3 MOTION IN A PLANE

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M + 4M = 7 M]

CONCEPTS & DEFINITIONS

1.0 In this chapter we study the motion of objects in a plane (2D). For this we need to understand the concepts of vectors.

1.1 The main contents of this chapter are (i) Scalars and vectors (ii) Motion in a plane (iii) Projectile motion (iv) Uniform circular motion

2.1 **Scalar :** A physical quantity which has only magnitude but no direction, is called a scalar.
Ex : Length, Mass, Time, Distance etc.

2.2 **Vector :** A physical quantity which has both magnitude and direction and obeys law of vector addition is called a vector.
Ex : Force, Displacement, Velocity, Accleration,Momentum etc

3 Geometrical representation of vectors :

Geometrically, a vector is represented by a **directed line segment** :

If the vector $\overline{AB}$ is denoted by $\bar{a}$ then

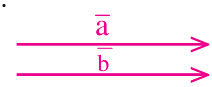
(i) A is called the initial point of the vector $\bar{a}$

(ii) B is called the terminal point of the vector $\bar{a}$

(iii)The magnitude of $\bar{a}$ is the length AB.It is written as $|\overline{AB}|$ or AB or $|\bar{a}|$

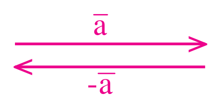
(iv)The direction of $\bar{a}$ is from the initial point A to the terminal point B.

4. Types of vectors :



4.1 Equality of vectors:
Two vectors are said to be equal if they have the same magnitude & direction.
The units of equal vectors are also equal.

4.2 Negative of a vector :



Let $\bar{a}$ be a vector then the vector having magnitude as that of $\bar{a}$ and direction opposite to that of $\bar{a}$ is called negative of $\bar{a}$ and it is denoted by $-\bar{a}$.

4.3 Zero vector (Null vector):

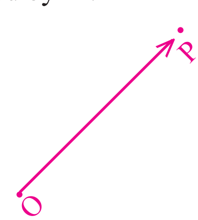
A vector of zero magnitude and arbitrary direction is called a zero vector, it is denoted by $\bar{0}$ or **O**.

4.4 Unit vector:
A vector of magnitude one unit is called a unit vector.

If $\bar{a}$ is a non-zero vector then the unit vector in the direction of $\bar{a}$ is denoted by $\hat{a}$.

The unit vector in the direction of the non-zero vector $\bar{a}$ is $\hat{a} = \dfrac{\bar{a}}{|\bar{a}|}$

4.5 Position vector :
If O is a point of reference and P is a point in space then the vector $\overline{OP}$ is called the position vector of P with reference to O.



5.1 Addition of vectors :

To add two vectors $\bar{a}, \bar{b}$ geometrically, the arrows are drawn to a scale with their directions. Next, keep the initial point of the second vector $\bar{b}$ at the final point of the first vector $\bar{a}$. Then draw an arrow from the initial point of $\bar{a}$ to the final point of the second vector $\bar{b}$. This new arrow represents the resultant vector $\bar{c}$, which is the sum of the two vectors.

5.2 Laws of vector addition :

(i) Vector addition obeys Commutative Law : $\vec{a}+\vec{b}=\vec{b}+\vec{a}$

(ii) Vector addition obeys Associative Law : $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$

(iii) Vector addition obeys Distributive Law : $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$, where k is scalar.

If k_1 and k_2 are two scalars then $(k_1+k_2)\vec{a}=k_1\vec{a}+k_2\vec{a}$

6.1 Resolution of a vector in a plane into two rectangular components:

Let a vector $\bar{A}$ makes an angle θ with the X-axis.

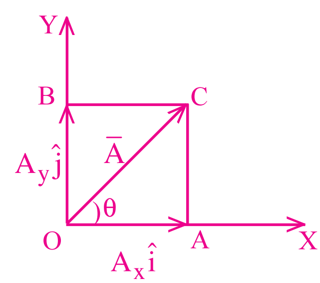
This Vector $\bar{A}$ can be resolved into two rectangular components, which are at right angles to each other.

If A_x, A_y are the components of the vector $\bar{A}$ along X, Y directions then vector $\bar{A}$ can be represented as

$\bar{A}=A_x\bar{i}+A_y\bar{j}$.

Here, Horizontal component of $\bar{A}$ is $A_x = A\cos\theta$.

Vertical component of $\bar{A}$ is $A_y = A\sin\theta$.



6.2 Resolution of a vector in space into three rectangular components:

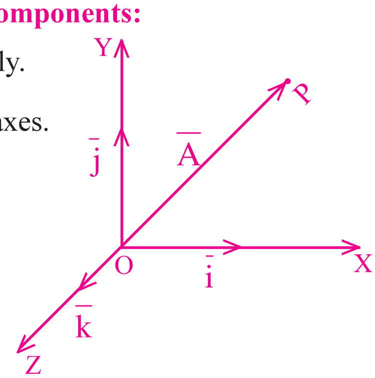
The unit vectors along X, Y, Z -axes are $\bar{i}$, $\bar{j}$, $\bar{k}$ respectively.

Vector $\bar{A}$ can be resolved into 3 components along X, Y, Z axes.

If A_x, A_y and A_z are the components of the vector $\bar{A}$ along X, Y, Z directions then vector $\bar{A}$ can be represented as

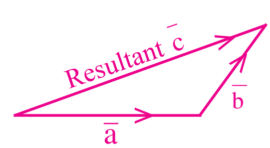
$\bar{A}=A_x\bar{i}+A_y\bar{j}+A_z\bar{k}$.

Magnitude of the vector $\bar{A}$ is $|\bar{A}|=\sqrt{A_x^2+A_y^2+A_z^2}$



7. Triangle law of vectors :

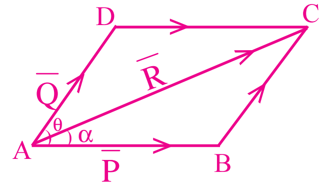
If two vectors are represented in magnitude and direction by the two sides of a triangle taken in an order, then their resultant is represented in magnitude and direction by the third side taken in the reverse order.



8. Parallelogram law of vectors : If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented in magnitude and direction by the diagonal passing through the same point.

Let θ be the angle between the vectors $\overline{P}, \overline{Q}$ and $\overline{R}$ be the resultant vector.

Let the resultant vector $\overline{R}$ makes an angle α with $\overline{P}$ then



(i) $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ (ii) $\alpha = \text{Tan}^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

i) If $\vec{P}$ and $\vec{Q}$ are parallel to each other then (i) $R = \sqrt{P^2 + Q^2 + 2PQ} = P + Q$ (ii) $\alpha = 0^0$.

ii) If the angle between the two vectors θ =180⁰ then (i) $R = \sqrt{P^2 + Q^2 - 2PQ} = P - Q$ (ii) $\alpha = 0^0$

iii) If the angle between the two vectors θ =90⁰ then (i) $R = \sqrt{P^2 + Q^2}$ (ii) $\text{Tan}\,\alpha = \frac{Q}{P}$

iv) If $\theta = 90^0$ and both the vectors have same magnitude then (i) $R = \sqrt{P^2 + P^2} = \sqrt{2}P$ (ii) $\alpha = 45^0$.

v) If two vectors have same magnitude i.e $|\vec{P}| = |\vec{Q}|$ then (i) R = 2P cos θ/2 (ii) α = θ /2

vi) If the resultant of two vectors is equal to either, i.e $|\vec{P}| = |\vec{Q}| = |\vec{R}|$ then $\theta = 120^0$.

9.1 Projectile and Trajectory: A 'body projected in to air' with some initial velocity ,making an oblique angle to the horizontal, is called a projectile and its path is called Trajectory.

Note : The path of a projectile is a Parabola.

Ex: The **'sixer'** hit by a batsman in the cricket game, a **Javellin** thrown by an athlete, a **Bullet** fired from a gun, a **Bomb** thrown from a moving Aeroplane.

9.2 Equations of motion for a Projectile (Here, θ is the angle of projection):

1. Horizontal component $u_x = u\cos\theta$; Vertical component $u_y = u\sin\theta$.

2. Velocity of projectile $v = \sqrt{v_x^2 + v_y^2}$, where $v_x = u_x = u\cos\theta$, $v_y = u\sin\theta - gt$

3. Time of ascent $t = \frac{u\sin\theta}{g}$ = Time of descent

4. Time of flight $T = 2t = \frac{2u\sin\theta}{g}$

5. Maximum height $h_{max} = \frac{u^2\sin^2\theta}{2g}$

6. Range $R = \frac{u^2\sin 2\theta}{g}$; $R_{max} = \frac{u^2}{g}$, ($\because \theta = 45^o$)

10.1 Uniform circular motion: If a particle moves along a circle with constant speed, then it is said to be in uniform circular motion.

10.2 Period of revolution (T): The time taken for one revolution

10.3 Frequency of revolution (n): The number of revolutions per second.

10.4 Angular displacement (θ): The angle described by the radius vector at the centre, in a given interval of time is called angular displacement.
SI unit: radian.
Dimensional formula: $[M^0L^0T^0]$

10.5 Angular velocity (ω): The rate of angular displacement is called Angular velocity. $\omega = \frac{\theta}{t}$
SI unit: rad s^{-1}.
Dimensional formula: $[M^0L^0T^{-1}]$

Ex : Angular velocity of seconds hand of a watch, $\omega = \frac{2\pi}{60} \text{ rad s}^{-1}$ $(\because \omega = \frac{\theta}{t} = \frac{2\pi}{T})$

10.6 Angular acceleration (α): The rate of angular velocity is called Angular acceleration.
SI unit : rad s^{-2}
Dimensional formula : $[M^0L^0T^{-2}]$

10.7 Centripetal force (CPF): The force required to maintain a body in a circular path is called centripetal force.
It is directed along the radius and **towards** the centre.

10.8 Centrifugal force(CFF): It is the force experienced by a particle in circular motion along the radius and **away** from the centre.

Formula: $\frac{mv^2}{r} = mv\omega = mr\omega^2$

10.9 Centripetal acceleration (ar): The linear acceleration directed along the radius towards the centre of the circle is called centripetal (or) radial acceleration.

Formula : $a_r = \frac{v^2}{r} = r\omega^2 = v\omega$

Imp. Formulae

1. Resultant of $\bar{P}, \bar{Q}$ $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$; $\alpha = \text{Tan}^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$
2. Horizontal component $u_x = u\cos\theta$; Vertical component $u_y = u\sin\theta$.
3. Velocity of projectile $v = \sqrt{v_x^2 + v_y^2}$, where $v_x = u_x = u\cos\theta$, $v_y = u\sin\theta - gt$
4. Time of ascent $t = \frac{u\sin\theta}{g}$ = Time of descent
5. Time of flight $T = 2t = \frac{2u\sin\theta}{g}$
6. Maximum height $h_{max} = \frac{u^2\sin^2\theta}{2g}$
7. Range $R = \frac{u^2\sin2\theta}{g}$; $R_{max} = \frac{u^2}{g}$, $(\because \theta = 45^\circ)$