OMQ

SOLUTIONS

1. SETS

1. **(3)**

Sol: $1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64\ (\times)$ $(\because x^3 \le 50)$ $\therefore$ Given set A= {1, 2, 3}

2. **(2)**

Sol: From (2), $x \in Z^+, x^2 + 1 < 20 \Rightarrow 1^2 + 1 = 3;\ 2^2 + 1 = 5;\ 3^2 + 1 = 10;\ 4^2 + 1 = 17;\ 5^2+1=26\ (\times)$

$\therefore$ the given set = {1, 2, 3, 4}

3. **(4)**

Sol: The set element {2, 3} is itself is a member of the set {1, {2, 3}, 4, 5}

4. **(1)**

Sol: From (1), $x \in R, + x \in (-4, 5) \Rightarrow x \le -4$ $(\because 4$ is included); $x > 5$ $(\because 5$ is excluded)

☛ In the textual options there is a correction. x should be Real but not Integer.

5. **(2)**

Sol: $A - B = \{1, 2, 3, \not{4}, \not{5}\} - \{4, 5, 6, 7, 8\} = \{1, 2, 3\};$

$B - A = \{\not{4}, \not{5}, 6, 7, 8\} - \{1, 2, 3, 4, 5\} = \{6, 7, 8\}$

$\therefore (A - B) \cup (B - A) = \{1, 2, 3, 6, 7, 8\}$

6. **(1)**

Sol: $B \cup C = \{2, 3, 4\} \cup \{3, 4, 5, 6\} = \{2, 3, 4, 5, 6\}$

$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{2, 3, 4, 5, 6\} = \{2, 3, 4\}$

7. **(2)**

Sol: Infact $A \cup A' = U$

8. **(3)**

Sol: $\varnothing' \cap A = U \cap A = A$, for any set A. $(\because \varnothing' = U)$

9. **(2)**

Sol: $U = \{x : x \text{ is a natural } 1 < x < 15\} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$A = \{x : x \text{ is a prime number } 1 < x < 15\} = \{2, 3, 5, 7, 11, 13\}$

$A' = U - A = \{\not{2}, \not{3}, 4, 5, 6, \not{7}, 8, 9, 10, 11, 12, 13, 14\} - \{2, 3, 5, 7, 11, 13\}$

$= \{4, 6, 8, 9, 10, 12, 14\}.$

10. **(3)**

Sol: In the set {d,e}, the element e is not a member of the given set {a, b, c, d}

2. RELATIONS & FUNCTIONS

1. **(1)**

Sol: $\left(2x+1, \frac{y}{2}\right) = (3, 3) \Rightarrow 2x+1=3 \Rightarrow x=1; \frac{y}{2}=3 \Rightarrow y=6$

2. **(3)**

Shortcut Solution: $n(A) = 3, n(B) = 2 \Rightarrow n(A \times B) = 3 \times 2 = 6$

Among the given, option (3) has 6 elements.

3. **(4)**

A relation is basically a set of ordered pairs.

So it can be represented as roster method, Set-builder method and an arrow diagram.

4. **(2)**

Sol: Since the relation is defined from A to A, then co-domain of R is A= {1, 2, 3, 4}

5. **(3)**

$f(0) = \frac{|0|}{0} = \frac{0}{0}$ which is undefined

6. **(1)**

Sol: $\frac{1}{\sqrt{x^2 - 25}}$ is defined $\Rightarrow x^2 - 25 > 0 \Rightarrow (x + 5)(x - 5) > 0 \Rightarrow x < -5$ or $x > 5$

$\therefore$ Domain = $(-\infty, -5) \cup (5, \infty)$

7. **(1)**

Sol: Let $y = f(x) = x^2$ Square of any real is non-negative. $\therefore x \in R \Rightarrow x^2 \geq 0 \Rightarrow f(x) \geq 0 \Rightarrow y \geq 0$

$\therefore$ Range of $f(x) = [0, \infty)$

8. **(3)**

Sol: $f(x) = x^2 + 2x - 7 \Rightarrow f(3) = 9+6-7 = 8.$

9. **(2)**

Sol: Let $f : N \rightarrow N$ be $f(x) = ax + b$

Given $f = \{(0, 1), (1, 3), (2, 5)\} \Rightarrow f(0) = 1 \Rightarrow a(0) + b = 1 \Rightarrow b = 1; f(1) = 3$

$\Rightarrow a(1) + b = 3 \Rightarrow a + 1 = 3 \Rightarrow a = 2 \quad \therefore f(x) = ax + b = 2x + 1$

10. **(1)**

Sol: $\sqrt{|x| - x}$ is defined when $|x| - x \geq 0 \Rightarrow |x| \geq x \Rightarrow x \in R. \quad \therefore$ domain = R.

TRIGONOMETRIC FUNCTIONS

1. **(3)**

Sol: $\cos 5\pi = \cos(4\pi + \pi) = \cos\pi = -1$.

2. **(2)**

Sol: We have $\cos 90^\circ = 0$ in the middle of the given product. Hence the given value is 0.

3. **(3)**

Sol: Given $\sin\theta + \operatorname{cosec}\theta = 2 \Rightarrow (\sin\theta + \operatorname{cosec}\theta)^2 = 2^2 \Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta = 4$

$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta + 2(1) = 4 \Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 4 - 2 = 2$

4. **(4)**

Sol: $\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \dfrac{3+2}{6-1} = 1 = \tan\dfrac{\pi}{4} \quad \therefore \theta+\phi = \dfrac{\pi}{4}$

5. **(3)**

Sol: $\dfrac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ} = \cos 2(15^\circ) = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

6. **(4)**

Sol: **Shortcut Solution:** When $\theta = 0^\circ$ we have $\sin 45^\circ - \cos 45^\circ = \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} = 0$ **(or)**

$\sin(45^\circ + \theta) - \cos(45^\circ - \theta) = (\sin 45^\circ \cos\theta + \cos 45^\circ \sin\theta) - (\cos 45^\circ \cos\theta + \sin 45^\circ \sin\theta)$

$= \left(\dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta\right) - \left(\dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta\right) = 0$

7. **(3)**

Sol: **Shortcut Solution:** When $\theta = 0^\circ$ we have $\cot\left(\dfrac{\pi}{4}+\theta\right)\cot\left(\dfrac{\pi}{4}-\theta\right) = \cot\left(\dfrac{\pi}{4}\right)\cot\left(\dfrac{\pi}{4}\right) = 1\times 1 = 1$

8. **(2)**

Sol: $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$= \cos 2\theta \cos 2\phi + \sin[(\theta - \phi) + (\theta + \phi)] \sin[(\theta - \phi) - (\theta + \phi)]$

$= \cos 2\theta \cos 2\phi + \sin 2\theta \sin(-2\phi) = \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi = \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$

9. **(2)**

Sol: $(\sin 50^\circ - \sin 70^\circ) + \sin 10^\circ = 2\cos\left(\dfrac{50^\circ + 70^\circ}{2}\right)\sin\left(\dfrac{50^\circ - 70^\circ}{2}\right) + \sin 10^\circ$

$= 2\cos 60^\circ \sin(-10^\circ) + \sin 10^\circ = -\sin 10^\circ + \sin 10^\circ = 0$

10. **(3)**

Sol: $\sin\theta + \cos\theta = 1 \Rightarrow (\sin\theta + \cos\theta)^2 = 1 \Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\Rightarrow 1 + \sin 2\theta = 1 \Rightarrow \sin 2\theta = 0$

Shortcut Solution: When $\theta = 0^\circ$ we have $\sin\theta + \cos\theta = \sin 0 + \cos 0 = 0 + 1 = 1$

$\therefore \sin 2\theta = \sin 2(0) = \sin 0 = 0$

4. COMPLEX NUMBERS

1. **(3)**

Sol: $i^{-999} = \frac{1}{i^{999}} = \frac{i}{i^{1000}} = \frac{i}{(i^2)^{500}} = \frac{i}{(-1)^{500}} = \frac{i}{1} = i$

2. **(1)**

Sol: Multiplicative inverse of $1+i$ is $\frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{1+1} = \frac{1}{2}(1-i)$

3. **(3)**

Sol: $|5+4i| = \sqrt{25+16} = \sqrt{41}$

4. **(2)**

Sol: $\sqrt{-25} \times \sqrt{-9} = 5i \times 3i = 15i^2 = 15(-1) = -15$

5. **(3)**

Sol: Conjugate of $\frac{2-i}{1-2i} = \overline{\frac{2-i}{1-2i}} = \frac{2+i}{1+2i} = \frac{(2+i)(1-2i)}{(1+2i)(1-2i)} = \frac{2-4i+i-2i^2}{1+4} = \frac{2-3i-2(-1)}{1+4} = \frac{4-3i}{5}$

6. **(1)**

Sol: $(z+3)(\bar{z}+3) = (z+3)(\bar{z}+\bar{3}) = (z+3)\overline{(z+3)} = |z+3|^2$ $[\because \text{for real } a, \bar{a} = a]$

7. **(4)**

Sol: If $y = 0$ then $x + iy = x + 0 = x \in R$ $\therefore$ $x + iy$ is a non-real complex number when $y \neq 0$

8. **(4)**

Sol: Given $a + ib = c + id$ $\Rightarrow a = c, b = d$ $\Rightarrow a^2 = c^2, b^2 = d^2$ $\Rightarrow a^2 + b^2 = c^2 + d^2$

9. **(2)**

Sol: $i + i^2 + i^3$ ....up to 1000 terms $= \frac{i(i^{1000}-1)}{i-1} = \frac{i\left[(i^2)^{500}-1\right]}{i-1} = \frac{i[(-1)^{500}-1]}{i-1} = \frac{i(1-1)}{i-1} = 0$

10. **(2)**

Sol: $\left|\frac{i+z}{i-z}\right| = 1 \Rightarrow |i+z| = |i-z| \Rightarrow |i+x+iy| = |i-(x+iy)| \Rightarrow |x+i(1+y)| = |-x+(1-y)i|$

$\Rightarrow x^2+(1+y)^2 = x^2+(1-y^2) \Rightarrow (1+y)^2 = (1-y)^2$ $\Rightarrow 4y = 0 \Rightarrow y = 0$. This is equation of x-axis.

$\therefore$ Locus of z lies on the x-axis.

PERMUTATIONS & COMBINATIONS

1. **(4)**

Sol: The student can select a pant from 5 pants in 5 ways and a shirt from 8 shirts in 8 ways.

$\therefore$ Required number of ways $= 5 \times 8 = 40$.

2. **(2)**

$^nP_4 = 1680 = 8 \times 7 \times 6 \times 5 = {}^8P_4$

3. **(1)**

Sol: $^nP_r = 1320 = 12 \times 11 \times 10 = {}^{12}P_3 \Rightarrow r = 3$.

4. **(1)**

The number of ways that he can send his 3 sons to 6 schools if no two of them are in the same school $= {}^6P_3$.

5. **(2)**

Sol: The required number of arrangements $= 7! = 5040$

6. **(4)**

Sol: The required number of arrangements $= \dfrac{12!}{4!3!5!}$

7. **(3)**

Sol: The number of 4-digit numbers $= 6^4$.

8. **(3)**

Sol: $(8 + 4)! = 12!$

9. **(3)**

Sol: $1080 = 2^3 \times 3^3 \times 5^1$ Hence, the number of positive divisors $= (3+1)(3+1)(1+1) = 32$.

10. **(2)**

Sol: The no.of ways of selecting 4 boys and 3 girls from a group of 8 boys and 5 girls$= {}^8C_4 \times {}^5C_3$.

11. **(4)**

Sol: $^{12}C_{s+1} = {}^{12}C_{2s-5} \Rightarrow s + 1 = 2s - 5$ (or) $(s + 1) + (2s-5) = 12$

$\Rightarrow s = 6$ (or) $3s = 16 \quad \Rightarrow s = 16/3$

$\therefore s = 6 \quad [\because$ s cannot be a fraction$]$

12. **(1)**

Sol: Number of diagonals of a polygon of 10 sides $= \dfrac{10(10-3)}{2} = 35$

13. **(4)**

Sol: Number of diagonals $= 54 \Rightarrow \dfrac{n(n-3)}{2} = 54 \Rightarrow n(n-3) = 108 = 12 \times 9 = 12(12-3) \Rightarrow n = 12$

BINOMIAL THEOREM

1. **(1)**

Sol: Third term of $\left(3x-\frac{y^3}{6}\right)^4$ is $T_3 = T_{2+1} = {}^4C_2\,(3x)^{4-2}\left(\frac{-y^3}{6}\right)^2 = {}^4C_2(3x)^2\left(\frac{-y^3}{6}\right)^2$

2. **(3)**

Sol: $C_0 + 3.\,C_1 + 3^2\,.\,C_2 + ... + 3^n\,.\,C_n = (1+3)^n = 4^n.$

3. **(2)**

Sol: As n = 6 is even, the middle term is $T_{\left(\frac{6}{2}+1\right)} = T_{3+1} = {}^6C_3(4)^3(2x)^3 = 20\times 64\times 8x^3 = 10240x^3$

4. **(3)**

Sol: $(x+1)^2\,(xy+1)^2 = (x^2+2x+1)(y^2+3y^2+3y+1).$ $\therefore$ Coeff. of x^2y^2 is 2.

5. **(3)**

Sol: In $(1+x)^{20},$ the coefficients of r^{th} and $(r+4)^{th}$ terms are equal

$\Rightarrow {}^{20}C_{r-1} = {}^{20}C_{r+3} \Rightarrow (r-1)+(r+3) = 20 \Rightarrow 2r = 18 \Rightarrow r = 9.$

6. **(3)**

Sol: $8^{48} = 64^{24} = (63+1)^{24} = 63q + 1$ for some integer q. $\therefore$ Remainder = 1.

7. **(2)** Σ form of binomial epxansion $= \Sigma\ r^{th}$ term

8. **(4)**

Sol: Coefficient of first term is ${}^nC_0 = 1$ and Coefficient of last term is ${}^nC_n = 1$

9. **(2)**

Sol: $(xy+2)^2 = (xy)^2 + 2(xy)(12) + 2^2 = x^2\,y^2 + 4xy + 4.$

10. **(3)** The index n should be 91.

11. **(1)**

Sol: The coefficient of $x^8\,y^{10}$ in $(x+y)^{18}$ is ${}^{18}C_{10} = {}^{18}C_8.$

7) SEQUENCE & SERIES

1. **(3)** In the given options only option (3) has finite number of terms.

2. **(1)** From option (1), in $a_n = a_{n-1} + a_{n-2}$, $n > 2$ put n= 3,4,5,.... then $a_3 = a_2+a_1$; $a_4 = a_3+a_2$; Thus, each number is the sum of two previous numbers. Hence,**Fibonacci sequence.**

3. **(1)** Fibonacci sequence is 0,1,1,2,3,5,......Here first term is 0.

☛The textual given option (2) is a mistake.

4. **(4)** $a_n = 4n + 6 \Rightarrow T_{15} = a_{15} = 4(15) + 6 = 66.$

5. **(2) Series :** $a_1 + a_2 + ... + a_n = \Sigma a_n$

6. **(4)** Sum of first five terms of $2 + 4 + 6 + ... = 2 + 4 + 6 + 8 + 10 = 30.$

7. **(4)** $\sum_{n=1}^{4}(2n+3) = 2\left(\frac{4\times5}{1\times2}\right) + 3(4) = 20 + 12 = 32.$

8. **(2)** $a_{n+1} = a_n r \Rightarrow r = \frac{a_{n+1}}{a_n} = \text{common ratio}$. Hence the sequence is G.P.

9. **(3)** n^{th} term of G.P. is $a_n = ar^{n-1}$

10. **(1)** In the G.P.: a, ar, ar^2, ..., ar^{n-1} putting r = 1 we get a + a + a, ..., + a = na

11. **(4)** a = 4, r = 2. Sum of five terms $S_5 = \frac{4(2^5-1)}{2-1} = 4\times31 = 124$

12. **(1)** $1 + \frac{1}{2} + \frac{1}{4} + ...$upto 6 terms $S_6 = \frac{(1-(1/2)^6)}{1-1/2} = 2\left(1-\frac{1}{2^6}\right) = 2 - \frac{1}{32} = \frac{63}{32}.$

13. **(2)** Geometric mean of 3 and 12 is $\sqrt{3\times12} = \sqrt{36} = 6$

14. **(2)** Let the two numbers be a, b. A.M. of a, b is $\frac{15}{2} \Rightarrow \frac{a+b}{2} = \frac{15}{2} \Rightarrow a+b = 15$ ....(1)

G.M. of a, b is $6 \Rightarrow \sqrt{ab} = 6 \Rightarrow ab = 36$ ....(2)

$(a-b)^2 = (a+b)^2 - 4ab = (15)^2 - 4(36) = 225 - 144 = 81 \Rightarrow a - b = 9$ ......(3)

$(1) + (3) \Rightarrow 2a = 24 \Rightarrow a = 12, b = 12 - 9 = 3$

15. **(2)** A.M. of two positive numbers is always greater than or equal to G.M.

16. **(1)** Ratio of A.M, G.M of a and b is 5 : 3

$\Rightarrow \frac{a+b}{2} : \sqrt{ab} = 5:3 \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{3} \Rightarrow 9(a+b)^2 = 25(4ab) \Rightarrow 9a^2 + 18ab + 9b^2 = 100ab$

$\Rightarrow 9a^2 - 82ab + 9b^2 = 0 \Rightarrow (9a-b)(a-9b) = 0 \quad \Rightarrow 9a - b = 0 \text{ (or) } a - 9b = 0$

$\Rightarrow \frac{a}{b} = \frac{1}{9} \text{ (or) } \frac{a}{b} = \frac{9}{1} \Rightarrow a:b = 9:1$

8) STRAIGHT LINES

1. **(2)** Slope of the line is m = tan 45° = 1; y-intercept is c = 3

Equation of the line is y = mx + c $\Rightarrow$ y = 1(x) + 3 $\Rightarrow$ y = x + 3

2. **(3)** Slope of $OA = \frac{2}{5}$, slope of $OB = \frac{-15}{6} = -\frac{5}{2}$

Product of slopes= –1 $\therefore \angle AOB = \pi/2$

3. **(3)** 3x + 4y –12 = 0 $\Rightarrow$ a = 3, b = 4, c= –12

$\therefore$ Area of triangle$=\frac{c^2}{2|ab|}=\frac{(-12)^2}{2|(3)(4)|}=6$ sq.units

4. **(3)** Area of triangle$=\frac{(-2)^2}{2|\cos\alpha\cdot\sec\alpha|}=\frac{4}{2(1)}=2$

5. **(2)** Area$=1\Rightarrow\frac{(-a)^2}{2|(3)(4)|}=1\Rightarrow a^2=24\Rightarrow a=2\sqrt{6}.$

6. **(4)** We get the image of (3, 6) w.r. to 2x–y –5 = 0 as (7, 4)

From the L.H.S of given options we have 4x–3y = 4(7)–3(4) = 28–12 = 16. Hence, option (4)

7. **(1)** From Page 15/35 we have $\frac{x}{2p}+\frac{y}{2q}=2\Rightarrow\frac{x}{3}+\frac{y}{4}=2\Rightarrow\frac{4x+3y}{12}=2\Rightarrow 4x+3y=24$

8. **(1)** Lines y = 4 – 3x, 2y + bx + 9 = 0 are parallel $\Rightarrow m_1 = m_2 \Rightarrow b/3 = 2/1 \Rightarrow b = 6$

Lines y = 4 – 3x, ay = x + 10 are $\perp$r $\Rightarrow m_1m_2 = -1 \Rightarrow (-3)(1/a) = -1 \Rightarrow a = 3 \therefore ab = (3)(6) = 18.$

9. **(3)** Let ends of the rod be A (a, 0), B(0, b); Length of rod = $l \Rightarrow a^2 + b^2 = l^2$

Let $P(x_1, y_1)$ divides the join of A (a, 0), B(0, b) in the ratio 1:2.

$\Rightarrow (x_1,y_1)=\left(\frac{2a}{3},\frac{b}{3}\right)\Rightarrow a=\frac{3x_1}{2};b=3y_1 \therefore a^2+b^2=l^2\Rightarrow\left(\frac{3x_1}{2}\right)^2+(3y_1)^2=l^2\Rightarrow 9x_1^2+36y_1^2=4l^2$

10. **(1)** The perpendicular distance from (1, 2) to the line 12x + 5y – 7 = 0 is $\frac{|12+10-7|}{\sqrt{144+25}}=\frac{15}{13}$

11. **(1) Ref. Page No. 29/32**

12. **(4)** Diameter, $2r=\frac{|15-5|}{\sqrt{4^2+3^2}}=\frac{10}{5}=2\Rightarrow r=1$ $\therefore$ Area $=\pi r^2=\pi(1)=\pi$

13. **(3)** a, b, c are in A.P $\Rightarrow$ 2b = a + c $\Rightarrow$ a – 2b + c = 0 $\Rightarrow$ a(1) + b(–2) + c = 0.

Fixed point is (1, –2)

14. **(1)** Same as above.

15. **(2)** Let P(α, β) be a point on the line 2x–3y = 5 $\Rightarrow 2\alpha-3\beta=5$....(1); A = (1, 2), B = (3, 4)

$PA = PB \Rightarrow PA^2 = PB^2 \Rightarrow \alpha+\beta=5$.......(2) By solving (1) & (2) we get P = (4, 1)

9)CONIC SECTIONS

CIRCLES

1. **(3)** Point of intersection of diameters $2x + y - 7 = 0$, $x + 3y - 11 = 0$ is centre C(2, 3).

Also P(5, 7) ⇒ Radius = $CP = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{9+16} = 5$

∴ Equation of the circle is $(x-2)^2 + (y-3)^2 = 5^2 \Rightarrow x^2 + y^2 - 4x - 6y - 12 = 0$.

2. **(4)** Centre of the circle $x^2 + y^2 - 4x - 2y + c = 0$ is A = (2, 1) and P = (10, 7).

PA meets the circle in Q with PQ = 5.

$PA = \sqrt{(10-2)^2 + (7-1)^2} \Rightarrow PQ + QA = \sqrt{64+36} \Rightarrow 5 + r = 10 \Rightarrow r = 5$

$\Rightarrow \sqrt{4+1-c} = 5 \Rightarrow \sqrt{5-c} = 5 \Rightarrow 5 - c = 25 \Rightarrow c = -20$

3. **(1)** Let $A = (x_1, y_1)$, $B = (x_2, y_2)$. x_1, x_2 are the roots of $x^2 + 2ax - b^2 = 0$

$\Rightarrow x^2 + 2ax - b^2 = (x - x_1)(x - x_2)$

y_1, y_2 are the roots of $y^2 + 2py - q^2 = 0 \Rightarrow y^2 + 2py - q^2 = (y - y_1)(y - y_2)$

Equation of the circle with AB as diameter is $x^2 + 2ax - b^2 + y^2 + 2py - q^2 = 0$

$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$.

Radius of the circle $= \sqrt{a^2 + p^2 + (b^2 + q^2)} = \sqrt{a^2 + b^2 + p^2 + q^2}$

4. **(1)** Equation of the given circle is $x^2 + y^2 - 6x + 12y + 15 = 0$.

Radius $r_1 = \sqrt{9+36-15} = \sqrt{30}$. Area of the given circle = 30π.

Equation of the required circle is $x^2 + y^2 - 6x + 12y + k = 0$.

Radius $r_2 = \sqrt{9+36-k} = \sqrt{45-k}$. Area of the required circle = $(45 - k)\pi$.

Given that $(45 - k)\pi = 2(30\pi) \Rightarrow k = -15$.

Equation of the required circle is $x^2 + y^2 - 6x + 12y - 15 = 0$

5. **(4)** Centre of the circle is (1, –2). Radius of the circle is $\sqrt{1+4-3} = \sqrt{2}$

Since the sides of the square are parallel to the coordinate axes, vertices of the square do not lie on horizontal and vertical lines through (1, –2). The given points are not the vertices.

6. **(2)** Equation of the circle with centre (cosα, sinα) and radius 1

is $(x - \cos\alpha)^2 + (y - \sin\alpha)^2 = (1)^2 \Rightarrow x^2 + y^2 - 2x\cos\alpha - 2y\sin\alpha + (\cos^2\alpha + \sin^2\alpha) = 1$

$\Rightarrow x^2 + y^2 - 2x\cos\alpha - 2y\sin\alpha = 0$. $[\because \cos^2\alpha + \sin^2\alpha = 1]$

7. **(1)** Radius of $x^2 + y^2 - 4x - 8y - 44 = 0$ is $r_1 = \sqrt{4+16+44} = \sqrt{64} = 8$

Radius of $x^2 + y^2 + 6x + 8y - 96 = 0$ is $r_2 = \sqrt{9+16+96} = \sqrt{121} = 11$

$\therefore r_1 : r_2 = 8 : 11$

8. **(4)** If r is the radius of the circle touching the parallel lines $3x - 4y + 5 = 0$, $6x - 8y - 9 = 0$ then

$$2r = \frac{|10+9|}{\sqrt{36+64}} = \frac{19}{10} \Rightarrow r = \frac{19}{20}.$$

9. **(3)** Equation of the given circle is $3(x^2+y^2)=16 \Rightarrow x^2+y^2=\frac{16}{3} \Rightarrow r=\frac{4}{\sqrt{3}}$

Perimeter of the circle $= 2\pi r = 2\pi\left(\frac{4}{\sqrt{3}}\right) = \frac{8}{\sqrt{3}}\pi$

10. **(4)** Equation of given circle is $3(x^2 + y^2) - 3x + 9y + 10 = 0 \Rightarrow x^2+y^2-x+3y+\frac{10}{3}=0$

Radius of the circle $\sqrt{\frac{1}{4}+\frac{9}{4}-\frac{10}{3}} = \sqrt{\frac{3+27-40}{12}} = \sqrt{\frac{-10}{12}}$,which is not defined.

PARABOLA

11. **(1)** $S=(0, 0), Z=(2,0) \Rightarrow A=\left(\frac{0+2}{2},\frac{0+0}{2}\right)=(1,0)$

12. **(1) Shortcut Solution:** Focus $(0,-a) = (0,-3) \Rightarrow a = 3 \Rightarrow 4a = 12$; directrix $y=3$ is above x-axis.

$\therefore$ Equation of the parabola is $x^2 = -12y$.

13. **(2)** Parabola $y^2 = 4ax$ passes through $(3, 2) \Rightarrow (2)^2 = 4a(3) \Rightarrow 4a = \frac{4}{3}$.

14. **(1) Shortcut Solution:** Vertex lies on the parabola. Among the given options $(-3, 0)$ is satisfied by $y^2 = 8(x + 3)$. $\left[\because 0^2 = 8(-3+3)\right]$

ELLIPSE

15. **(2)** $e = 4/5$, $(ae, 0) = (4, 0) \Rightarrow a e = 4 \Rightarrow a = 5$. $b^2 = 25\left(1-\frac{16}{25}\right)=9$

$\therefore$ Equation of ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$

16. **(4)** Given $S = (0, 0)$, $e = 1/2$ equation of directrix is $x - 4 = 0$.

We have $CZ=\frac{a}{e} \Rightarrow CS+SZ=\frac{a}{e} \Rightarrow ae+SZ=\frac{a}{e} \Rightarrow SZ=\frac{a}{e}-ae$

$\Rightarrow$ Distance from focus to directrix $=a\left(\frac{1}{e}-e\right) \Rightarrow |0-4| = a\left(2-\frac{1}{2}\right) \Rightarrow a\left(\frac{3}{2}\right)=4 \Rightarrow a=\frac{8}{3}$

$\therefore$ Length of the semi-major axis is $a = 8/3$

17. **(1)** $2a = 3(2b) \Rightarrow a = 3b \quad \therefore e^2 = \frac{a^2 - b^2}{a^2} = \frac{8b^2}{9b^2} = \frac{8}{9} \Rightarrow e = \frac{2\sqrt{2}}{3}$.

18. **(2)** Given ellipse is $x^2 + 3y^2 = 6 \Rightarrow \frac{x^2}{6} + \frac{y^2}{2} = 1 \quad e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{6-2}{6}} = \sqrt{\frac{4}{6}} = \sqrt{\frac{2}{3}}$

HYPERBOLA

19. **(1)** Given hyperbola is $9y^2 - 4x^2 = 36 \Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1; \; e = \sqrt{\frac{a^2 + b^2}{b^2}} = \sqrt{\frac{9+4}{4}} = \frac{\sqrt{13}}{2}$

Foci of the hyperbola $= (0, \pm be) = \left(0, \pm 2\left(\frac{\sqrt{13}}{2}\right)\right) = (0, \pm\sqrt{13})$

20. **(3)** $x^2 - 3y^2 - 4x - 6y - 11 = 0 \Rightarrow (x^2 - 4x + 4) - 3(y^2 + 2y + 1) = 12 \Rightarrow (x-2)^2 - 3(y+1)^2 = 12$

$\Rightarrow \frac{(x-2)^2}{12} - \frac{(y+1)^2}{4} = 1 \Rightarrow a^2 = 12, b^2 = 4 \Rightarrow e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{12+4}}{\sqrt{12}} = \frac{4}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

Distance between foci $= 2ae = 2\sqrt{12}\left(\frac{2}{\sqrt{3}}\right) = 8$

21. **(1)** $\frac{x^2}{9-c} + \frac{y^2}{5-c} = 1$ represents a hyperbola if $9 - c > 0,\ 5 - c < 0 \Rightarrow 9 > c,\ 5 < c$

$\Rightarrow 5 < c,\ c < 9 \Rightarrow 5 < c < 9$.

22. **(2)** Eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$\sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$. Foci are (3, 0), (–3, 0).

Eccentricity of the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1 = \sqrt{\frac{4+b^2}{4}}$. Foci are $(\sqrt{4+b^2}, 0)(-\sqrt{4+b^2}, 0)$

Given that $\sqrt{4+b^2} \Rightarrow 9 = 4 + b^2 \Rightarrow b^2 = 5$

10)3D GEOMETRY

1. **(2)**

Sol: On the y-axis, the x-coordinates and z-coordinates are zeros.

2. **(1)**

Sol: The foot of the perpendicular from P(13, 5, 12) on to the x - axis is Q(13, 0, 0)

$\therefore PQ = \sqrt{(13-13)^2 + (5-0)^2 + (12-0)^2} = \sqrt{0+25+144} = \sqrt{169} = 13.$

3. **(4)**

Sol: Length of the diagonal $= \sqrt{(4-1)^2 + (5-2)^2 + (8-5)^2} = \sqrt{9+9+9} = 3\sqrt{3}.$

4. **(3)**

Sol: On the ZX- Plane we have y = 0

5. **(3)**

Sol: $P(0,a,3),\ Q(3,0,7), PQ = \sqrt{41} \Rightarrow \sqrt{(0-3)^2 + (a-0)^2 + (3-7)^2} = \sqrt{41}$

$\Rightarrow 9 + a^2 + 16 = 41 \Rightarrow a^2 = 16 \Rightarrow a = \pm 4$

6. **(3)**

Sol: Let $P(2,4,5), Q(3,5,-9)$

PQ divides yz- plane in the ratio $-x_1 : x_2 = -2 : 3$

7. **(3)** Mid point of AC $= \left(\frac{1+4}{2}, \frac{1+5}{2}, \frac{1+1}{2}\right) = \left(\frac{5}{2}, 3, 1\right)$;

Mid point of BD $= \left(\frac{4+1}{2}, \frac{1+5}{2}, \frac{1+1}{2}\right) = \left(\frac{5}{2}, 3, 1\right)$

Mid points coincide $\rightarrow$ Diagonal bisects $\rightarrow$ ABCD form a parallelogram.

8. **(1)** On the x-axis we have y = 0, z = 0

9. **(3)**

Sol: XZ - plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio –3 : 7

10. **(2)**

On the YZ- plane, x-coordinate is zero. Hence (0, 4, 5)

11. **(1)**

Sol: S1 : $(3, a^2 - 1, a^2 - 3a + 2)$ lies on x- axis $\Rightarrow a^2 - 1 = 0,\ a^2 - 3a + 2 = 0$

$\Rightarrow (a + 1)(a - 1) = 0,\ (a - 1)(a - 2) = 0 \Rightarrow a = 1.$

$\therefore$ S1 is true.

S2: S2 is true. Further S2 is correct explanation of S1.

11) LIMITS & DERIVATIVES

1. **(4)** $\underset{x\to1^+}{Lt}\dfrac{x^2-1}{|x-1|}=\underset{x\to1^+}{Lt}\dfrac{x^2-1}{|x-1|}=\underset{x\to1^+}{Lt}(x+1)=2\,;\ \underset{x\to1^-}{Lt}\dfrac{x^2-1}{|x-1|}=\underset{x\to1^-}{Lt}\dfrac{x^2-1}{|x-x|}=-\underset{x\to1^-}{Lt}(x+1)=-2.$

R.H.L $\neq$ L.H.L $\therefore$ Limit does not exist.

2. **(4)** By taking $x=\cos\theta$ and simplifying we get R.H.L = 1 and L.H.L does not exist. Hence, the given limit does not exist.

3. **(2)** $\underset{x\to0}{Lt}\left(\dfrac{a^x+b^x}{2}\right)^{\frac{1}{x}}=e^{\underset{x\to0}{Lt}\left(\frac{a^x+b^x}{2}-1\right)^{\frac{1}{x}}}=e^{\underset{x\to0}{Lt}\left(\frac{a^x+b^x-2}{2x}\right)}=e^{\frac{1}{2}\underset{x\to0}{Lt}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}\right)}=e^{\frac{1}{2}(\log a+\log b)}$

4. **(1)** $\underset{x\to0}{Lt}(1+\tan^2x)^{\frac{1}{2x^2}}=e^{\underset{x\to0}{Lt}(1+\tan^2x-1)\cdot\frac{1}{2x^2}}=e^{\underset{x\to0}{Lt}\frac{\tan^2x}{2x^2}}=e^{\frac{1}{2}\underset{x\to0}{Lt}\left(\frac{\tan x}{x}\right)^2}=e^{\frac{1}{2}(1)}=e^{\frac{1}{2}}$

5. **(1)** $\underset{x\to0}{Lt}\dfrac{a^x-b^x}{d^x-1}=\underset{x\to0}{Lt}\dfrac{\frac{(a^x-1)-(b^x-1)}{x}}{\left(\frac{d^x-1}{x}\right)}=\underset{x\to0}{Lt}\dfrac{\left(\frac{a^x-1}{x}\right)-\left(\frac{b^x-1}{x}\right)}{\left(\frac{d^x-1}{x}\right)}=\dfrac{\log a-\log b}{\log d}=\dfrac{\log(a/b)}{\log d}$

6. **(1)** $\underset{x\to0}{Lt}\left(\dfrac{1+\tan x}{|1+\sin x|}\right)^{\frac{1}{\sin x}}=e^{\underset{x\to0}{Lt}\left(\frac{1+\tan x}{1+\sin x}-1\right)\frac{1}{\sin x}}=e^{\underset{x\to0}{Lt}\frac{(1+\tan x-1-\sin x)}{(1+\sin x)\sin x}}=e^{\underset{x\to0}{Lt}\frac{(\tan x-\sin x)}{(1+\sin x)\sin x}}=e^0=1$

7. **(4)** $\underset{x\to0}{Lt}\dfrac{\tan x-\sin x}{x^3}=\underset{x\to0}{Lt}\dfrac{\tan x}{x}\cdot\dfrac{(1-\cos x)}{x^2}=1.\dfrac{1}{2}=\dfrac{1}{2}$

8. **(3)** **Applying L- Hospital Rule:** $\underset{x\to0}{Lt}\dfrac{\frac{1}{2\sqrt{1+2x}}(2)-\frac{1}{2\sqrt{1-2x}}(-2)}{2\cos2x}=\dfrac{1+1}{2(1)}=\dfrac{2}{2}=1$

9. **(2)** If $x\in(1,2)$ the $[x]=1$ and hence $f(x)=x+1\Rightarrow f'(x)=1\quad\therefore f'(3/2)=1$

10. **(1)** $f(x)=x\cos x\Rightarrow f'(x)=\cos x-x\sin x\Rightarrow f'(\pi/2)=0-(\pi/2)\times1=-\pi/2.$

11. **(4)** $f(x)=1+x+x^2+x^3+x^4+\ldots+x^n\Rightarrow f'(x)=1+2x+3x^2+4x^3+\ldots+nx^{n-1}$

$\therefore f'(1)=1+2+3+\ldots+n=\dfrac{n(n+1)}{2}=\dfrac{n^2+n}{2}$

12) PROBABILITY

1. **(2)** $P(A \cup B) \leq 1 \Rightarrow P(A) + P(B) - P(A \cap B) \leq 1 \Rightarrow 0.8 + 0.7 - P(A \cap B) \leq 1$

$\Rightarrow 1.5 - 1 \leq P(A \cap B) \Rightarrow P(A \cap B) \geq 0.5.$

2. **(2)** Let A be the event of containing 53 Sundays in leap year and S be the sample space.

A leap year contains 366 days i.e., 52 weeks and 2 days extra.

The extra two days may be (Sun, Mon) or (Mon, Tues) or (Tues, Wed) or (Weds, Thu) or (Thu, Fri) or (Fri, Sat) or (Sat, Sun). $\therefore$ n(S) = 7, n(A) = 2, $\therefore$ P(A) = 2/7.

3. **(4)** $A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} \Rightarrow n(A) = 6, n(S) = 36 \quad P(E) = \frac{6}{36} = \frac{1}{6}$

4. **(1)** $A = \{(1,7), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} \Rightarrow n(A) = 6, n(S) = 36 \quad P(E) = \frac{6}{36} = \frac{1}{6}$

5. **(1)** $P(A \cup B) = P(A) + P(B) \Rightarrow P(A) + P(B) - P(A \cap B) = P(A) + P(B) \Rightarrow P(A \cap B) = 0$

$\Rightarrow$ A and B are mutually exclusive.

6. **(3)** Prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17, 19 $\Rightarrow$ n(A) = 8, n(S) = 20.

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

7. **(4)** A, B are mutually exclusive $\Rightarrow A \cap B = \varnothing$

$P(A' \cap B') = P(A \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) = 1 - 0.5 - 0.03 = 0.2$

8. **(3)** Number of Red card = 26 $\Rightarrow$ n(A) = 26, n(S) = 52 $\quad \therefore P(A) = \frac{26}{52} = \frac{1}{2}$

Probability of not getting red card $= P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}.$

9. **(2)** Actually $\mathbf{0 \leq P(E) \leq 1}$

10. **(4)** $P(E_1 \cup E_2) = P(E_1) + P(E_2)$