4 COMPLEX NUMBERS

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M + 4 M = 7 M]

CONCEPTS & FORMULAS

1) Complex Number: It is a number of the form z= a+ib, where a,b are real numbers, $i=\sqrt{-1}$.

Here, a is called the real part of z, denoted by Rez and b is called the imaginary part,Imz.

- $z = a + ib$ where $a, b \in R$, $i=\sqrt{-1}$ i.e., $i^2=-1$

Example : Let **$z = 1 + 2i$** then **Rez = 1** and **Imz = 2**;

Note 1: Integral powers of i: $i=\sqrt{-1} \Rightarrow i^2=-1,\ i^3=i^2(i)=-1(i)=-i,\ i^4=(i^2)^2=(-1)^2=1$

- $i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i;\quad i^4 = 1, i^5 = i, i^6 = -1, i^7 = -i;\quad i^8 = 1$.....
- For any Integer k, $i^{4k} = 1;\ i^{4k+1} = i;\ i^{4k+2} = -1;\ i^{4k+3} = -i$

Note 2: For $a \in R^+$, $\sqrt{-a} = i\sqrt{a}$, **Ex:** $\sqrt{-5}=\sqrt{-1}\sqrt{5}=i\sqrt{5}$, $\sqrt{-4}=\sqrt{-1}\sqrt{4}=2i$

Note 3: $\sqrt{-a}.\sqrt{-b}=i\sqrt{a}.i\sqrt{b}=i^2\sqrt{ab}=-\sqrt{ab}$

2) Equality of complex numbers: $a + ib = c + id \Leftrightarrow a = c$ and $b = d$

If two complex numbers are equal then their respective 'real parts' & 'imaginary parts' are equal.

3) FOUR FUNDAMENTAL OPERATIONS OF COMPLEX NUMBERS:

3.1) ADDITION: If $z_1 = a + ib$, $z_2 = c + id$ then $z_1 + z_2 = (a + c) + i(b + d)$

Example : If $z_1 = 1 + 2i$, $z_2 = 3 + 4i$ then $z_1 + z_2 = (1 + 3) + i(2 + 4) = 4 + 6i$

3.2) SUBTRACTION: If $z_1 = a + ib$, $z_2 = c + id$ then $z_1 - z_2 = (a - c) + i(b - d)$

Example : If $z_1 = 2 - i$, $z_2 = 6+3i$ then $z_1 - z_2 = (2 - 6) + i(-1 - 3) = -4 - 4i$

3.3) MULTIPLICATION:

If $z_1 = a + ib$, $z_2 = c + id$ then $z_1z_2 = (a + ib)(c + id) = (ac - bd) + i(bc + ad)$;

$(a + ib)(c + id) = ac + a(id) + (ib)c + (ib)(id) = ac + i(ad + bc) + (i^2)bd$

$= ac + i(ad + bc) - bd = (ac - bd) + i(ad + bc)$

Example : **(1 + 2i) (3 + 4i)** $= 1(3) + 1(4i) + (2i)3 + (2i)(4i) = 3 + 4i + 6i + (i^2)8$

$= 3 + 10i + (-1)8 = 3 + 10i - 8 = -5 + 10i$

Note 1: $(a + ib)(c - id) = (ac + bd) + i(bc - ad)$;

Note 2.1: $(a+ib)^2=(a^2-b^2)+i(2ab)$; **Note 2.2:** $(a-ib)^2=(a^2-b^2)-i(2ab)$

Note 3: $(a+ib)(a-ib)=a^2+b^2$

Tip Top Chit Bits

- $(-1)^{even} = +ve$; **Ex:** $(-1)^2=1$; $(-2)^4=4$
- $(-1)^{odd} = -ve$; **Ex:** $(-1)^1 = -1$; $(-2)^3 = -8$
- In real numbers $(a + b)(a - b) = a^2 - b^2$
- In complex numbers $(a+ib)(a-ib)=a^2+b^2$

3.4) DIVISION : If $z_1 = a + ib$, $z_2 = c + id$ then

$$\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \left(\frac{ac+bd}{c^2+d^2}\right) + i\left(\frac{bc-ad}{c^2+d^2}\right)$$

Example: If $z_1 = 2 + 3i$, $z_2 = 4 + 5i$ then find z_1 / z_2.

Solution: $$\frac{z_1}{z_2} = \frac{2+3i}{4+5i} = \frac{(2+3i)(4-5i)}{(4+5i)(4-5i)} = \left(\frac{2(4)+3(5)}{4^2+5^2}\right) + i\left(\frac{3(4)-2(5)}{4^2+5^2}\right) = \frac{23}{41} + i\frac{2}{41}$$

4) MULTIPLICATIVE INVERSE:

If $z = a + ib$ then its multiplicative inverse is $z^{-1} = \frac{1}{a+ib} = \frac{a-ib}{a^2+b^2}$

Example: The multiplicative inverse of 7 + 24i is $\frac{7-24i}{7^2+24^2} = \frac{7}{625} - \frac{24i}{625}$

$$\frac{1}{7+24i} = \frac{1(7-24i)}{(7+24i)(7-24i)} = \frac{7-24i}{7^2+24^2} = \frac{7-24i}{49+576} = \frac{7-24i}{625} = \frac{7}{625} - \frac{24i}{625}$$

5) MODULUS & CONJUGATE OF A COMPLEX NUMBER:

For the complex number $z = a + ib$,

(i) Modulus is $|z| = |a+ib| = \sqrt{a^2+b^2}$ **(ii) Conjugate** is $\bar{z} = a - ib$

To write the conjugate of a complex number, we have to **change the sign of imaginary part.**

Example: Write the modulus & conjugate of (i) $z_1 = 2 + 3i$ (ii) $z_2 = -3 + 4i$.

Solution: (i) $|z_1| = |2+3i| = \sqrt{2^2+3^2} = \sqrt{13}$. Conjugate of $z_1 = 2+3i$ is $\bar{z}_1 = 2-3i$;
(ii) $|z_2| = |(-3)+4i| = \sqrt{(-3)^2+4^2} = 5$. Conjugate of $z_2 = -3+4i$ is $\bar{z}_2 = -3-4i$

6) Properties of Modulus & Conjugate:

(i) $|z_1 z_2| = |z_1|\,|z_2|$ (ii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $|z_2| \neq 0$ (iii) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ (iv) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

7) Quadratic Equation($ax^2+bx+c=0$):

The **roots** of $ax^2+bx+c=0$ are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, $a \neq 0$

7.1) The **sum** of the roots of $ax^2+bx+c=0$ is $\alpha + \beta = \frac{-b}{a} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

7.2) The **product** of the roots of $ax^2+bx+c=0$ is $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Tip Top Chit Bits

Steps in the Division of Complex numbers : • Rationalise the denominator. • Multiply the Nr. & Dr. with RF of Dr.

• RF of a+ib is a–ib • RF of a–ib is a+ib • In Dr, $(a+ib)(a-ib) = a^2+b^2$

• $(a+b)^2 = (a-b)^2 + 4ab$ • $(a-b)^2 + 4ab = (a+b)^2$

• $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ • $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$