13 OSCILLATIONS

1 VSAQ + 1 LAQ [2M + 8M = 10M]

CONCEPTS & DEFINITIONS

1.0. Periodic Motion: A motion that repeats itself at regular intervals of time is called periodic motion.'Motion of Planets around the Sun' are 'Periodic or Harmonic '.

1.1. Oscillations: If a body is given a small displacement from the equilibrium position, a force comes into play which tries to bring the body back to the equilibrium point and it leads to oscillations or vibrations.

If frequency is very small, we call it an oscillation. **Ex:** Oscillation of a branch of tree.

If the frequency is high, we call it a vibration. **Ex:** vibration of a string of a musical instrument

The 'To & Fro' Vibrations of the prongs of a tuning tork are vibratory.

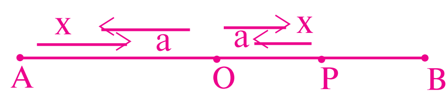
The'Up & Down'motion of the needle of a Sewing Machine is vibratory.

2.0 A kind of motion which is involved with a combination of 'to & fro Oscillatory motion' and 'Periodic motion' is called Simple Harmonic Motion.

In this chapter we study S.H.M w.r. to (i) Reference circle (ii)Simple pendulum (iii)Loaded Spring .

2.1 Simple Harmonic Motion : The 'to and fro motion' of a particle along a straight line, about a fixed point is said to be "Simple Harmonic", when the **direction of its acceleration** is always towards that **fixed point** and the **magnitude of the acceleration** is proportional to its **displacement** from that fixed point.

Thus, acceleration $\propto$ - displacement



$\Rightarrow$ $a \propto -x$

$\Rightarrow$ $a = -kx$, Here k is the Proportionality constant.

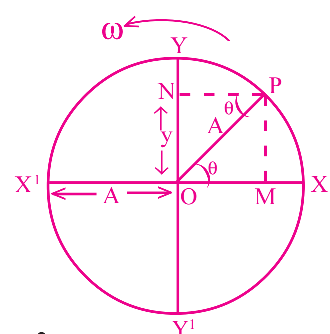
The negative sign indicates that acceleration and displacement are opposite in direction.

Ex : Motion of a loaded spring which is stretched and released, vibrations of the prongs of a tuning fork,The oscillations of a simple pendulum with small displacement

2.2 Simple harmonic motion w.r.t a Reference Circle :

Consider a particle P moving on the circumference of a circle of radius A with angular velocity ω in anticlockwise direction.

M and N are the projections of P on XX^1 and YY^1 resp.



As the particle moves along the circumference of the circle, its projections move along the diameters.

When the particle completes one revolution, the projections M and N make one oscillation on the diameters.

Displacement of the projection on y-axis is $y = A \sin\omega t$

Centripetal acceleration $(a_r) = A\omega^2$.

Acceleration of the projection on YY^1, $a = a_r \sin\theta = A\omega^2 \sin\theta = A\omega^2 \sin\omega t$

$= \omega^2 (A\sin\omega t) = -\omega^2 y \;\therefore\; a \propto -y$

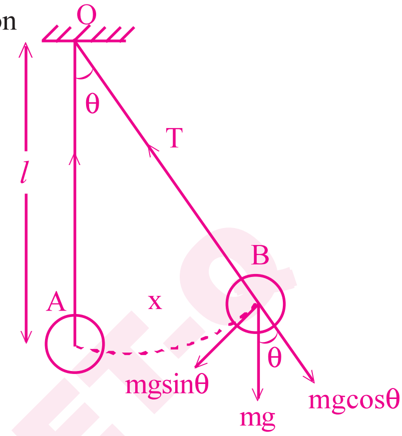
The'–' sign indicates that acceleration and displacement are in opposite directions.

Therefore, the motion of the projection on the diameter is Simple Harmonic.

3.0 Simple pendulum :
A metallic bob suspended by a light, torsionless, inextensible string is called simple pendulum.

3.1 Length of the pendulum :
The distance between the point of suspension and centre of mass of the bob.

3.2 Expression for the Time period of a simple pendulum:
If l is the length of simple pendulum, g is the acceleration due to gravity,



then Time period $T = 2\pi\sqrt{\frac{l}{g}}$

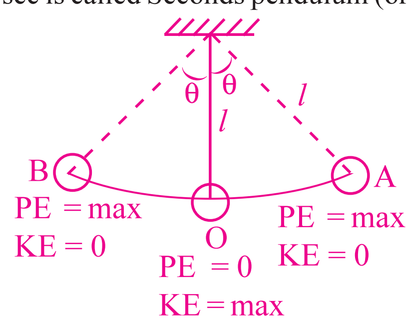
Note1: At a given place, the time period of simple pendulum is directly proportional to square root of its length. $T \propto \sqrt{l}$

Note2 : The time period of a pendulum is inversely proportional to square root of acceleration due to gravity at that place. $T \propto \frac{l}{\sqrt{g}}$

Note3 : Time period is independent of the "mass of the bob" and "amplitude"

3.3 Seconds pendulum :
A pendulum whose time period is 2 sec is called Seconds pendulum (or) Ticks pendulum (or) Beats pendulum.

Length of Seconds pendulum :



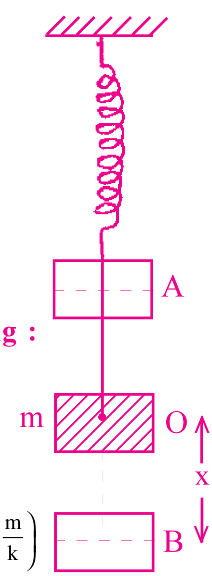
$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow 2 = 2\pi\sqrt{\frac{l}{g}} \Rightarrow 4 = 4\pi^2\frac{l}{g} \Rightarrow l = \frac{g}{\pi^2}$$

Note : On the surface of the Earth, the length of the Seconds pendulum is approximately equal to 1m.

3.4 Law of conservation of energy :
During the oscillations of a simple pendulum, the total energy remains constant.

PE + KE = constant

4.0 Definition of force constant or spring constant of a loaded spring :
Force constant is defined as restoring force per unit displacement.



Formula : $k = \frac{F}{x}$

SI unit: Nm^{-1}.

4.1 Expression for the time period of the oscillations of a loaded spring :
If m is the mass of the block , k is Force constant then $T = 2\pi\sqrt{\frac{m}{k}}$

4.2 Time period of the oscillations of the spring :

$$T = 2\pi\sqrt{\frac{\text{extension produced}}{\text{acceleration due to gravity}}} = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{m}{k}}$$

$\left(\because kx = mg \Rightarrow \frac{x}{g} = \frac{m}{k}\right)$

5. **Forced Oscillations:** When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations.

6. **Damped Oscillations:** The mechanical energy in a real oscillating system decreases during oscillations because external forces, such as drag, inhibit the oscillations and transfer mechanical energy to thermal energy. The real oscillator and its motion are then said to be damped oscillations.

7. **Resonance**: The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called resonance.

Imp.Formulae

I. With respect to a **Reference Circle :**

A particle executing simple Harmonic Motion has

(i) Displacement $y = A\sin(\omega t + \theta)$

(ii) Velocity $v = A\omega\cos\omega t$ (or) $v = \omega\sqrt{A^2 - y^2}$

(iii) Acceleration $a = -\omega^2 y$

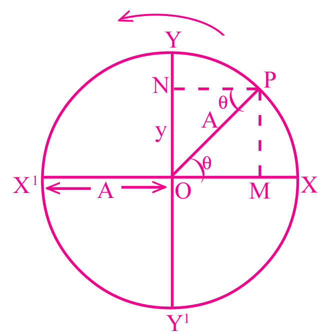
(iv) Time period $T = \frac{2\pi}{\omega}$ (or) $T = 2\pi\sqrt{\frac{y}{a}}$

(v) Frequency $n = \frac{1}{T} = \frac{\omega}{2\pi}$ (or) $n = \frac{1}{2\pi}\sqrt{\frac{a}{y}}$

(vi) KE $= \frac{1}{2}m\omega^2(A^2 - y^2)$

(vii) $P.E = \frac{1}{2}m\omega^2 y^2$

(viii) TE $= \frac{1}{2}m\omega^2 A^2$



II. With respect to a **Simple Pendulum :**

The Time period of a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

Length of Seconds pendulum $l = \frac{g}{\pi^2} \approx 1m$

Total Energy $TE = \frac{1}{2}m\omega^2 A^2$