Previous IPE

SOLVED PAPERS

MARCH -2025(TS)

PREVIOUS PAPERS

IPE: MARCH-2025(TS)

Time : 3 Hours

MATHS-2B

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 41 = 0$
2. Locate the position of the point P (4, 2) w.r.to the circle $2x^2 + 2y^2 - 5x - 4y - 3 = 0$.
3. Find the equation of the common chord of the circles $x^2+y^2+2x+3y+1=0$, $x^2+y^2+4x+3y+2=0$
4. Find the coordinates of the point on the parabola $y^2 = 2x$ whose focal distance is 5/2
5. Find the foci, eccentricity of the Hyperbola $16y^2 - 9x^2 = 144$.
6. Find $\int \sqrt[3]{2x^2}\, dx$ on $x \in (0, \infty)$ dx
7. Find $\int 2x\, e^{x^2} dx$ on $x \in R$
8. Evaluate $\int_0^4 |2 - x|\, dx$
9. Evaluate $\int_0^{\pi/2} \sin^7 x\, dx$
10. Find the order of the differential equation of the family of all circles with their centres at the origin.

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. Find the equation of tangent and normal at (3, 2) on the circle $x^2 + y^2 - x - 3y - 4 = 0$.
12. Find the equation of the circle which passes through the origin and intersects the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 - 8y + 12 = 0$ orthogonally.
13. If the length of the major axis of an ellipse is 3 times the length of its minor axis, then find the eccentricity of the ellipse.
14. Find the value of k if $4x + y + k = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 3$.
15. Find the equation to the hyperbola whose foci are (4, 2) and (8, 2) and eccentricity 2.
16. Find the area enclosed between the parabola $y = x^2$ and the line $y = 2x$.
17. Solve $(xy^2 + x)dx + (yx^2 + y)\, dy = 0$

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. Find the equation of the circle whose centre lies on the X-axis and passing through (–2, 3) and (4, 5).

19. Find the transverse common tangents to the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$

20. Find the coordinates of vertex, focus, equation of the directrix and axis for the parabola $y^2 + 4x + 4y - 3 = 0$

21. Evaluate $\int \frac{1}{1 + \sin x + \cos x} dx$

22. Obtain reduction formula for $I_n = \int \tan^n x \, dx$, for an integer $n \geq 2$ and deduce the value of $\int \tan^6 x \, dx$.

23. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

24. Solve the differential equation $\frac{dy}{dx} + y \tan x = \sin x$

IPE TS MARCH-2025
SOLUTIONS

SECTION-A

1. Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 41 = 0$.

A: Given circle is $x^2 + y^2 - 4x - 8y - 41 = 0$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$,

We get $2g = -4$; $2f = -8$; $c = -41 \Rightarrow g = -2, f = -4, c = -41$

Centre $C = (-g, -f) = (-(-2), -(-4)) = (2, 4)$

Radius $r = \sqrt{g^2 + f^2 - c} = \sqrt{2^2 + (4)^2 - (-41)} = \sqrt{4 + 16 + 41} = \sqrt{61}$

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2. Locate the position of the point P (4, 2) w.r.to the circle $2x^2 + 2y^2 - 5x - 4y - 3 = 0$.

Sol: Given point $P(x_1, y_1) = (4, 2)$ and circle $S \equiv 2x^2 + 2y^2 - 5x - 4y - 3 = 0$

Now, $S_{11} = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 2 \times 4^2 + 2 \times 2^2 - 5(4) - 4(2) - 3$

$= 32 + 8 - 20 - 8 - 3 = 9 > 0$

$\therefore$ P(4, 2) lies outside the circle

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3. Find the equation and length of the common chord of the circles $x^2+y^2+2x+2y+1 = 0$, $x^2+y^2+4x+3y+2 = 0$

Sol: Given circles are $S \equiv x^2 + y^2 + 2x + 2y + 1 = 0$ and

$S' \equiv x^2 + y^2 + 4x + 3y + 2 = 0$

Equation of the common chord is $S - S' = 0 \Rightarrow -2x - y - 1 = 0 \Rightarrow 2x + y + 1 = 0$

For the circle $S \equiv x^2 + y^2 + 2x + 2y + 1 = 0$

centre $C(-1, -1)$

radius $r = \sqrt{1^2 + 1^2 - 1} = \sqrt{1 + 1 - 1} = \sqrt{1} = 1$

Length of the perpendicular from $C(-1,-1)$ to the line $2x + y + 1 = 0$ is

$$p = \frac{|2(-1) - 1 + 1|}{\sqrt{4+1}} = \frac{|-2|}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$\therefore$ Length of the common chord

$$= 2\sqrt{r^2 - p^2} = 2\sqrt{1 - \frac{4}{5}} = 2\sqrt{\frac{1}{5}} = \frac{2}{\sqrt{5}} \text{ units}$$

4. Find the co-ordinates of the points on the parabola $y^2 = 2x$ whose focal distance is 5/2.

Sol: Given Parabola is $y^2 = 2x \Rightarrow 4a = 2$

$\Rightarrow a = 2/4 = 1/2$

Given focal distance SP = 5/2

Formula: Focal distance SP = $x_1 + a$

$$\Rightarrow x_1 + \frac{1}{2} = \frac{5}{2} \Rightarrow x_1 = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$$

But, $y_1^2 = 2x_1 \Rightarrow y_1^2 = 2(2) = 4 \Rightarrow y_1 = \pm 2$

$\therefore$ $P(x_1, y_1) = (2, \pm 2)$

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5. Find the foci, eccentricity of the Hyperbola $16y^2 - 9x^2 = 144$.

Sol: Equation of hyperbola is $16y^2 - 9x^2 = 144$

$$\Rightarrow \frac{16y^2}{144} - \frac{9x^2}{144} = 1 \Rightarrow \frac{y^2}{9} - \frac{x^2}{16} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = -1$$

This is in standard form II. Here, $a^2 = 16$, $b^2 = 9$

(i) Foci = $(0, \pm be) = \left(0, \pm 3\left(\frac{5}{3}\right)\right) = (0, \pm 5)$

(ii) Eccentricity $e = \frac{\sqrt{a^2+b^2}}{b} = \frac{\sqrt{16+9}}{3} = \frac{\sqrt{25}}{3} = \frac{5}{3}$

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6. Find $\int \sqrt[3]{2x^2}\, dx$ on $x \in (0, \infty)$ dx

Sol: $I = \int \sqrt[3]{2x^2}\, dx$

$$= \sqrt[3]{2} \int x^{2/3} dx = \sqrt[3]{2} \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c = \sqrt[3]{2} \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c = \sqrt[3]{2} . \frac{3}{5} x^{\frac{5}{3}} + c$$

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7. Find $\int 2x e^{x^2} dx$ on $x \in R$

Sol: Put $x^2 = t \Rightarrow 2x\, dx = dt$ $\therefore I = \int 2x e^{x^2} dx = \int e^t dt = \int e^t dt = e^t + c = e^{x^2} + c$

8. Evaluate $\int_0^4 |2-x|\,dx$

Sol: $|2-x| = 2-x$ when $2-x \geq 0 \Rightarrow x \leq 2$

$= -(2-x)$ when $2-x < 0 \Rightarrow x > 2$

$$\therefore \int_0^4 |2-x|\,dx = \int_0^2 |2-x|\,dx + \int_2^4 |2-x|\,dx$$

$$= \int_0^2 (2-x)\,dx + \int_2^4 (x-2)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$= \left(4 - \frac{4}{2}\right) + \left[(8-8)-(2-4)\right]$$

$$= 2 + 0 + 2 = 4$$

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9. Evaluate $\int_0^{\pi/2} \sin^7 x\,dx$

Sol: When n is odd,

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{(n-1)(n-3)....}{n(n-2)....}\frac{2}{3}.1$$

$$\therefore \int_0^{\pi/2} \sin^7 x\,dx = \frac{(6)(4)(2)}{(7)(5)(3)}(1) = \frac{16}{35}$$

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10. Find the order of the differential equation of the <u>family of all circles</u> with their centres at the origin.

Sol: The equation of the family of circles with centres at origin is $x^2 + y^2 = r^2$, (r)

Differentiating w.r.to x, we get $2x + 2y\frac{dy}{dx} = 0 \Rightarrow x + y\frac{dy}{dx} = 0$ $\therefore$ order is 1

SECTION-B

11. Find the equation of tangent and normal at (3, 2) on the circle $x^2 + y^2 - x - 3y - 4 = 0$.

Sol: The equation of the tangent at (3, 2) on the circle $S = x^2 + y^2 - x - 3y - 4 = 0$ is $S_1 = 0$

$\Rightarrow 3x + 2y - \frac{1}{2}(x+3) - \frac{3}{2}(y+2) - 4 = 0 \Rightarrow 6x + 4y - x - 3 - 3y - 6 - 8 = 0$

$\Rightarrow 5x + y - 17 = 0$

The slope of the above tangent is –5

$\Rightarrow$ slope of its normal is 1/5

$\therefore$ Equation of the normal at (3, 2) with slope $\frac{1}{5}$ is $y - 2 = \frac{1}{5}(x-3)$

$\Rightarrow 5y - 10 = x - 3 \Rightarrow x - 5y + 7 = 0$

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12. Find the equation of the circle which passes through the origin and intersects the circles $x^2 + y^2 - 4x - 6y - 3 = 0$, $x^2 + y^2 - 8y + 12 = 0$ orthogonally.

Sol: Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

It cuts $x^2 + y^2 - 4x - 6y - 3 = 0$; $x^2 + y^2 - 8y + 12 = 0$

$g_1 = -2, f_1 = -3, c_1 = -3 \quad g_2 = 0, f_2 = -4, c_2 = 12$

Let g, f, c be constants of required circle.

Required circle passes through origin $\therefore c = 0$

Required circle is orthogonal to both circles.

$\therefore 2g(-2) + 2f(-3) = -3 + 0$ ——— (i)

$2g(0) + 2f(-4) = 12 + 0$ ——— (ii)

Solving (i) and (ii) we get $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Condition of orthogonality $f = -\frac{3}{2}, g = \frac{6}{2}$

Required circle be $x^2 + y^2 + 6x - 3y = 0$

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13. If the length of the major axis of an ellipse is 3 times the length of its minor axis then find the eccentricity of the ellipse.

Sol: Given that, length of the major axis = 3(length of minor axis) $\Rightarrow 2a = 3(2b) \Rightarrow a = 3b \Rightarrow a^2 = 9b^2$

Now, $b^2 = a^2(1-e^2) \Rightarrow \cancel{b^2} = 9\cancel{b^2}(1-e^2)$

$\Rightarrow 1 = 9(1-e^2) \Rightarrow 1 - e^2 = \frac{1}{9} \Rightarrow e^2 = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow e = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$

14. Find the value of k if 4x + y + k =0 is a tangent to the ellipse $x^2 + 3y^2 = 3$.

Sol: Equation of the given ellipse is $x^2+3y^2=3 \Rightarrow \frac{x^2}{3}+\frac{y^2}{1}=1 .....(1) \Rightarrow a^2=3, b^2=1$

Given line is $4x + y + k = 0$

$\Rightarrow y = -4x - k$ ....(2)

Comparing with $y = mx + c$ we get $m = -4$ and $c = -k$

Now applying the tangential condition $c^2 = a^2m^2 + b^2$, we get

$k^2 = 3(-4)^2 + 1^2 = 3(16) + 1 = 48 + 1 = 49 \Rightarrow k = \pm 7$

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15. Find the equation to the hyperbola whose foci are (4, 2) and (8, 2) and eccentricity 2.

Sol: The given Foci are S (4, 2) and S' (8, 2).

Centre C is the mid point of the foci

$\therefore C = \left(\frac{4+8}{2}, \frac{2+2}{2}\right) = (6,2) = (h, k)$

The distance between C(6, 2) and S(4, 2) is $ae = \sqrt{(6-4)^2+(2-2)^2} = \sqrt{4} = 2$

Given that $e = 2$ $\quad \therefore ae = 2 \Rightarrow a(2) = 2 \Rightarrow a = 1$

Now, $b^2 = a^2 (e^2 - 1) = 1(4 - 1) = 3$

$\therefore$ Equation of the hyperbola is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \Rightarrow \frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$

$\Rightarrow 3(x-6)^2 - (y-2)^2 = 3$

$\Rightarrow 3(x^2 - 12x + 36) - (y^2 - 4y + 4) = 3$

$\Rightarrow 3x^2 - 36x + 108 - y^2 + 4y - 4 - 3 = 0$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$

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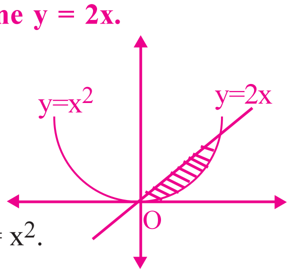
16. Find the area enclosed between the parabola $y = x^2$ and the line $y = 2x$.

Sol: The given curves are $y = x^2$.... (1); $y = 2x$ ......(2)

Solving (1), (2) we have $x^2 = 2x$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x - 2) = 0 \Rightarrow x = 0, 2$



The upper boundary curve is $y = 2x$, the lower boundary curve is $y = x^2$.

$\therefore$ The area enclosed between the curves is

$$A = \int_0^2 (2x - x^2)dx = \left[2.\frac{x^2}{2} - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3} \text{ sq.units}$$

17. Solve $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$

Sol: The given D.E is in the variables and separable form

$\therefore (xy^2 + x)dx + (yx^2 + y)dy = 0 \Rightarrow (xy^2 + x)dx = -(yx^2 + y)dy$

$\Rightarrow x(y^2 + 1)dx = -y(x^2 + 1)dy \Rightarrow \frac{x}{x^2+1}dx = -\frac{y}{y^2+1}dy$

$\Rightarrow \int \frac{2x}{x^2+1}dx = -\int \frac{2y}{y^2+1}dy \Rightarrow \log(x^2+1) = -\log(y^2+1) + \log c$

$\Rightarrow \log(x^2+1) + \log(y^2+1) = \log c \Rightarrow \log(x^2+1)(y^2+1) = \log c \Rightarrow (x^2+1)(y^2+1) = c$

$\therefore$ The solution is $(x^2+1)(y^2+1) = c$

SECTION-C

18. Find the equation of circle whose centre lies on the x–axis and passing through (–2, 3) and (4, 5).

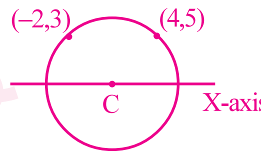
Sol: Let A = (–2, 3), B = (4, 5)

We take $S(x_1, y_1)$ as the centre of the circle

$\Rightarrow SA = SB \Rightarrow SA^2 = SB^2 \Rightarrow (x_1+2)^2 + (y_1-3)^2 = (x_1 - 4)^2 + (y_1 - 5)^2$

$\Rightarrow (\cancel{x_1^2} + 4x_1 + 4) + (\cancel{y_1^2} - 6y_1 + 9) = (\cancel{x_1^2} - 8x_1 + 16) + (\cancel{y_1^2} - 10y_1 + 25)$

$\Rightarrow 12x_1 + 4y_1 - 28 = 0 \Rightarrow 4(3x_1 + y_1 - 7) = 0 \Rightarrow 3x_1 + y_1 - 7 = 0$ ....(1)



But, centre (x_1, y_1) lies on y = 0 (X-axis) $\therefore y_1 = 0$

Now, (1) $\Rightarrow 3x_1 + 0 - 7 = 0 \Rightarrow 3x_1 = 7 \Rightarrow x_1 = 7/3$

$\therefore$ Centre of the circle $S(x_1, y_1) = (7/3, 0)$.

Also, we have A = (–2, 3) So, radius $r = SA \Rightarrow r^2 = SA^2$.

$$\therefore r^2 = \left(\frac{7}{3}+2\right)^2 + (3-0)^2 = \left(\frac{13}{3}\right)^2 + 9 = \frac{169}{9} + 9 = \frac{169+81}{9} = \frac{250}{9}$$

$\therefore$ circle with centre (7/3,0) and $r^2 = 250/9$ is

$$\left(x - \frac{7}{3}\right)^2 + (y-0)^2 = \frac{250}{9} \Rightarrow \frac{(3x-7)^2}{9} + y^2 = \frac{250}{9} \Rightarrow \frac{(3x-7)^2 + 9y^2}{\cancel{9}} = \frac{250}{\cancel{9}}$$

$\Rightarrow (3x-7)^2 + 9y^2 = 250 \Rightarrow (9x^2 - 42x + 49) + 9y^2 = 250 \Rightarrow 9x^2 + 9y^2 - 42x - 201 = 0$

$\Rightarrow 3(3x^2 + 3y^2 - 14x - 67) = 0 \Rightarrow 3x^2 + 3y^2 - 14x - 67 = 0$

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19. Find the equation to the pair of transverse common tangents to the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$

Sol: For the circle $x^2+y^2-4x-10y+28=0$, centre $C_1=(2,5)$, radius $r_1 = \sqrt{(-2)^2 + (-5)^2 - 28} = \sqrt{1} = 1$

For the circle $x^2+y^2+4x-6y+4=0$, centre $C_2=(-2,3)$, radius $r_2 = \sqrt{2^2 + (-3)^2 - 4} = \sqrt{9} = 3$

The internal centre of similitude, I divides C_1C_2 internally in the ratio $r_1 : r_2 = 1:3$

$$\therefore I = \left(\frac{1(-2)+3(2)}{1+3}, \frac{1(3)+3(5)}{1+3}\right) = \left(\frac{4}{4}, \frac{18}{4}\right) = \left(1, \frac{9}{2}\right)$$

The equation to the pair of transverse common tangents is $S_1^2 = S_{11}(S)$

$$\Rightarrow \left[x + \frac{9}{2}y - 2(x+1) - 5\left(y + \frac{9}{2}\right) + 28\right]^2 = \left(1 + \frac{81}{4} - 4 - 45 + 28\right)(x^2 + y^2 - 4x - 10y + 28)$$

$$\Rightarrow \left(-x - \frac{y}{2} + \frac{7}{2}\right)^2 = \frac{1}{4}(x^2 + y^2 - 4x - 10y + 28) \Rightarrow \frac{(2x - y + 7)^2}{4} = \frac{1}{4}(x^2 + y^2 - 4x - 10y + 28)$$

$\Rightarrow (2x + y - 7)^2 = x^2 + y^2 - 4x - 10y + 28$

$\Rightarrow 4x^2 + y^2 + 49 + 4xy - 28x - 14y = x^2 + y^2 - 4x - 10y + 28 \Rightarrow 3x^2 + 4xy - 24x - 4y + 21 = 0$

20. Find the coordinates of vertex, focus, equation of the directrix and axis for the parabola $y^2 + 4x + 4y - 3 = 0$

Sol: Given parabola is $y^2 + 4x + 4y - 3 = 0 \Rightarrow y^2 + 4y = -4x + 3$. Add 4 on both sides

$\Rightarrow y^2 + 4y + 4 = -4x + 7 \Rightarrow (y+2)^2 = -4\left(x - \frac{7}{4}\right)$ This is a horizontal left side parabola.

Comparing it with $(y-k)^2 = -4a(x-h)$, we get $4a = 4 \Rightarrow a = 1$ Also, $h = 7/4$, $k = -2$

(i) Vertex $= (h, k) = \left(\frac{7}{4}, -2\right)$

(ii) Focus $= (h - a, k) = \left(\frac{7}{4} - 1, -2\right) = \left(\frac{3}{4}, -2\right)$

(iii) Equation of the directrix is $x = h + a \Rightarrow x = \frac{7}{4} + 1 = \frac{7+4}{4} = \frac{11}{4} \Rightarrow 4x = 11 \Rightarrow 4x - 11 = 0$

(iv) Equation of the axis is $y = k \Rightarrow y = -2 \Rightarrow y + 2 = 0$

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21. Evaluate $\int \frac{1}{1 + \sin x + \cos x} dx$.

Sol: Put, $\tan\frac{x}{2} = t$ then $\sin x = \frac{2t}{1+t^2}$; $\cos x = \frac{1-t^2}{1+t^2}$ and $dx = \frac{2dt}{1+t^2}$

$$\therefore I = \int \frac{1}{1+\sin x + \cos x} dx = \int \frac{1}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \cdot \left(\frac{2dt}{1+t^2}\right) = \int \frac{1}{\frac{(1+t^2) + 2t + (1-t^2)}{\cancel{1+t^2}}} \cdot \frac{2dt}{\cancel{1+t^2}}$$

$$= \int \frac{2dt}{2+2t} = \frac{\cancel{2}}{\cancel{2}} \int \frac{dt}{1+t} = \log|1+t| + c = \log\left|1 + \tan\left(\frac{x}{2}\right)\right| + c$$

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22. Obtain reduction formula for $I_n = \int \tan^n x\, dx$, for an integer $n \geq 2$ and deduce the value of $\int \tan^6 x\, dx$.

Sol: Let $I_n = \int \tan^n x dx = \int (\tan^{n-2} x) \tan^2 x dx$

$= \int (\tan^{n-2} x)(\sec^2 x - 1) dx$

$= \int \tan^{n-2} x \sec^2 x dx - \int \tan^{n-2} x dx$

$= \frac{\tan^{n-1}}{n-1} - I_{n-2}$ .....(1) $\quad \left[\because f(x) = \tan x, f'(x) = \sec^2 x\right]$

Put, n = 6, 4, 2, 0 successively in (1), we get

$$I_6 = \int \tan^6 x dx = \frac{\tan^5 x}{5} - I_4 = \frac{\tan^5 x}{5} - \left[\frac{\tan^3 x}{3} - I_2\right]$$

$$= \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - I_0 = \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - x + c \quad [\because I_0 = x]$$

23. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

Sol: We know $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\therefore I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$$

$$= \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}dx$$

$$= \int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}dx - \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$$

$$= \int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}dx - I$$

$$\Rightarrow I + I = \int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx \Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

Put, cosx = t $\Rightarrow$ sinxdx = –dt

Also, x = 0 $\Rightarrow$ t = cos0 = 1

and x= π $\Rightarrow$ t = cosπ = –1

$$\therefore I = \frac{\pi}{2}\int_1^{-1} \frac{-dt}{1+t^2} = \frac{\pi}{2}\int_{-1}^{1} \frac{dt}{1+t^2} = \frac{\pi}{2}[\mathrm{Tan}^{-1}t]_{-1}^{1}$$

$$= \frac{\pi}{2}[\mathrm{Tan}^{-1}(1) - \mathrm{Tan}^{-1}(-1)]$$

$$= \frac{\pi}{2}\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = \frac{\pi}{2}.\frac{\pi}{2} = \frac{\pi^2}{4}$$

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24. Solve the differential equation $\frac{dy}{dx} + y\tan x = \sin x$

Sol: Given D.E is in the form $\frac{dy}{dx} + yP(x) = Q(x)$. This is a linear D.E in y.

Here, $P = \tan x \Rightarrow \int Pdx = \int \tan x dx = \log\sec x$ $\therefore I.F = e^{\int Pdx} = e^{\log\sec x} = \sec x$

Hence, the solution is $y.(I.F) = \int (I.F)Qdx$

$$\Rightarrow y.\sec x = \int \sec x.\sin x dx = \int \tan x dx = \log\sec x + c$$