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## PROBABILITY

1 OMQ + 1 SAQ + 1 LAQ [1 M + 4M + 8 M = 13 M]

## CONCEPTS &amp; FORMULAS

- Random Experiment:** An experiment is said to be a random experiment if
  - The outcome of the experiment is never predicted in advance.
  - It had more than one out come.
  - The experiment can be conducted any number of times, under similar conditions.

**Ex:** Tossing a coin, rolling two dice at once, drawing a card from a pack of cards, selecting a number from a set of given numbers. etc.,

**Non- Ex:** Deterministic experiments like Hitting a ball with a bat, measuring the temperature of a body, the experiments that are conducted in physics labs or chemistry labs etc.,
- Sample space (S):** It is the set of all possible outcomes of a random experiment.
 

**Ex 1:** In the random experiment of tossing a coin once, sample space  $S = \{H, T\}$

**Ex 2:** In the random experiment of throwing a die once, sample space  $S = \{1, 2, 3, 4, 5, 6\}$
- Event (E):** Any subset of the sample space is called an event.
 

**Ex:** When a die is rolled once, the event of getting an odd number is  $E = \{1, 3, 5\}$
- Elementary Event:** Any outcome of a trial of a random experiment is called an elementary event. The total number of elementary events in S is denoted by  $n(S)$ .
 

**Ex 1:** If a coin is tossed once then the elementary events are Head (H), Tail (T)  
Hence  $S = \{H, T\}$  and  $n(S) = 2$

**Ex 2:** If two coins are tossed at a time then  $S = \{HH, HT, TH, TT\}$  and  $n(S) = 2^2 = 4$

**Ex 3:** If an ordinary unbiased fair die is thrown once then  $S = \{1, 2, 3, 4, 5, 6\}$  and  $n(S) = 6$
- Sure Event:** The event which is sure to happen is called a 'Sure event' or a 'certain event'.
 

**Ex:** Any one of the numbers 1 or 2 or 3 or 4 or 5 or 6 turning up when a die is rolled once.
- Impossible Event:** The event which is impossible to happen is called an Impossible event.
 

**Ex:** The number 7 turning up when an ordinary die is thrown, is an impossible event.
- Complementary events (E')**: If E is the event of happening of an event then the event of non-happening of the event E is called its complementary event and it is denoted by  $E'$  or  $\bar{E}$  or  $E^c$ 
  - Complementary event of E is 'not E'. Also  $E' = S - E$  or  $E \cup E' = S$

**Ex :** Let E be the event of getting an odd number when a die is thrown. Thus  $E = \{1, 3, 5\}$   
Then its complementary event  $E'$  is the event of 'not getting an odd number'.  
Thus,  $E'$  is the event of getting an even number and  $E' = \{2, 4, 6\}$ . Here  $E \cup E' = S$
- Equally likely events:** Events of a Random experiment are said to be equally likely events if all of them have equal chance of happening.
 

**Note:** The elementary events of a random experiment are always equally likely.

**Ex :** In tossing a coin, turning up Head(H) or Tail(T) are equally likely events.

**Non-Ex:** Passing or failing in an examination may not be equally likely.

## TIT BITS

- Event A or B = Set  $A \cup B$
- Event A and not B = Set  $(A - B)$
- A, B are **Mutually Exclusive** events  $\Leftrightarrow A \cap B = \Phi$
- Event A and B = Set  $A \cap B$
- $P(\text{not } A) = P(A') = 1 - P(A)$

9. **Favourable Events:** The outcomes that are **concerned with the happening** of a "given event E" are called Favourable events. The number of favourable events to E is denoted by  $n(E)$ .

**Ex :** If two coins are tossed at a time then  $S = \{HH, HT, TH, TT\}$  and  $n(S) = 2^2 = 4$   
Here favourable event to get atleast a head is  $E = \{HH, HT, TH\}$  and  $n(E) = 3$ .

10. **Mutually Exclusive events:** Events are said to be mutually exclusive (disjoint) if, happening of any one of them prevents the happening of all other events.

No two of them occur at once. If A and B are mutually exclusive then  $A \cap B = \Phi$

**Ex :** In tossing a coin once, the events of getting H or T are mutually exclusive events.

**Non-Ex:** When a die is rolled, the sample space is  $S = \{1, 2, 3, 4, 5, 6\}$

Now consider the events of

(i) Getting an odd number  $A = \{1, 3, 5\}$       (ii) Getting an Even number  $B = \{2, 4, 6\}$

(iii) Getting a prime number  $C = \{1, 3, 5\}$ . Here A, B are mutually exclusive as  $A \cap B = \Phi$

But A, C are not mutually exclusive as  $A \cap C = \{3, 5\} \neq \Phi$

11. **Exhaustive Events:** A set of events is said to be exhaustive if their union results S.

**Note:** All the elementary events of a random experiment are always exhaustive.

**Ex 1:** In tossing a coin, turning up Head (H) or Tail (T) are exhaustive events.

**Ex 2:** When a die is rolled, the sample space  $S = \{1, 2, 3, 4, 5, 6\}$ .

Let  $A = \{1, 3, 5\}$  and  $B = \{2, 4, 6\}$ .  $A \cup B = \{1, 3, 5\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6\} = S$ .  $\therefore$  A, B are exhaustive

12. **Probability of an event:** For a finite sample space (S) with equally likely outcomes probability of an event A is  $P(A) = \frac{n(A)}{n(S)}$ , where  $n(A)$  = number of elements in the set A.

13. **Axioms of axiomatic definition of probability:**

If S is the sample space with outcomes  $\omega_1, \omega_2, \dots, \omega_n$  then

(i)  $P(\omega_i) \geq 0$  and  $P(\omega_i) \leq 1$  (or)  $0 \leq P(\omega_i) \leq 1$  for each  $\omega_i \in S$  (Axiom of non-negativity)

(ii)  $P(\omega_1) + P(\omega_2) + \dots + P(\omega_n) = 1$  (or)  $\sum_1^n P(\omega_i) = 1$  (or)  $P(S) = 1$  (Axiom of certainty)

(iii)  $P(A) = \sum P(\omega_i)$ ,  $\omega_i \in A$  (Axiom of union)

14. (i)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  where A, B are any 2 events of sample space S.

(ii)  $P(A \cup B) = P(A) + P(B)$  where A, B are mutually exclusive events.

