

## 1

## SETS

1 OMQ + 1 VSAQ + 1 SAQ [1 M + 2M + 4M = 7 M]

## CONCEPTS &amp; FORMULAS

**1.1 Set:** A set is a 'well defined collection' of **elements** / objects / ideas.

In a set, (i) **order** of elements (ii) **repetition** of elements are ignored.

**Ex 1:**  $A = \{1, 2, 3\}$

**Ex 2:** The rivers of India

**Non-Ex :** The collection of ten most talented writers of India.

- If  $x$  is an element of a set  $A$ , then we write,  $x \in A$ ; if  $x$  is not a member of  $A$ , then we write  $x \notin A$ .

**1.2.0** Here we have a **Big list of few sets**. Using this we can understand the concepts of sets easily.

**1.2.1 Sets in Roster form:**

$A = \{1, 2, 3\}$ ,  $B = \{2, 3, 5, 7\}$ ,  $C = \{4, 6, 8, 9, 10\}$ ,

$D = \{\text{Sun, Mon, Tue, Wed, Thu, Fri, Sat}\}$ ,  $E = \{\}$ ,  $F = \{1, 5\}$ ,  $M = \{5, 10, 15, \dots\}$ ,

$N = \{1, 2, 3, 4, 5, \dots, \infty\}$ ,  $W = \{0, 1, 2, 3, \dots, \infty\}$ ,  $O = \{0\}$ ,  $P = \{3, 2, 1\}$ ,  $S = \{1\}$ ,  $V = \{a, e, i, o, u\}$ .

**Non-Ex:**  $\{a, 1, \text{Friend}\}$ ,  $\{\text{best cricket player, Jan 1, Top Hero, India}\}$

**1.2.2 Sets in Set builder form:** The set builder forms of the sets given in (1.2.0) are as follows:

$A = \{x/x \text{ is a natural number less than } 4\}$ ,  $B = \{x: x \text{ is a prime number below } 8\}$ ,

$C = \{x: x \text{ is a composite number less than } 12\}$ ,  $D = \{x/x \text{ is a day of week}\}$ ,  $E = \{x/x \neq x\}$ ,

$F = \{x/x \text{ is a factor of } 5\}$ ,  $M = \{x/x \text{ is a multiple of } 5\}$  **[Upto Exercise 1(a)]**

**1.3 Empty Set( $\emptyset$ ):** A set which does not contain any element is called the empty set or null set.

**Ex:**  $A = \{x : x \text{ is an even prime number greater than } 2\}$

**1.4.1 Finite Sets:** Sets with **definite(countable)** number of elements are finite sets.

In (1.2.0), the finite sets are  $A, B, C, D, E, F, O, P, S, V$ ;

- The empty set  $\emptyset$  is finite.
- **Singleton set:** It is a finite set with a single element.

**Ex:** In (1.2.0), the singleton sets are  $O, S$ .

**1.4.2 Infinite Sets:** Sets with **infinite (uncountable)** number of elements are infinite sets.

In (1.2.0), the infinite sets are  $M, N, W$ .

**1.5 Equal Sets:** Sets with **exactly the same elements** are equal sets.

In (1.2.0),  $A = P$ ;  $A \neq B \neq C$  [Order of elements may differ in equal sets.]

**Cardinality of a set  $n(A)$ :** The **number of elements** in a set is called its cardinality.

In (1.2.0) we have,  $n(A) = 3$ ,  $n(B) = 4$ ,  $n(C) = 5$ ,  $n(D) = 7$ ,  $n(\emptyset) = 0$ ,  $n(F) = 2$ ,  $n(O) = 1$ ,  $n(S) = 1$

**[Upto Exercise 1(b)]**

**1.6. Subset ( $A \subset B$ ):** If all the elements of a set A are also present in set B, then A is a subset of B.

In (1.2.0),  $A \subset N, N \subset W, S \subset A, A \subset P$ ; ;  $A \not\subset B, B \not\subset C$

- $\emptyset$  is subset to every set.
- Every set is subset to itself.

**Power set  $P(A)$ :** The set of **all subsets of A** is called the power set of A .

- If  $n(A) = m$  then the number of elements in its power sets is  $n(P(A)) = 2^m$ .
- In (1.2.0),  $F = \{1, 5\} \Rightarrow P(F) = \{\{1\}, \{5\}, \{1,5\}, \emptyset\}$ . Here,  $n(F) = 2 \quad \therefore n(P(F)) = 2^2 = 4$

**1.6.1 Subsets of real numbers:**  $N \subset W \subset Z \subset Q \subset R$ ;  $Q' \subset R$

**1.6.2 Intervals as subsets of R:**

- Closed interval  $[a, b] = \{x : a \leq x \leq b\}$ . Here, both the end numbers a,b are included.
- Open interval  $(a, b) = \{x : a < x < b\}$ . Here, both the end numbers a,b are not included.
- Left- closed, right-open interval  $[a, b) = \{x : a \leq x < b\}$ . Here, a is included and b is excluded.
- Left- open, right- closed interval  $(a, b] = \{x : a < x \leq b\}$ . Here, a is excluded and b is included.

**1.7 Universal set (U):** It is the **super set** containing all the related sets.

Ex : Set of reals R is a universal set to N, W, Z, Q, Q'. [Upto Exercise 1(c)]

**1.8 Venn Diagrams** depict various relationships between sets diagrammatically.

Here the Universal set is denoted by a rectangle and sets are denoted by circles.

**1.9 Operations on Sets:**

**1.9.1 Union of Sets ( $A \cup B$ ):** It is a set with **all the elements** from A,B.

In (1.2.0),  $A \cup B = \{1, 2, 3, 5, 7\}$ ;  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- Repetition of elements should be dropped in  $A \cup B$
- $A \cup B = \{x/ x \in A \text{ or } x \in B\}$



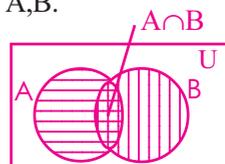
**1.9.2 Intersection of Sets ( $A \cap B$ ):** It is a set with the **common elements** of A,B.

In (1.2.0),  $A \cap B = \{2, 3\}$ ,  $B \cap M = \{5\}$ ,  $B \cap C = \emptyset$

- $A \cap B = \{x/ x \in A \text{ and } x \in B\}$
- If  $A \cap B = \emptyset$  then A, B are disjoint.

**Disjoint Sets:** Sets with **no common elements** are disjoint sets.

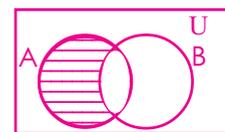
In (1.2.0), the disjoint sets: A&D; A&E; A&M; D&N; C&F.....



**1.9.3 Difference of Sets ( $A - B$ ):** Elements in the first set A but not in the second set B.

In (1.2.0),  $A - B = \{1\}$ ;  $B - A = \{5, 7\}$ ;  $N - W = \emptyset$ ;  $W - N = \{0\}$

- $A - B = \{x/ x \in A \text{ and } x \notin B\}$ ; •  $A - B = A - (A \cap B)$  (**only A** but not B)



**Laws on Sets:**

i) **Idempotent Laws:**  $A \cup A = A, A \cap A = A$

ii) **Associative Laws:**  $(A \cup B) \cup C = A \cup (B \cup C)$ ;  $(A \cap B) \cap C = A \cap (B \cap C)$

iii) **Commutative Laws:**  $A \cup B = B \cup A$ ;  $A \cap B = B \cap A$

iv) **Distributive Laws:**  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ ;  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

v) **Identity laws:**  $A \cup \emptyset = A, A \cup U = U$ ;  $A \cap U = A, A \cap \emptyset = \emptyset$

vi) **Laws of absorption:**  $A \cup (A \cap B) = A$ ;  $A \cap (A \cup B) = A$  [Upto Exercise 1(d)]

**1.9 Complement of a set ( $A'$ ):** It is the set with all elements in the universal set but **not A**.

Ex: In Real numbers (R), the complementary set of all rationals (Q) is the set of all Irrationals (Q')

i) **Complement Laws:**  $A \cup A' = U, A \cap A' = \emptyset$ ;  $(A')' = A, U' = \emptyset, \emptyset' = U$

ii) **De-Morgan's Laws:**  $(A \cup B)' = A' \cap B'$ ;  $(A \cap B)' = A' \cup B'$  [Upto Exercise 1(e&f)]