

# 5 WORK, ENERGY AND POWER

1 VSAQ + 1 LAQ [ 2M + 8M = 10 M]

## CONCEPTS & DEFINITIONS

1 In this chapter we study about (i) Scalar Product (ii) Work (iii) K.E and P.E (iv) Work energy Theorem (v) Law of conservation of energy (vi) Power (vii) Collisions

2.1 **Scalar product** : The scalar product of two vectors  $\vec{a}, \vec{b}$  is defined as  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$   
Here,  $\theta$  is the angle between the vectors  $\vec{a}, \vec{b}$ .

2.2 If  $\theta$  is the angle between the vectors  $\vec{a}, \vec{b}$  then  $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$

2.3 If  $\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$  and  $\vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$  then  $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

2.3 If  $\vec{a}$  is perpendicular to  $\vec{b}$  then  $\vec{a} \cdot \vec{b} = 0 \Rightarrow a_x b_x + a_y b_y + a_z b_z = 0$

2.4  $\vec{i} \cdot \vec{j} = 0, \vec{j} \cdot \vec{k} = 0, \vec{k} \cdot \vec{i} = 0$  (Since  $\cos 90^\circ = 0$ )  
Also  $\vec{i} \cdot \vec{i} = 1, \vec{j} \cdot \vec{j} = 1, \vec{k} \cdot \vec{k} = 1$  (Since  $\cos 0 = 1$ ).

3. **Work (W)** : Work is said to be done, when a force acting on a body moves it, through some distance in the direction of the force.

If a force  $F$  acts on a body along a direction making an angle  $\theta$  with the direction of displacement, then the workdone  $W = (F \cos \theta) S$

$$W = FS \cos \theta$$

In vector form,  $\mathbf{W} = \vec{F} \cdot \vec{S}$

**Note** : If  $\theta = 0^\circ$  then  $\cos 0 = 1 \Rightarrow W = FS$ .

Also, if  $\theta = 90^\circ$  then  $\cos 90^\circ = 0 \Rightarrow W = 0$

**Units of work** :

**One joule**: Work done on a body is said to be one joule, when a force 1N displaces it through 1m in the direction of the force.

SI unit : **joule (J)**;  $1 \text{ J} = 1 \text{ N} \times 1 \text{ m}$

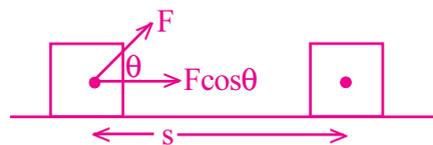
**One erg**: Work done on a body is said to be one erg, when a force 1 dyne displaces it through 1cm in the direction of the force.

CGS unit: **erg** ;  $1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm}$

**Conversions** :  $1 \text{ joule} = 10^7 \text{ erg}$ .

**Dimensional formula** :  $[M^1 L^2 T^{-2}]$

**Note**: Work done is a scalar.



### 'Your Power'

Suppose **YOU** go to your college on a bicycle. Initially, your bicycle is with **INERTIA**.

To make your bicycle move, you need to apply some **FORCE** on the pedals.

As a result, your bicycle starts moving and you acquire some **MOMENTUM**.

To achieve this, you did some **Work**. For this, you need to spend some **ENERGY**.

And 'how fast' you ride your bicycle is your **POWER**.

**Nature of workdone in various situations :** Depending on the angle between  $\vec{F}$  and  $\vec{S}$ , workdone can be positive, negative or zero.

(i) If the angle between  $\vec{F}$  and  $\vec{S}$  is **acute**, then workdone is **positive**.

**Ex:** 1) When a body falls freely under gravitational pull, the workdone by the gravity is positive.

2) When a horse pulls a cart on a level road, the workdone by the pulling force is positive.

(ii) If the angle between  $\vec{F}$  and  $\vec{S}$  is **obtuse**, then work done is **negative**.

**Ex :** 1) When a body is made to slide over a rough surface, the workdone by frictional force is negative.

2) When a positive charge is moved towards another positive charge, the workdone by electrostatic force between them is negative.

(iii) If the angle between  $\vec{F}$  and  $\vec{S}$  is **90°**, then workdone is **zero**.

Workdone will also be zero when either  $\vec{F}$  or  $\vec{S}$  or both  $\vec{F}$  and  $\vec{S}$  are zero.

**Ex:** 1) When a coolie travels on a platform with a load on his head, workdone by the coolie is zero.

2) When a body is moved along a circular path with the help of a string, the workdone by tension in the string is zero.

#### 4. ENERGY : The capacity to do work is called Energy.

Energy is a scalar quantity.

It is measured in the same units as work.

**Units of Energy :**

SI unit : **Joule**.

CGS unit : **erg**.

**Dimensional formula :**  $[M^1L^2T^{-2}]$

Mechanical energy is of two types 1. Potential energy (P.E or U) 2. Kinetic energy ( K.E or K)

#### 4.1 Potential energy (U) : It is the energy possessed by a body by virtue of its position.

**Ex :** A 'stretched bow', water in a reservoir ,a compressed spring

**Formula :** P.E =  $mgh$  or  $U = mgh$

#### 4.2 Kinetic Energy : It is the energy possessed by a body by virtue of its motion.

**Ex :** A bullet fired from a gun,a train in motion ,a ball in motion possess KE

**Formula :** Kinetic energy  $K.E = \frac{1}{2}mv^2$  (or)  $K = \frac{1}{2}mv^2$

#### 4.3 Relation between Kinetic energy (K) and Linear momentum (p) :

We know that  $K = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 \frac{m}{m} = \frac{1}{2} \frac{m^2v^2}{m} = \frac{1}{2} \frac{(mv)^2}{m} = \frac{1}{2} \frac{p^2}{m}$  ( $\because p = mv$ )  $\therefore K = \frac{p^2}{2m}$

#### 4.4 Work-Energy theorem: "Workdone by a constant resultant force acting on a body is equal to change in its kinetic energy".

$$W = \vec{F} \cdot \vec{S} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

**5. Various forms of energy:**

(i) Heat (ii) Chemical energy (iii) Electrical energy (iv) nuclear energy.

**6. The principle of conservation of energy :**

“If the internal forces of a system are conservative, then the total mechanical energy (PE+KE) of the system always remains constant”. Thus, in a conservative system **PE + KE = constant**

**7 Power (P):** The **rate of doing work** (transferring energy) is called power. Power (P) =  $\frac{\text{work}}{\text{time}}$ 

**Instantaneous power:** It is measured as the dot product of the instantaneous velocity and the

force acting on it at that instant  $P = \frac{W}{t} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{V} = FV \cos \theta$

Hence, power is a scalar, which is equal to the dot product of force and velocity.

**Units of Power :**

**Watt :** The power of an agent is said to be 1 watt, when 1Joule of work is done in 1second.

**SI unit :** watt (W) (or) joule/sec

**CGS unit :** erg / sec

**Conversions:**

1 watt =  $10^7$  erg/sec

1 Horse - power (1HP) = 746 W, (This unit is used in Automobile-Engineering calculations)

1 kilo watt (KW) = 1000 W

1 mega-watt (MW) =  $10^6$  W

**Dimensional formula :** D.F of power =  $[ML^2T^{-3}]$

**8.0 Collisions:**

If two bodies collide in a short time interval then

(i) Velocities of colliding bodies change

(ii) Total Momentum will be conserved

(iii) Total Kinetic energy will be changed or conserved

**Examples :** Hitting a cricket ball with a bat, collision between a Striker and a coin in the Carom board, collisions of Gas molecules, collision of  $\alpha$ -particle with Gold nucleus in Rutherford's Scattering experiment

**8.1 Types of collisions :** There are 2 types of collisions.

They are **(i) Elastic collision** **(ii) Inelastic collision (Perfect, Partial)**

**8.2 Elastic collision :** The collision in which both Momentum and Kinetic energy are conserved, is known as elastic collision.

**Ex :** Collision between 2 smooth Billiard balls, collisions between gas molecules

**8.3 Inelastic collision:** The collision in which only Momentum is conserved but not kinetic energy, is called as inelastic collision.

**Ex: Hitting** a ball with a bat, collision between a bullet and its target, Rebounds of a ball dropped on the floor

**9. One dimensional elastic collisions:** If the **velocities** of colliding bodies, are confined along a straight line, **before and after** the collision, then such collisions are called one dimensional collisions (or) head -on collisions.

**Imp. Formulae**

$$1.1 \quad \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$1.2. \quad \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$2.1 \quad \text{Work done } W = \vec{F} \cdot \vec{S} = FS \cos \theta$$

$$2.2 \quad \text{Workdone in lifting a body of mass 'm' to a height 'h' is } W = PE = mgh$$

$$3. \quad \text{Kinetic Energy K.E} = \frac{1}{2} mv^2$$

$$4. \quad \text{Potential energy P.E} = mgh$$

$$5. \quad \text{Relation between K.E & momentum P is K.E} = \frac{p^2}{2m}$$

$$6. \quad \text{Work energy theorem: } W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

$$7. \quad \text{Law of conservation of energy: K.E+P.E} = \text{constant.}$$

$$8. \quad \text{Power } P = \frac{w}{t} \text{ or } \frac{mgh}{t} \text{ or } \frac{\frac{1}{2}mv^2}{t}$$

$$9. \quad \text{Coefficient of restitution } e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

10. i) For an elastic collision  $e = 1$   
 ii) for partial inelastic collision  $e < 1$   
 iii) for perfectly inelastic collision  $e = 0$