2 MOTION IN A STRAIGHT LINE

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M + 4M = 7 M]

CONCEPTS & DEFINITIONS

1.0 In this chapter we study the motion of bodies along a straight line.

1.1 The main contents of this chapter are (i) Displacement, velocity, acceleration of bodies moving in a straight line, freely falling bodies (ii) kinetic equations of motion (iii) Graphs

2.1 **Rectilinear motion** is the motion of bodies along a **straight line.**

2.2 **Uniform motion:** A body is said to be in uniform motion if it covers equal distances in equal intervals of time. **Ex:** Body moving with a constant speed.

2.3. **Non-uniform motion:** A body is said to be in non uniform motion it it covers unequal distances in equal intervals of time. **Ex:** A ball thrown upwards.

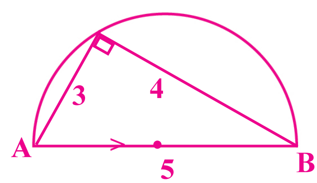
3.1 **Rest and Motion :**

A body is said to be **at rest** if it does not change its position with respect to time & surroundings.
A body is said to be **in motion** if it changes its position with respect to time and surroundings.
The terms rest and motion are **relative**.

Ex 1 : Trees, hills and buildings are in motion with respect to Sun, and are at rest w.r.t. Earth.
Ex 2: If we are travelling in a train then we are in motion w.r.t surroundings, but we are at rest w.r.t our co-passengers.

3.2 **Path length (Distance) Vs Displacement:**

- The total length of any path covered by a body between two points is called **path length** or **distance** . It is a scalar.
- The change of position a body in a specified direction is called **displacement**. (or)
  The shortest distance between the starting point and the ending point is called **displacement.** It is a vector.



Ex: In the diagram, a path length between A and B is 7cm; another path length along the semicircle is 2.5π cm. The displacement (shortest distance) between A and B in the above diagram is 5cm. Here, pathlength ≥ displacement.

3.3 **Speed Vs Velocity:**

Speed is the rate of change of path length (distance).
It has only magnitude but no specific direction. It is a scalar quantity.
Velocity is the rate of change of displacement.
It has both magnitude and a specific direction. It is a vector quantity.

☛ The concept of scalars and vectors is given in the next chapter.

3.4 **Average speed Vs Average Velocity:**

Average speed is the ratio of change in path length (Δx) to the time interval(Δt).
Average velocity is the ratio of change in displacement (Δx) to the time interval(Δt).

- Average speed $= \frac{\text{Pathlength}(\Delta x = x_2 - x_1)}{\text{Time interval }(\Delta t = t_2 - t_1)}$ • Average velocity $= \frac{\Delta x}{\Delta t}$

☛ Average speed is ≥ average velocity in a given time. (∵ pathlength ≥ displacement.)

3.5 Instantaneous speed Vs Instantaneous velocity (simply velocity v):

Instantaneous speed is the speed of the body at particular instant of time.

Ex: Speed of a moving car observed in the speedometer denotes the instantaneous speed.

Instantaneous velocity is the **'rate of change of displacement'** of a body **with time.**

If the motion of a body is given by x=f(t) then its $\text{velocity}(v) = \frac{dx}{dt}$ (or) $v = \underset{t\to 0}{Lt}\frac{\Delta x}{\Delta t}$

Units of velocity: ms^{-1} ; **dimension formula:** $[LT^{-1}]$

☛ **Average velocity** tells us how fast a body is moving over a 'given interval of time',whereas **instantaneous velocity** tells us how fast the body is moving at a 'particular instant of time'.

3.6 Acceleration, Positive acceleration and Negative acceleration (Retardation):

Acceleration: It is the **rate of change of velocity** of a body with **time**.

Ex: A freely falling body, Motion of a moving train in the beginning.

Units of acceleration: ms^{-2} ; **dimension formula:** $[LT^{-2}]$

Positive acceleration : If the velocity of a body is increasing with time, then the body is said to be moving with **positive acceleration (or)** simply **acceleration.**

Ex: A train starting from a station moves with positive acceleration in the beginning.

Negative acceleration : If the velocity of a body is decreasing with time, then the body is said to be moving with **Negative acceleration** or **Retardation** or **deceleration**.

Ex : A train reaching a station stops in a retardation.

3.7 Instantaneous acceleration(simply acceleration a):

$$\text{Acceleration}(a) = \frac{dv}{dt} \text{ (or) } a = \underset{t\to 0}{Lt}\frac{\Delta v}{\Delta t}$$

Imp Note: If the motion of a body is expressed as x=f(t) then to find its velocity or acceleration we prefer to apply the **concept of derivatives** rather than the concept of limits.

3.8. Uniform Acceleration Vs Non-uniform Acceleration:

- **Uniform Acceleration :** A body is said to have **uniform Acceleration** if it has equal changes in velocity in equal intervals of time, however small the time intervals may be.

  Ex : Motion of a freely falling body (magnitude changes)

- **Non-uniform Acceleration:** A body is said to have **non-uniform Acceleration**, if the magnitude or direction of the acceleration changes with time.

  Ex : Motion of moving electrons around the nucleus.(direction changes)

3.9 Acceleration due to gravity(g): It is the uniform acceleration produced in the bodies due to earth's gravitational force. It is produced both in freely falling or projected bodies.

- When a body is falling towards Earth , its velocity increases. Hence, g is taken as positive.
- When a body is projected upwards , its velocity decreases. Here, g is taken as negative.
- The general value of g is on the surface of Earth is 9.8 ms^{-2}.
- The value of g is at the centre of the earth is zero.

3.10 Freely falling body and body projected vertically up :

Freely falling body : When a body is dropped from a height , then its velocity increases gradually and attains a maximum velocity when it hits the ground.

Here, the **initial velocity** is **zero** and acceleration is + **g**.

Body projected vertically up: When a body is projected vertically up from the ground then it moves against gravity. Its velocity decreases gradually.Hence,its acceleration is taken as **-g**. At the **peak point** its **velocity becomes zero**.

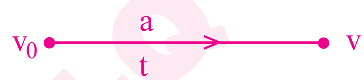
The height at which the velocity becomes zero, is called **maximum height**.

4. Derivations of Equations of Motions in a straight line :

4.1 Derive the equation $v=v_0+at$, with usual notation.

Proof: Consider a body starting with initial velocity 'v_0' and moving with uniform acceleration 'a'. After time 't', let its final velocity be 'v'.

From the definition of acceleration,



$$a = \frac{\text{Change in velocity}}{\text{time}} = \frac{v - v_0}{t} \Rightarrow v - v_0 = at$$

$\therefore$ $\mathbf{v=v_0+at}$

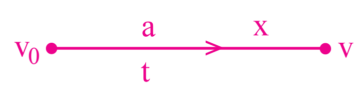
4.2 Derive the equation $x = v_0 t + \frac{1}{2}at^2$

Proof: Consider a body starting with initial velocity 'v_0' and moving with uniform acceleration 'a'. After time 't', let its final velocity be 'v'. Let 'x' be the distance travelled by the body in time 't'.

Distance travelled x = Average velocity $\times$ time

$$x = \left(\frac{v_0 + v}{2}\right)t = \left(\frac{v_0 + (v_0 + at)}{2}\right)t, [\because v = v_0 + at]$$

$$= \left(\frac{2v_0 t + at^2}{2}\right) = v_0 t + \frac{1}{2}at^2$$



$$\therefore \mathbf{x = v_0 t + \frac{1}{2}at^2}$$

4.3 Derive the equation $v^2 = v_0^2 + 2ax$

Proof: Consider a body starting with initial velocity 'v_0' and moving with uniform acceleration 'a'. After time 't',let its final velocity be 'v'.Let 'x' be the distance travelled by the body in time 't'.

Distance travelled x = Average of velocity $\times$ time

$$x = \left(\frac{v_0 + v}{2}\right)t = \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right), \quad \left(\because a = \frac{v - v_0}{t} \Rightarrow t = \frac{v - v_0}{a}\right)$$

$$= \left(\frac{v^2 - v_0^2}{2a}\right) \qquad \therefore \ v^2 - v_0^2 = 2ax \Rightarrow \mathbf{v^2 = v_0^2 + 2ax}$$

Equations for a freely falling body (Here , v_0 = 0, a =+g , x=h)

1. $v = gt$, (From $v=v_0+at$)
2. $h = \frac{1}{2}gt^2$, [From $x = v_0 t + \frac{1}{2}at^2$]
3. $v^2 = 2gh \Rightarrow v = \sqrt{2gh}$, [From $v^2 = v_0^2 + 2ax$]
4. Time $t = \sqrt{\frac{2h}{g}}$, [From $h = \frac{1}{2}gt^2$]

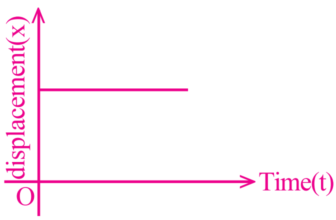
5. GRAPHICAL REPRESENTATION OF LINEAR MOTION:

5.1 Displacement-Time graph: A graph drawn taking 'time (t)' on the X-axis and 'displacement(x)' of a moving particle on the Y-axis, is called 'displacement-time' graph.

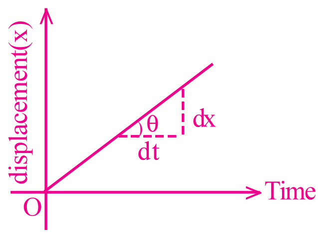
The 'Slope' of the tangent at a point on this curve gives 'velocity' at that point.

$$\text{Slope} = \tan\theta = \frac{dx}{dt} = \text{Velocity } (v)$$

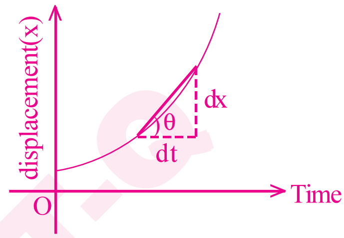
Various cases:



(i) Body is at rest (Slope=0)



(ii) Body is moving with constant velocity



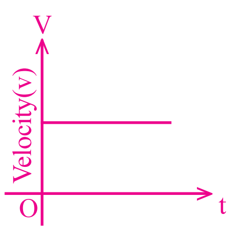
(iii) Body is moving with increasing velocity

5.2 Velocity-time Graph: A graph drawn taking 'time (t)' on the X-axis and 'velocity(v)' of a moving body on Y-axis is called 'velocity-time' graph.

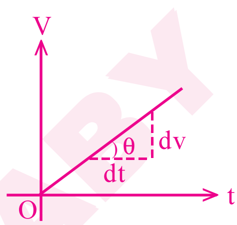
The 'Slope' of the tangent at a point on this curve gives 'acceleration'.

$$\text{Slope} = \tan\theta = \frac{dv}{dt} = \text{acceleration } (a)$$

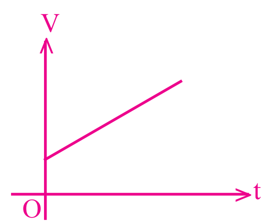
Various cases:



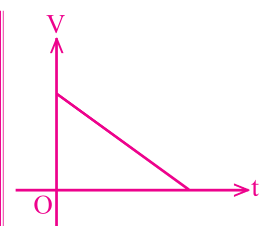
(i) Uniform velocity



(ii) Uniform acceleration starting from Rest



(iii) Uniform acceleration with some initial velocity



(iv) Uniform retardation

Imp.Formulae

1. Speed $= \dfrac{\text{Total pathlength}(\Delta x)}{\text{Total time }(\Delta t)}$
2. Velocity $v = \dfrac{dx}{dt}$
3. Acceleration $a = \dfrac{dv}{dt}$
4. $v = v_0 + at$
5. $x = v_0 t + \dfrac{1}{2}at^2$
6. $v^2 = v_0^2 + 2ax$