2 RELATIONS AND FUNCTIONS

1 OMQ + 1 VSAQ + 1 SAQ + 1 LAQ [1 M + 2M +4M +8M= 15 M]

CONCEPTS & FORMULAS - I

I) RELATIONS

2.1 Ordered Pair (a, b): A **pair of elements** arranged in a **particular order**.

Ex : (1, 2), (2, 1), (1, 1), (x, y)....

- **Equality of Ordered Pairs:** If (a, b) = (x, y) then a = x and b = y.
- **Ordered Pair Vs Set :** • $(1, 2) \neq (2, 1)$, but $\{1, 2\} = \{2, 1\}$;

2.2 Cartesian Product of Sets (A × B): The cartesian product of two non-empty sets A and B is the set of **all are ordered pairs** of elements from A to B.

- $A \times B = \{(a, b) / a \in A, b \in B\}$
- **If n(A) = p and n(B) = q** then **n(A × B) = pq**

Ex 1: Let A = {a, b}, **B =** {1, 2, 3} then

i) $A \times B = \{a, b\} \times \{1, 2, 3\} = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

ii) $B \times A = \{1,2,3\} \times \{a,b\} = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

Here, n(A) = 2, n(B) = 3 $\therefore n(A \times B) = 2 \times 3 = 6$. Thus, A × B contains 6 elements.

Ex 2: In the co-ordinate system, the cartesian product of a set of all elements (R) on the X-axis and the set of all elements (R) on the Y-axis is **the xy plane**.

Thus, $R \times R = \{(x, y): x, y \in R\}$ = co-ordinates of all points in the xy plane.

[Upto Exercise 2(a)]

2.3 Relation : In general, a relation describes a connection (link) between the elements of two sets.

Ex 1: Students in a class and their respective friends form a relation(friendship).

Ex 2: A group of people and their respective age form a relation (people - age relation).

Relation (R : A→B): Mathematically, any subset of A × B is called a relation from A to B. In other words, relation is any element (set) of power set of A × B.

- If an element a is related with b by the relation R then it is denoted by a R b (or) $(a, b) \in R$
- If n(A)=p, n(B)=q then (i) number of elements in $A \times B = pq$ (ii) number of relations from A to B $= 2^{pq}$

Ex : Let A ={a,b},B ={1,2}.

Then $A \times B = \{a,b\} \times \{1, 2\} = \{(a,1), (a,2), (b,1), (b,2)\} \Rightarrow n(A \times B) = 4$

$\therefore$ no. of relations from A to B $= 2^{2\times2} = 2^4 = 16$

Representation of a Relation: Relation is basically a set. So, it is represented in the following ways.

(i) Roster form (ii) Set-builder form (iii) Tabular form

(iv)Arrow diagram (v) Expression between 2 variables (like y = f(x)) (vi) Graph

Domain & Range of a Relation: If a relation is expressed as a set of ordered pairs then the set of all **first elements** of ordered pairs is called **domain** and the set of all **second elements** is called **range**.

- If $R : A \to B$ is said to be a relation then **A** is called a **domain** and **B** is called **codomain** of R.

Note: Range of a relation is always a subset to its codomain.

[Upto Exercise 2(b)]

II)FUNCTIONS

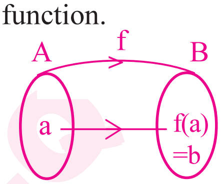
2.4 1) A function f:A→B is a **special type of relation** satisfying the following two conditions.

i) Domain totality: 'All the elements' of the domain A must be paired.
So, if there is any element with 'no mapping image' then that relation is not a function.

ii) Uniqueness: Any element of A should have **one and only one** (unique) image.
That means, any element of A 'should not be paired' with two or more elements of B.
Ofcourse, two or more elements of A may have the same image in B..

☞ In a relation, if any one of the above two conditions fails then it is not a function.
Thus, every function is a relation but every relation need not be a function.

2) Function: A relation f : A → B is said to be a function, if **every** element in A is related to a **unique** image in B.
If a ∈ A then f(a) = **b** is called the **f image of a** and **a** is known as the **pre- image of f(a)**



(a, b) ∈f

3) Domain (A), Codomain (B) & Range f(A) of the function (f : A→B):

If f : A → B is a function then A is called its domain and B is called codomain.

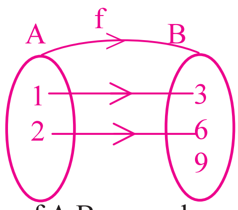
• **Range:** In the function f : A → B, the set of all images is called the **range** of f. It is denoted by f(A).
Thus, f(A) = {f(a) / a∈ A} (or) f(A) = { b / b = f(a)}.

Note: f(A) ⊆ B. That means, range of a function is always a subset to its codomain.

Ex: Let A = {1, 2}, B = {3, 6, 9} and f : A → B is defined as f = {(1, 3), (2, 6)} then

i) Domain of f is A={1,2) ii) Codomain of f is B = {3,6,9}

iii) Range of f is f(A)= {3,6} Here, f(A)⊆B



4) Real Function: f : A → B is said to be a real function when all the elements of A,B are reals.

5) Algebra of real functions:

Let f : A → R, g : B → R be 2 real functions such that A ∩ B ≠ ∅ then f,g can be operated algebraically in the following manner with domains indicated in the brackets

(i) (f+g)(x) = f(x)+g(x), (∀ x ∈ A∩B);

(ii) (f g)(x) = f(x)g(x), (∀ x ∈ A∩B)

(iii) (kf)(x) = kf(x), (∀ x∈ A);

(iv) $f^n(x) = [f(x)]^n$, (∀ x∈ A, n∈ N)

(v) $\sqrt{f}(x) = \sqrt{f(x)}$, (∀ x∈ A such that f(x)≥0);

(vi) $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$ $(\forall x \in A \cap B, g(x) \neq 0)$

6) Hints to find the domains of real functions:

Type I: If the function is of the form $\sqrt{f(x)}$ then its domain is {x / f(x)≥0}

Type II: If the function is of the form $\frac{1}{\sqrt{f(x)}}$ (or) log f(x) then its domain is {x / f(x)>0}

Type III: If the function is of the form $\frac{1}{f(x)}$ then its domain is R−{x / f(x)=0}

[Upto Exercise 2(c)& 2(d)]

Note: Refer P.Nos. 52,53 **Concepts & Formulas -II** for **'Real functions and Graphs'** .