12 LIMITS & DERIVATIVES

1 OMQ + 1 VSAQ+ 1 SAQ + 1 LAQ [1 M + 2M+ 4M +8 M = 15M]

CONCEPTS & FORMULAS

I)LIMITS

[1 OMQ + 1 VSAQ+ 1 SEMI LAQ =7 Marks]

1. **Functional value & Limit of a function:** If f(x) is a real function and a is a real number then (i) f(a) denotes the value of f(x) exactly at x=a.

 (ii) $\underset{x\to a}{\text{Lt}}\ f(x)$ denotes the value of f(x) in the neighbourhood of a.

2. **Left Hand Limit(L.H.L):** If x approaches a from the left i.e., through values just smaller than a then the limit is called the left hand limit of f(x) and it is written as $\underset{x\to a-}{\text{Lt}}\ f(x)$

3. **Right Hand Limit(R.H.L):** If x approaches a from the right i.e., through values just larger than a then the limit is called the right hand limit of f(x) and it is written as $\underset{x\to a+}{\text{Lt}}\ f(x)$

 Note: If L.H.L=R.H.L then we say the limit exists, otherwise the limit **does not exist.**

 - Limit of a function at a point is the **common value** of the left and right hand limits if they coincide.
 - f(a) and $\underset{x\to a}{\text{Lt}}\ f(x)$ may be equal or may not be equal.

 Evenif one is defined then the other one may or may not be defined.

4. **Indeterminate forms:** $\frac{0}{0}, \frac{\infty}{\infty}, 1^{\infty}, 0x\infty, \infty-\infty, 0^{\infty}, \infty^{0}, 0^{0}$

5. **Evaluation of $\underset{x\to a}{\text{Lt}}\ f(x)$:** First, check whether f(a) exists as a real number or not. If f(a) exists as a real then $\underset{x\to a}{\text{Lt}}\ f(x)=f(a)$. For if, f(a) takes the indeterminate form such as $\frac{0}{0}, \frac{\infty}{\infty}, 1^{\infty}$ etc., then reduce the given limit into standard limit forms (or) rationalise the numerator / denominator or both accordingly (or) factorise; (or) take proper substitutions and proceed accordingly.

6. **Standard Limits:**

 6.1) $\underset{x\to a}{\text{Lt}}\frac{x^n-a^n}{x-a}=na^{n-1}$ **6.2)** $\underset{x\to 0}{\text{Lt}}\frac{\sin x}{x}=1;\ \underset{x\to 0}{\text{Lt}}\frac{\tan x}{x}=1;$ ☞ $\underset{x\to 0}{\text{Lt}}\frac{\sin(kx)}{x}=k$

FEW MORE IMP. LIMITS

$\underset{x\to 0}{\text{Lt}}\frac{e^x-1}{x}=1;\quad \underset{x\to a}{\text{Lt}}\frac{a^x-1}{x}=\log_e a;\quad \underset{x\to 0}{\text{Lt}}\frac{\log(1+x)}{x}=1$

II) DERIVATIVES

[1 OMQ + 1 VSAQ+ 1 SEMI LAQ =8 Marks]

1. Derivative :

The derivative of a real function f(x) denotes the **rate of change of y** at any given value of x.

- Derivative of y=f(x) is denoted by $f'(x)$ (or) $\frac{dy}{dx}$
- Geometrically, f'(x) denotes the slope of the curve y=f(x) at any point on it.

  f '(a) denotes the slope of the curve = f(a) at exactly at x=a.
- In Physics, the derivative of displacement (s) is velocity and derivative of velocity is acceleration .

2. Derivative using the First Principle:

- The derivative of a function f at any point x is defined as $f'(x) = \underset{h \to 0}{Lt} \frac{f(x+h)-f(x)}{h}$
- The derivative of a function f at a specific value a is $f'(a) = \underset{h \to 0}{Lt} \frac{f(a+h)-f(a)}{h}$

3. Standard Formulas on derivatives: If u,v are any two functions then

(i) $(u \pm v)' = u' \pm v'$ **(Addition rule)**

(ii) $(uv)' = u'v + uv'$ **(Product rule)** • $(uvw)' = u'vw + uv'w + uvw'$

(iii) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ **(Quotient rule)** (or) $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

(iv) If $y = f(g(x))$ then $\frac{dy}{dx} = f'(g(x)).g'(x)$ **(Chain rule)**

4. Standard Derivatives:

(i) $\frac{d}{dx} x^n = nx^{n-1}$ • $\frac{d}{dx}(f(x))^n = nf(x)^{n-1} f'(x)$ **[Chain rule]**

(ii) $\frac{d}{dx}(\sin x) = \cos x$ **(iii)** $\frac{d}{dx}(\cos x) = -\sin x$

(iv) $\frac{d}{dx}(\tan x) = \sec^2 x$ **(v)** $\frac{d}{dx} \cot x = -\csc^2 x$

(vi) $\frac{d}{dx}(\sec x) = \sec x \tan x$ **(vii)** $\frac{d}{dx}(\csc x) = -\csc x \cot x$

FEW MORE IMP. DERIVATIVES

- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx} kx^2 = 2kx$
- $\frac{d}{dx} kx^3 = 3kx^2$
- $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$
- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} \log x = \frac{1}{x}$