any diameter	$\frac{MR^2}{2}$
		(iii) A tangent perpendicular to its plane	$2MR^2$
		(iv) A tangent in its plane	$\frac{3MR^2}{2}$
2.	**Thin circular disc of radius R** [Mass: M; Radius:R]	(i) Axis passing through its centre and perpendicular to its plane	$\frac{MR^2}{2}$
		(ii) About any diameter	$\frac{MR^2}{4}$
		(iii) A tangent perpendicular to its plane	$\frac{3MR^2}{2}$
		(iv) A tangent in its plane	$\frac{5MR^2}{4}$
3.	**Hollow sphere** [Mass: M; Radius:R]	About its diameter	$\frac{2}{3}MR^2$
4.	**Solid Sphere** [Mass: M; Radius:R]	(i) About its diameter	$\frac{2}{5}MR^2$
		(ii) About a tangent to the surface	$\frac{7}{5}MR^2$
5.	**Hollow cylinder (tube) [Mass:M,Radius:R,Length:L]**	About its geometrical axis	MR^2
6.	**Solid cylinder** [Mass: M; Radius:R]	(i) About its geometrical axis	$\frac{MR^2}{2}$
		(ii) **Axis** passing through its centre and perpendicular to its own axis	$M\left[\frac{l^2}{12}+\frac{R^2}{2}\right]$
7.	**Thin uniform rod** of length L [Mass: M; Radius:R, Length :L]	(i) **Axis** passing through centre and perpendicular to the rod	$\frac{ML^2}{12}$
		(ii) About its end and perpendicular to the rod	$\frac{ML^2}{3}$
8.	**Rectangular plate or bar** [Mass: M; Length :L, Breadth:b]	Axis passing through its centre and perpendicular to its plane	$M\left[\frac{l^2+b^2}{12}\right]$

13.2 Comparison between Linear and Rotational Motion:

Linear Motion	Rotational Motion
1. Displacement x	**1.** Angular displacement θ
2. Velocity $v = \frac{ds}{dt}$	**2.** Angular velocity $\omega = \frac{d\theta}{dt}$
3. Acceleration $a = \frac{dv}{dt}$	**3.** Angular acceleration $\alpha = \frac{d\omega}{dt}$
4. Mass: $m=F/a$	**4.** Moment of inertia: $I = MK^2$
5. Force $F = ma = \frac{dp}{dt}$	**5.** Torque $\tau = I\alpha = \frac{dL}{dt}$
6. Work $W = FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	**6.** Work $W = \tau\theta = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$
7. Kinetic energy $K.E_T = \frac{1}{2}mv^2$	**7.** Kinetic energy $K.E_R = \frac{1}{2}I\omega^2$
8. Power $P = Fv$	**8.** Power $P = \tau\omega$
9. Linear momentum $p = mv$	**9.** Angular momentum $L = I\omega$

Imp.Formulae

1. $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$, $y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + ...}{m_1 + m_2 + m_3 + ...}$

2.1 $v_{cm} = \frac{m_1v_1 + m_2v_2 + m_3v_3 + ...}{m_1 + m_2 + m_3 + ...}$ 2.2. $a_{cm} = \frac{m_1a_1 + m_2a_2 + m_3a_3 + ...}{m_1 + m_2 + m_3 + ...}$

3. Vector Product of $\bar{a}, \bar{b}$ is $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin\theta\hat{n}$

4.1 Angular velocity (ω): $\omega = \frac{d\theta}{dt}$ 4.2. Angular acceleration $\alpha = \frac{d\omega}{dt}$

5. Moment of Inertia $I = Mk^2$

6. Torque (or) Moment of Force: $\bar{\tau} = \bar{r} \times \bar{F} = Fr\sin\theta = I\alpha$

7. Angular Momentum $L = mvr = mr^2\omega = I\omega = \bar{r} \times \bar{p}$

8. Relation between Torque and Angular Momentum $\tau = \frac{dL}{dt}$

9. Rotational Kinetic Energy $K.E_R = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

10. Parallel axes theorem: $I = I_G + Mx^2$

11. Perpendicular axes theorem : $I_z = I_x + I_y$

12. Law of Conservation of Angular Momentum : $I_1\omega_1 = I_2\omega_2$