6 PERMUTATIONS AND COMBINATIONS

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M +4M = 7 M]

CONCEPTS & FORMULAS

1) **Multiplication Principle / Fundamental Principle of Counting /Counting without counting :**
If an event A can occur in **m** different ways, following which another event B can occur in **n** different ways, then the number of occurences of both the events A and B one after another is **m×n.**
Ex 1: In a class room, there are benches in 6 rows and each row has two benches.
Then by the Multiplication principle, the total number of benches = 6×2=12.
Ex 2: In a college assembly, all the students stand in 10 lines with 40 in each line.
Then the total number of students in the college = 10×40=400.
Ex 3: A boy has 5 phants and 6 shirts. Then by the Multiplication principle, the number of ways that he can make up with different possible dresses = 5×6=30.

2) **Addition principle:** If an event A can occur in **m** ways, another event B which is independent of A can occur in **n** ways, then the number of occurrences of atleast one of A,B (i.e., A or B) is (m+n).
Ex: Let a question A can be solved in 2 ways (say, a_1, a_2) and another question B can be solved in 3 ways (say, b_1, b_2, b_3) then the total number of ways of solving either A or B is 2+3=5 ways i.e., a_1,a_2,b_1,b_2,b_3.
Here, we note that both the questions i.e.,A and B can be solved in 2×3= 6 different ways.
They are a_1b_1, a_1b_2, a_1b_3, a_2b_1, a_2b_2, a_2b_3.

3) **Permutations:** Suppose we have a **finite number of n things** and we take a few things say r from the given n things and arrange them in all possible ways. Then each such an arrangement is called a permutation. In a permutation order has importance.
Ex : We are given 3 numbers 1,2,3. Then all possible **2 digit permutations** that can be formed from 1,2,3 are (1,2), (1,3), (2,3), (2,1), (3,1),(3,2). (6 in total)
Note 1: In the given set 1,2,3 all are different/dissimilar.
Note 2: In each arrangement order is given importance. Hence 1,2 and 2,1 are treated differently.
Note 3: In any arrangement, the repetition is not allowed. Hence 11, 22, 33 are not admitted.
Permutation Formula: The number of permutations of **n** dissimilar things taken **r** at a time is

$${}^nP_r = \frac{n!}{(n-r)!},\ r\le n,\ n\ne 0,\ r=0,1,2,\ldots n \qquad \text{(or)} \qquad {}^nP_r = n(n-1)(n-2)\ldots\ldots r \text{ terms.}$$

Example 1: ${}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times\cancel{2\times1}}{\cancel{2\times1}} = 360$ (or) ${}^6P_4=6\times5\times4\times3=360$

Practice Q's: Find the values of **(i)** ${}^4P_2, {}^4P_3, {}^4P_4$ **(ii)** ${}^5P_5, {}^5P_4, {}^5P_3, {}^5P_2, {}^5P_1, {}^5P_0$
(iii) ${}^6P_1, {}^6P_2, {}^6P_3, {}^6P_4, {}^6P_5$ **(iv)** ${}^{100}P_2, {}^{100}P_1$

Example 2: Number of **5 letter words** that can be formed from the letters of the word MATHS is ${}^5P_5$ =5! =5×4×3×2×1=120. [MATHS, MATSH, TAMHS,....120 words]

4) Permutations of a few alike things: The number of permutations of 'n' things, in which there are p alike things of one kind, q alike things of the 2^{nd} kind, r alike things of the 3^{rd} kind and the rest are different is $\frac{n!}{p!q!r!}$.

Example : Number of 5 letter words that can be formed from the letters of the word INDIA is $\frac{n!}{p!q!r!} = \frac{5!}{2!} = \frac{120}{2} = 60$ [INDIA, IINDA, IDNAI, NDAII,....60 words]

5) Combination: It is simply a 'selection' of some things from a finite number of things.
Order makes the key difference between a permutation and a combination.
Just like elements in a set, in a combination the order of arrangement is ignored.
Hence, in a permutation (1,2) and (2,1) are treated as two different arrangements, whereas in a combination these both are treated as the same because order has no importance.
Ex: We are given 3 numbers 1,2,3. Then all possible **2 digit combinations** that can be formed from 1,2,3 are {1,2}, {1,3}, {2,3} (3 in total)

- **Combination Formula:** The number of combinations of **n** dissimilar things taken **r** at a time is

$$^nC_r = \frac{n!}{r!(n-r)!} \text{ (or) } ^nC_r = \frac{^np_r}{r!} \text{ (or) } ^nC_r = \frac{n(n-1)(n-2)\text{.......r terms}}{r!}$$

Ex 1: $^6C_4 = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!} = \frac{6\times5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}\times2\times1} = 15$ (or) $^6C_4 = \frac{6\times5\times\cancel{4\times3}}{\cancel{4\times3}\times2\times1} = 15$

Practice Q's: Find the values of **(i)** $^4C_2, ^4C_3, ^4C_4$ **(ii)** $^5C_5, ^5C_4, ^5C_3, ^5C_2, ^5C_1, ^5C_0$
(iii) $^6C_1, ^6C_2, ^6C_3, ^6C_4, ^6C_5$ **(iv)** $^{100}C_2, ^{100}C_1$

Ex 2: Let A = { a,b,c,d}. Then the number of two element subsets from A = $^4C_2$=6.
They are {a,b},{a,c},{a,d},{b,c},{b,d},{c,d}.

Important results on Combinations:

- $^nC_r = {^nC_{n-r}}$ **Ex:** $^{10}C_3 = {^{10}C_7}$; $^5C_3 = {^5C_2}$; $^4C_3 = {^4C_1}$; $^3C_3 = {^3C_0}$
- $^nC_0 = 1$; $^nC_n = 1$ **Ex:** $^{10}C_0 = 1$; $^{10}C_{10} = 1$.;
- $^nC_1 = n$; $^nC_{n-1} = n$ **Ex:** $^{10}C_1 = 10$, $^{10}C_9 = 10$

* If $^nC_r = {^nC_s}$ then r=s **(or)** r+s=n

Ex 1: If $^nC_4 = {^nC_5}$ then n=4+5=9. Here we have $^9C_4 = {^9C_5}$

Ex 2: If $^{15}C_r = {^{15}C_{2r}}$ then r+2r=15 ⇒ 3r=15⇒r=5. Here we have $^{15}C_5 = {^{15}C_{10}}$

☞ **Key words in combinations:** Selection, choosing, committees, sets, groups, teams.

TIT BITS

- 0! =1, 1! =1, 2! =2, 3! =6 • 4! =24, 5! =120, 6! =720
- $^nP_r = n(n-1)$..... r terms. • $^nP_0 = 1$; • $^nP_1 = n$ • $^nP_{n-1} = n!$ • $^nP_n = n!$
- $^nC_0 = 1$ • $^nC_1 = n$ • $^nC_{n-1} = n$ • $^nC_n = 1$ • $^nC_r = \frac{^np_r}{r!}$ hence $\frac{^nP_r}{^nC_r} = r!$