1 UNITS & MEASUREMENTS

1 OMQ + 1 VSAQ [1 M + 2M = 3 M]

CONCEPTS & DEFINITIONS

1.0 The important contents of the chapter:
i) Fundamental and derived quantities; their units, dimensions and simple applications.
ii) Significant figures - Rules of rounding off numbers.

1.1 Any quantity that can be 'measurable' is called a **Physical quantity**.
Physical quantities are of two types.They are a) **Fundamental** quantities b)**Derived** quantities

1.2 a) **Fundamental quantities** are independent of other physical quantities.
Ex: Length, Mass, Time.

1.3 b) **Derived quantities** can be derived from other physical quantities.
Ex: Area, Volume, Velocity, Acceleration, Force.

2.0 Measurement of any physical quantity consists of a **number** and a **unit**. (**Ex:** 50 kg)
The number indicates the **size** of the quantity and the unit indicates the **standard reference** of the quantity. There are two types of Units. They are a) Fundamental units b) Derived units.

2.1 a) **Fundamental units:**The units of fundamental quantities are called fundamental units.
Ex: metre, kilogram, second.

2.2 b) **Derived units:** The units of derived quantities are called **derived units.**
Ex: m^2 for area, ms^{-1} for velocity; Newton (or) dyne for force.

3.0 **Systems of units:** To measure various fundamental quantities there are 4 types of systems. They are C.G.S. system (Metric system), M.K.S system, F.P.S system and SI.

System \ Quantity	Length	Mass	Time
C.G .S	centimetre (cm)	gram (g)	second (s)
M.K.S	metre (m)	kilogram (kg)	second (s)
F.P.S	foot (ft)	pound (l b)	second (s)

3.1 In SI, there are 7 fundamental physical quantities and 2 supplementary physical quantities.

3.1.1 Fundamental Quantities in SI

	Fundamental Quantities	Units	Symbols
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd

3.1.2 Supplementary Quantities in SI

1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

☛ π rad =180°; 2π rad=360°; $\pi/2$ rad =90°;
1 degree = 60 minutes(1°=60'); 1 minute = 60 seconds (1' =60"); 1" = 4.85×10^{-6} rad;

4.0 Dimensions : The dimensions of a physical quantity are the powers ,to which the fundamental quantities are to be raised ,to represent that physical quantity.

4.1 Dimensional Formula(D.F) : It is the expression,showing the powers to which the fundamental quantities are to be raised, to represent a physical quantity.

Ex : D.F of velocity = $[M^0L^1T^{-1}]$ (or) $M^0L^1T^{-1}$ (or) L^1T^{-1}

Here the powers 0,1,–1 of M,L,T are called the dimensions of velocity.

4.2 Dimensional equation : It is the equation, containing the symbol of a physical quantity on L.H.S and its dimensional formula on R.H.S.

Ex : Velocity (v) = $M^0L^1T^{-1}$, Acceleration (a) =$M^0L^1T^{-2}$

4.3 Principle of Homogeneity of Dimensions: The physical quantities having the same dimensions may be either added or subtracted or equated.

Ex : Consider the equation, v = u+at

The term on L.H.S is velocity.

Hence each term on R.H.S, 'u' and 'at' should possess the dimensions of velocity.

4.4 Uses of Dimensional Analysis : Dimensional analysis is used,

1) To convert one system of units into other system.
   Here, we use the relation such as $\mathbf{n_1u_1 = n_2u_2}$
2) To verify the correctness of equations related to physical quantities.
   Here, we use the **principle of Homogeneity**.
3) To derive relations between different physical quantities.
   Ex: If power (P) is assumed to depend on force (F) and velocity (V) then we write $P \propto F^aV^b$

4.5 Limitations of Dimensional Analysis :

1) The values of proportionality constants cannot be found by dimensional methods.
2) The equations containing trigonometric, exponential and logarithmic functions can't be analyzed
3) Dimensional methods are not applicable to derive an equation ,which is a sum of 'some quantities'. **Ex:** v = u + at; $s = ut + \frac{1}{2}at^2$ ....etc
4) It is difficult to apply dimensional methods on the physical quantities which are involved with more than 3 fundamental quantities .

4.6 Dimensional Constants: These are quantities having a fixed value and a dimensional formula as well.

Ex : Planck's constant, Universal gravitational constant, Universal gas constant, speed of light.

4.7 Dimensionless Quantities: There are certain physical quantities, which do not possess any dimensions.Such quantities are called dimensionless quantities. 'Ratio' of same quantities, angles,certain proportionality constants are dimensionless.

Ex : Strain, refractive index, trigonometrical ratios,specific gravity (relative density)...etc.

Different Quantities with same D.F.

ML^2T^{-2} : Work, Energy (P.E,K.E), Moment of force, Moment of couple, Torque, Heat

MLT^{-2} : Force,Weight,Tension, Normal reaction, Frictional force, Viscous force

ML^0T^{-2} : Surface tension, Force constant, Spring constant

$ML^{-1}T^{-2}$: Stress, Pressure, Modulus of elasticity

☞ MEMORY TIP: Remember L^2, L, L^0, L^{-1} in order. [M,T^{-2} are common]

4.8. Some important physical quantities with formula, unit & D.F(dimensional formula):

S.No	Physical quantity	Formula	Unit(SI)	D – derivation	D.F
1.	Length	-----	m	-----	[L]
2.	Mass	-----	kg	-----	[M]
3.	Time	-----	s	-----	[T]
4.	Area (A)	lb	m^2	[L][L]	$[L^2]$
5.	Volume (V)	lbh	m^3	[L][L][L]	$[L^3]$
6.	Density(d)	Mass/Volume	kgm^{-3}	$[M] / [L^3]$	$[ML^{-3}T^0]$
7.	Frequency (ν)	Vibrations / Time	s^{-1} or Hz	1/[T]	$[T^{-1}]$
8.	Displacement (s)	Velocity $\times$ Time	m	[L]	[L]
9.	Velocity (v)	Displacement/Time	ms^{-1}	[L]/[T]	$[M^0LT^{-1}]$
10.	Angular velocity(ω)	Angle/ Time	rad s^{-1}	$[T^{-1}]$	$[M^0L^0T^{-1}]$
11.	Acceleration (a)	Velocity/Time	ms^{-2}	$[LT^{-1}] / [T]$	$[M^0LT^{-2}]$
12.	Linear momentum(P)	Mass $\times$ Velocity	kg ms^{-1}	$[M][LT^{-1}]$	$[MLT^{-1}]$
13.	Angular momentum(L)	Momentum $\times$ Arm	kg m^2s^{-1}	$[MLT^{-1}]$ [L]	$[ML^2T^{-1}]$
14.	Force (F)	Mass $\times$ Acceleration	N	$[M][LT^{-2}]$	$[MLT^{-2}]$
15.	Impulse (J)	Force $\times$ Time	Ns	$[MLT^{-2}][T]$	$[MLT^{-1}]$
16.	Torque(τ)	Force $\times$ Arm	kgm^2s^{-2}	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
17.	Pressure(P)	Force/Area	Nm^{-2} or Pa	$[MLT^{-2}][L^{-2}]$	$[ML^{-1}T^{-2}]$
18.	Work(W)	Force $\times$ Distance	J	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
19.	Power(P)	Work/Time	W or Js^{-1}	$[ML^2T^{-2}] / [T]$	$[ML^2T^{-3}]$
20.	Kinetic Energy(K.E)	$(1/2)\,mv^2$	J	$[M][LT^{-1}]^2$	$[ML^2T^{-2}]$
21.	Potential Energy(P.E)	mgh	J	$[M][LT^{-2}][L]$	$[ML^2T^{-2}]$
22.	Stress(s)	Force/Area	Nm^{-2} or Pa	$[MLT^{-2}] / [L^2]$	$[ML^{-1}T^{-2}]$
23.	Strain	$\frac{\text{Change in Length}}{\text{Original Length}}$	No units	-----	No dimensions
24.	Modulus of Elasticity	Stress/Strain	Nm^{-2} or Pa	$[ML^{-1}T^{-2}]$	$[ML^{-1}T^{-2}]$
25.	Gravitational constant (G)	$\frac{(\text{Force})\times(\text{Length})^2}{(\text{Mass})^2}$	N m^2 kg^{-2}	$\frac{[MLT^{-2}]\times[L]^2}{[M]^2}$	$[M^{-1}L^3T^{-2}]$
26.	Planck's constant(h)	Energy/Frequency	Js	$[ML^2T^{-2}] / [T^{-1}]$	$[ML^2T^{-1}]$
27.	Surface Tension(T)	Force/Length	Nm^{-1}	$[MLT^{-2}] / [L]$	$[ML^0T^{-2}]$
28.	Specific heat(s)	$\frac{\text{Energy}}{\text{Mass}\times\text{Temp.}}$	J $kg^{-1}K^{-1}$	$\frac{[ML^2T^{-2}]}{[M][K]}$	$[L^2T^{-2}K^{-1}]$
29.	Specific gravity	$\frac{\text{Density of substance}}{\text{Density of water}}$	No units	------	No dimensions
30.	Plane Angle(θ) (Angular displacement)	$\frac{\text{Length of arc}(l)}{\text{Radius(r)}}$	rad	------	No dimensions

5.0 'Better Measurement' & 'Reporting' the values of certain physical quantities is not a simple task.

To do so, we have to consider the following 4 factors.

(i) Accuracy & Precision in the measured values.

(ii) Certain 'uncertainties' in the measurements, called 'errors'.

(iii) 'Number of significant figures' in the 'reported values'.

(iv) 'Rounding off' the measured value to a 'desired degree of accuracy'.

5.1 'Significant figures' (sig.fig.):

Significant figures are the 'number of significant digits' in a value that are known to be reliable and certain. They express the precision of a measurement.

The process of finding the number of significant figures in a given number involves inclusion or exclusion of certain number of digits according to certain given rules.

5.2 Rules for determining the 'number of significant figures':

a) All non-zero digits are significant, irrespective of the location of the decimal point

Ex 1: 235 has 3 sig.fig ; 75.18 has 4 sig. fig.

Ex 2: The number of sig. fig. in **18452** ,1845.2 ,184.52,18.452, 1.8452 is 5.

b) Zeros between two non-zero digits are also significant.

Ex 1: 102 has 3 sig.fig ; 2009 has 4 sig.fig.

Ex 2: The number of sig. fig. in **100001**,10.2003,106.008, 1.06008, 106078 is 6

c) The **zero before the decimal point** is **not significant.**

Ex: The number of sig. fig. in 0.**143** is 3.

d) Leading Zeros (zeros before the first non-zero digit) are **not significant.**

The zeros to the right of the decimal point but to the left of the first non-zero digit are 'not significant'.

Ex 1: The number of significant figures in 0.07 is 1.

Ex 2: 0.00**25** has only 2 sig.fig but not 4

Ex 3: 0.00**2308** has only 4 sig.fig but not 6.

e) Trailing zeros (zeros put at the end of a number) are **significant**, if there is a decimal point. After the decimal point ,all zeros to the right of the last non - zero digit, are significant .

Ex 1: 200. has 3 sig. fig. (this decimal point makes the zeros significant)

Ex 2: The number of sig. fig. in 0.**200** is 3

Ex 3: 12.00 has 4 sig. fig. (zeros after the decimal point are significant)

Ex 4: The number of sig. fig. in 0.0**56000** is 5.

Ex 5: The number of sig. fig. in **5.00**, **5.10**, **50.0** is 3.

f) The **trailing zeros** are **not significant,** if there is **no decimal point.**

Ex 1: 200 has 1 sig. fig. (the zeros are not significant as there is no decimal)

Ex 2: 3000 has 1 sig. fig

g) All the digits in the digital term N of **scientific notation** ($N\times10^n$) are **significant**.

Ex : 5.02×10^2 has 3 sig.fig.

h) Exact measurements have **infinite number** of **significant figures**.

Ex 1: '10 bananas' in a basket has infinite number of significant figures.

Ex 2: 46 students in a class has infinite number of significant figures.

Ex 3: Speed of light in vacuum = 299,792,458 m/s. (exact) has infinite number of sig.fig.

Ex 4: $\pi = 3.1415926....$ has infinite number of significant figures.

6.1 Rounding numbers involves approximating a given value to a specified number of significant figures or decimal places.

Ex: The velocity of light 2.99792458×10^8 ms^{-1} is rounded off to 3×10^8 ms^{-1}, for convenient calculations.

6.2 The 3 Rules of Rounding numbers:

Rule1: The ' more than 5 rule'

If the last digit to be dropped, is more than 5 then raise the preceding digit by 1.

Ex 1: To round off 1.249 to 3 sig.fig. gives 1.25

Ex 2: Rounding off 1.3161 to 3 sig.fig. gives 1.32

Ex 3: If 4728 is rounded off to three significant figures , it becomes 4730

Ex 4: Round value of 3.7 is 4

Ex 5: Round value of 5.67 is 5.7

Ex 6: Round value of 0.249 is 0.25

Ex 7: Round value of 4562 is 4600.

Rule 2: The ' less than 5 rule'

If the last digit to be dropped is less than 5 then simply drop it and its successors.

Ex 1: To round off 5.234 by removing 4 , we have to round it to 5.23

Ex 2: Rounding off 1.2143 to two decimal places gives 1.21.

Ex 3: Round value of 78123 is 78000

Rule 3: The ' exactly 5 rule'. Here two sub cases arise.

a) If the preceding digit before 5 is even then it is to be unchanged and 5 is dropped.

Ex : Rounding off 4.7258 to two decimal places gives 4.72

b) If the preceding digit before 5 is odd then raise it by '1'.

Ex : Rounding off 4.7158 to two decimal places gives 4.72

6.3 Arithmetic operations (A, S, M, D) with significant figures :

Procedure: When certain measurements are given to be operated arithmetically, the final result after the arithmetic operations should have the same number of decimal places as the **measurement** with the **lowest significant figures.**

i) Addition(A) and Subtraction(S) of Significant Figures:

Ex: Let us find the value of 12.11 + 18.0 + 1.012

The actual sum of the given numbers is 31.122. The rounded answer is 31.1

Reason: Among the given three values 18.0 has the **least number of significant figures**.

It has only one digit after the decimal point.

Hence the result should be reported only upto one digit after the decimal point.

Thus, the final answer is 31.1.

ii) Multiplication(M) and Division(D) of Significant Figures :

Ex : Let us find the value of 2.5×1.25

The actual product of the given numbers is 3.125. The rounded answer is 3.1

Reason: Among the given two values, 2.5 has the **least number of significant figures**.

It has two significant figures.

Hence, the result should have only two significant figures.

Thus,the final answer is 3.1